Theory of Structure (1)

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By

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Chapter (1)
INTRODUCTION

1.1. Structures Definition

There are various types of structures in the world such as bridges, residential building, government building, parking building, educational building, religious building, industrial building, power stations, commercial electrical powers, and so on as shown in Fig. 1. 1. The structures constructed from many materials reinforced concrete, steel, stainless steel, aluminum and wood for example.

Fig. 1.1: Example of structures

1.2. Stages of Building the Structures

1. Architectural Layout

This stage is carried out by architecture engineers and in this stage; the structure is planed according the usage and the requirements of the building owners.

2. Structural Model and Behavior of structural Model

These two stages are called structural analysis. In the structural analysis processes stage, the loads on the structure and the structural model are indicated. Then the responses of the real structure under the excitation of expected loading and external environment during the service life of the structure is predict.

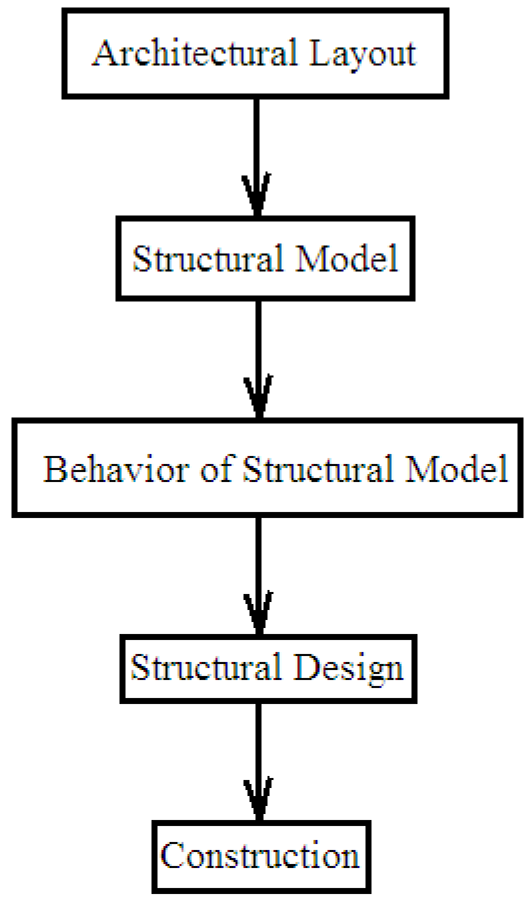


Fig. 1.2: Flow chart structural stages

3. Structural Design

In this stage, the type of used material is indicated, the amount of the used material are calculated.

4. Construction

In this stage, the building becomes real. This stage passes through several stages such as construction footing, column, beam………so on.

1.3. Structural Model

You must understand that when we deal with any structures, we must study three objects; loads, structure and the support of the structures as shown in Fig. 1.3. The three objects must be modeled to indicate the behavior of the structure.

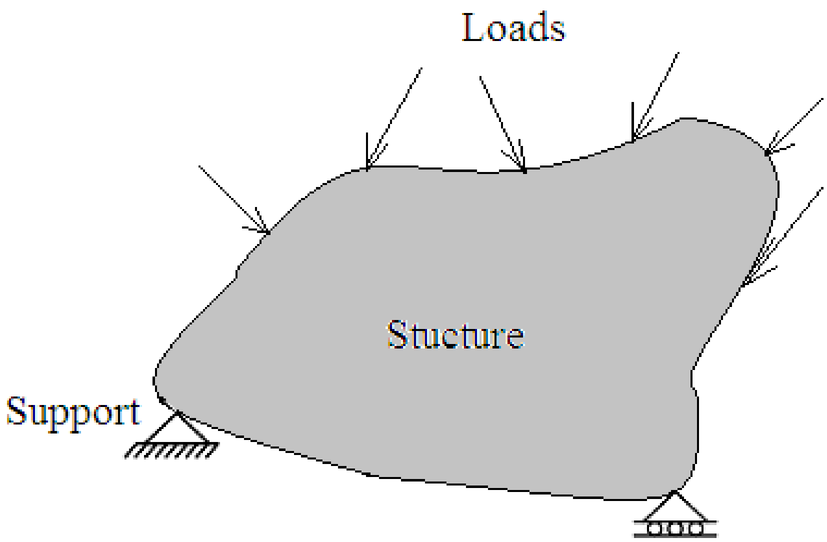


Fig. 1.3

1.4. Loads

Loads are the force that influnce on the structures. The loads on the structures are divided into:-

1. Dead loads: these loads are with a constant magnitude and remain in one position during the service life of the structures. They include the one weight of the structures and the loads that are permanently attached to the structures; floor covering.
2. Live loads: these loads very in their position and their magnitude may change. The live load included the wieght of the persons, furnitur, wind loads, earthquake loads, loads of trucks and cranes.

The two loads can be modeled (structural model) by concentrated load, uniform load on line, uniform load on area or body load as shown in Fig. 1.4.

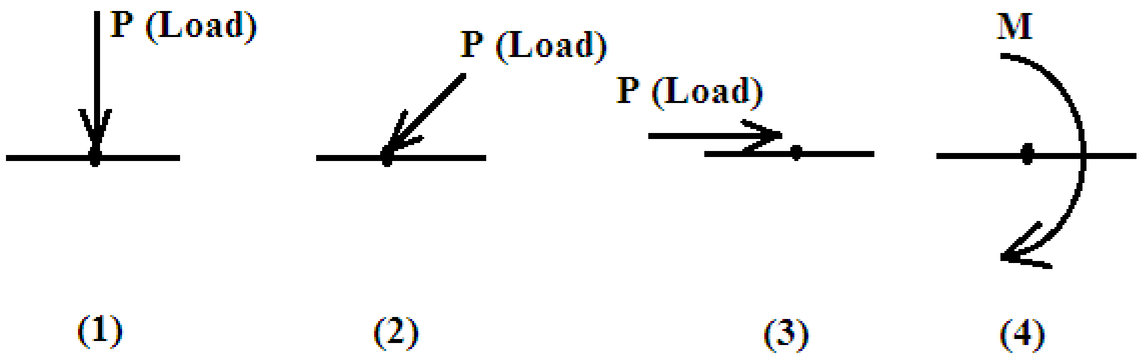
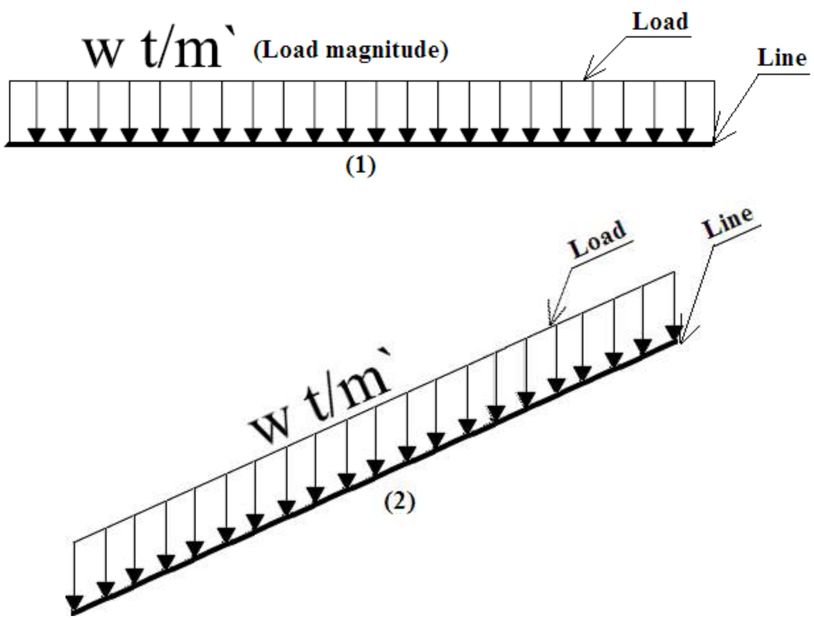
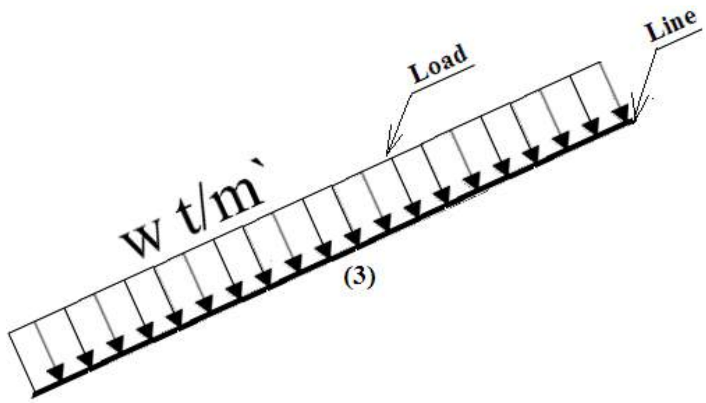


Fig. 1.4.a: The concentrated load





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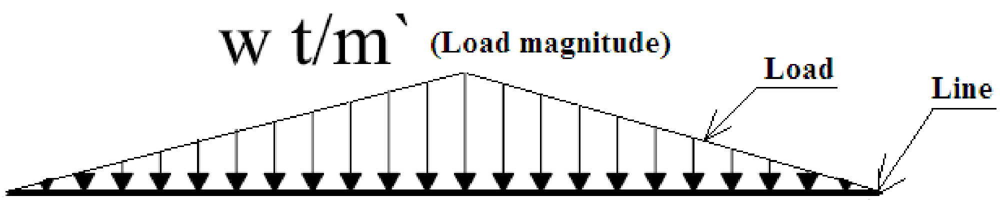


(4)

Fig. 1.4.b: The uniform load

1.5. Components of the Structures

The structure composes of some components as illustrated in Fig. 1.5. Each component subjects to loads and it transfers these loads to the next components (supports).

1. *Slab:* the first component is the slab. The slab subjected to its own weight, the floor covering and the live load and it supports on the beam. It was modeled by area as shown in Fig. 1.6.
2. *Beam:* The beams subject to their own weight and the load from slab. The beam supports on columns and it was modeled by line. This line has start and end points. These points were called nodes or joints as presented in Fig. 1.6.
3. *Columns:* The column is the support of the beam and it modeled as the beam.
4. *Footing:* The footing transfers load from the structure through the column to the soil. It was simulated by area as slab.

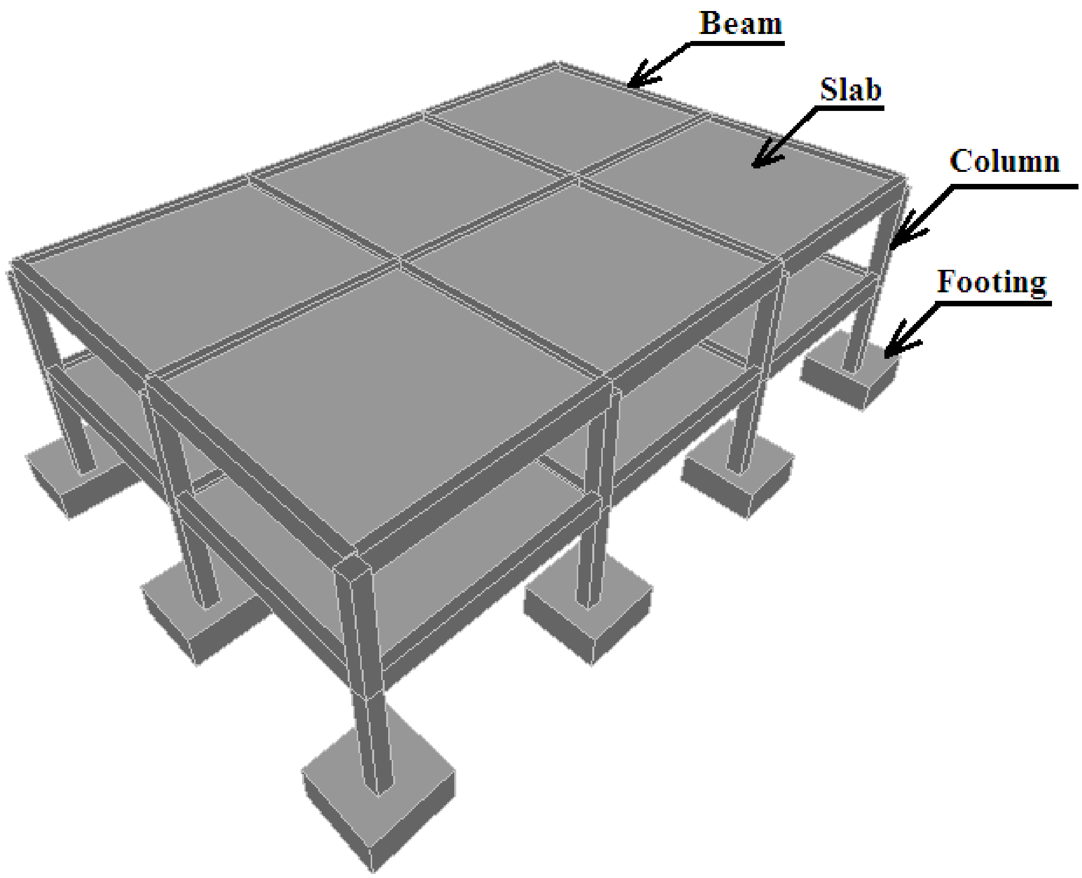
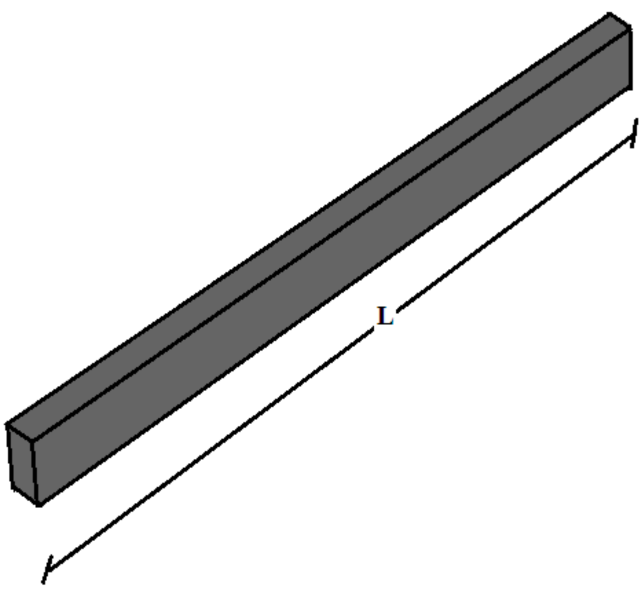
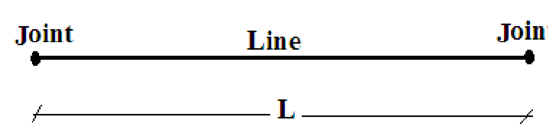


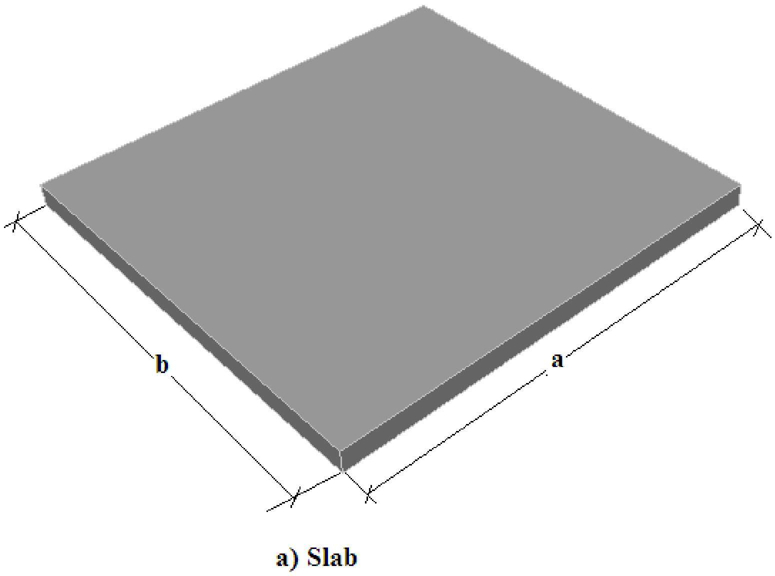
Fig. 5.1: Structure components





Beam

Beam model



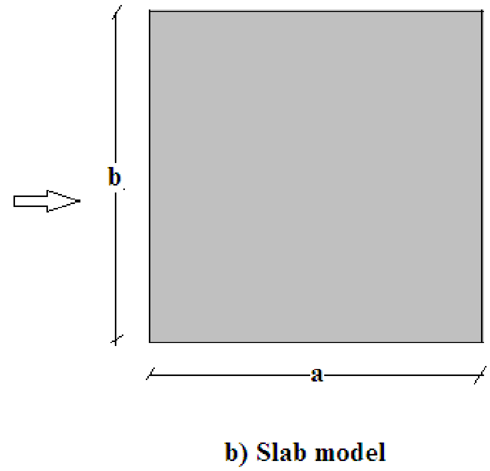


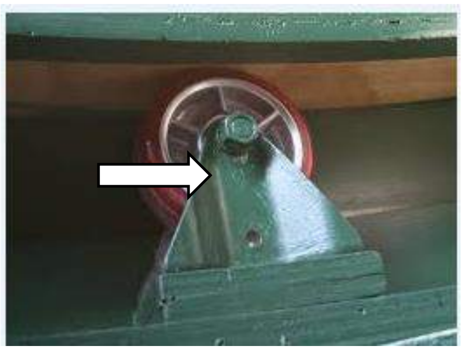
Fig. 1.6: Structural model of beam and slab

1.6. Supports

There are four types of supports. These supports are determined according to the type of the analysis and construction.

1. <u>Movable or Roller Support</u>

This support is constructed to permit a movement parallel to the supporting surface. It transfers only a single reaction perpendicular to the support surface. The roller support is presented in Fig. 1.7.



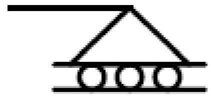
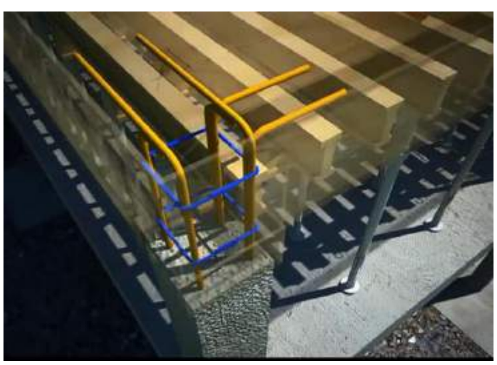


Fig. 1.7: Roller support

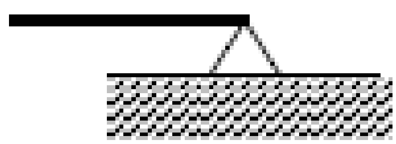
2. Hinged or Pin Support

This support prevents movement in a horizental and vertical directionand on the other hand it perimits rotation about the support. Hinged support transfers three reactions perpendicular to the support surface. The connection between the slab of the bridges and the retaining walls may be constructed to make the retaining wall is a hinged support to the bridge slab. Also you can see the hinged support in the connection between the beam and column. The column is a hinged support to the column because the reinforcements of the column do not insert the beam. The pin support is presented in Fig. 1.8.

a) In steel



b) In concrete

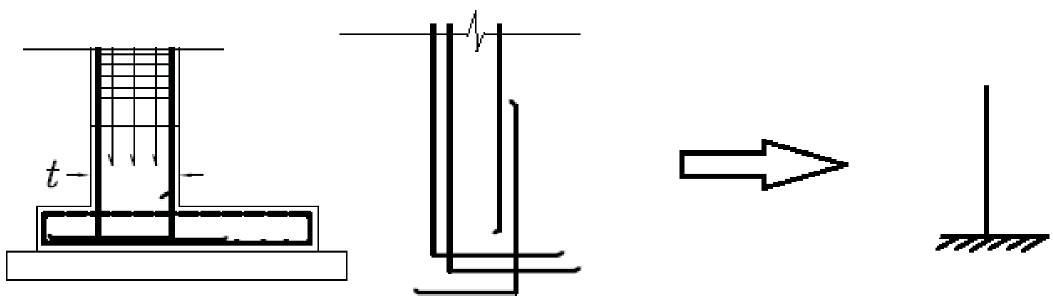


Model

Fig. 1.8: Details of hinged support

3. Fixed Support

This support does not allow the movements in all directions; x, y and z and rotation about x, y and z direction. This support transfers three reactions perpendicular to the support surface and three bending moments. The reinforcements of the concrete column were inserted in the footing so that the footing is fixed support to the column. This support is indicated in Fig. 1.9.



a) Real b) Model

Fig. 1.9: Details of fixed support

4. Link support

This support like the beam with hinged support at one end and intermediate hinge at the second end.

Chapter (2)

REACTIONS

2.1. Introduction

A structure must be designed to resist all the forces that act on it. If the structure cannot resist these forces, it may collapse as shown in Fig. 2.1. So that you must calculate the external forces acted on the structures and indicate the behavior of the structures under these loads (internal forces).

Fig. 2.1: The structure collapse

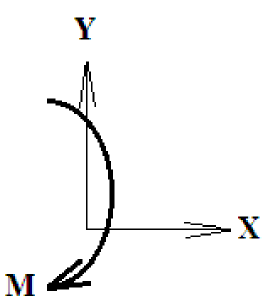
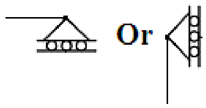
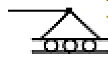
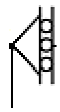
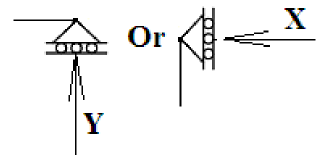
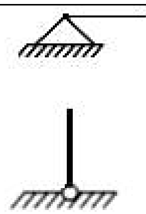
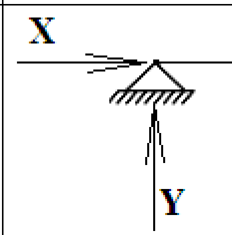
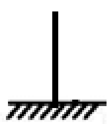
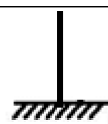
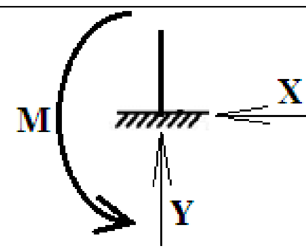
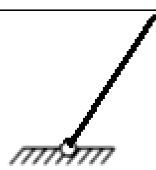
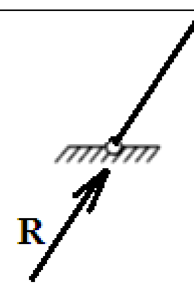
2.2. **Reactions**

The external forces that act on the structures included the loads reactions of the supports. Loads or applied forces referred to the load that have the tendency to move the structure (dead load, live load and so on). The reactions are forces exerted by supports of the structure and they are those forces applied to the structure to counteract the action of the applied force (they prevent its motion and keep it in equilibrium). The reactions are usually among the unknowns to be determined by analysis. To identify any force completely, there are three unknowns to that must be determined; the force magnitude, direction and the line of action of the force.

If a support prevents translation of body in a given direction, a force is developed on the body in that direction. This means that a support that prevents translation of the structure in a particular direction creates a reaction force on the structure in that direction. Also the support prevents rotation of the structure about a particular axis creates a reaction couple on the structure about that axis. All the types of the supports are presented in table 2.1.

From table 2.1, it can be seen that for roller support, the magnitude of reaction force that acts perpendicular to the supporting surface is unknown and it may be directed either away from or into the structure. Also for the *hinged support* the reactions consist of two components Y and X and the magnitude of the two components are unknown. Additionally the *fixed support* prevents the translation in x, y and one rotation about z direction so that it has three unknown components; X, Y and M. For the link support, the reaction force R is unknown and this force acts in the direction of the link and may be directed either into or away from the structure.

Table 2.1: Type of supports

Support type	Model	Movements	Reactions
Roller support 		 $x=?$ $y=0$ $\theta=?$  $x=0$ $y=?$ $\theta=?$	
Hinged support		$x=0$ $y=0$ $\theta=?$ 	
Fixed support		 $x=0$ $y=0$ $\theta=0$	
Link support		All the movements are allowed except the movement in the direction of the link member.	

2.3. Equations of static Equilibrium

A structure is considered to be in equilibrium if , initially at rest. It remains at rest when subjected to a system of forces couples. If a structure is in equilibrium, then all its members and parts are also in equilibrium. For a plane structure laying in the xy plane and subjected to a coplanar system of forces and couples, the necessary and sufficient conditions for equilibrium are:

1. The algebraic sum of the components of all forces parallel to the x axis is zero as presented in Eq.1.
2. The algebraic sum of the components of all the forces parallel to the y axis is zero as presented in Eq. 2.
3. 3. The algebraic sum of the moments of all forces about any point in the plane of forces is zero (see Eq. 3).

$$\sum F_x = 0 \quad (1)$$

$$\sum F_y = 0 \quad (2)$$

$$\sum M = 0 \quad (3)$$

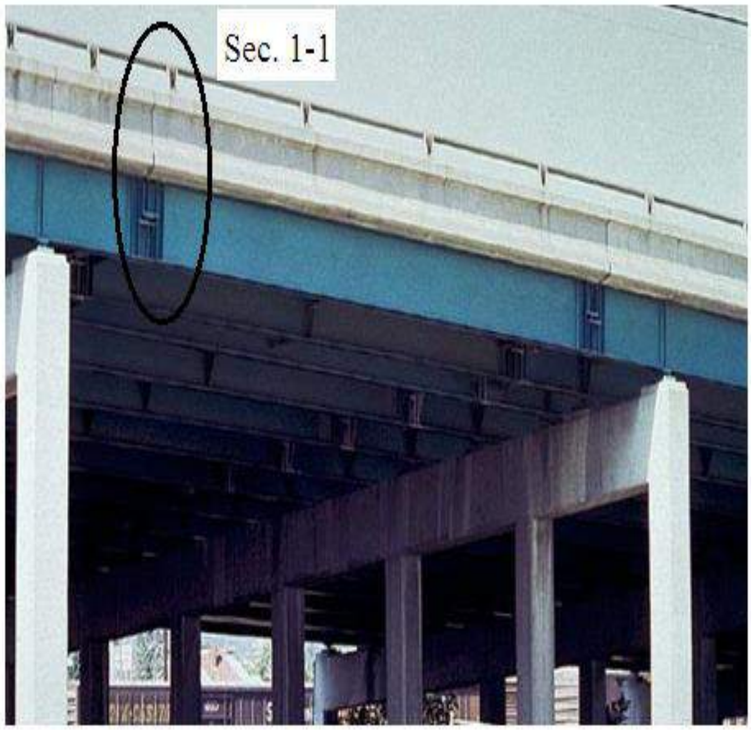
The three equations are called the equation of the equilibrium of structures in the plane direction. These equations are used to determine the unknown reactions.

2.4. Statically Determinate and Statically Indeterminate

From the previous sections, we can be concluded that there are two types of external forces; the applied forces and reactions of the supports. The applied forces are known; magnitude, direction and line of actions of the force. On the other hand for the reactions only the point of force and perhaps the directions are unknown as shown in table. 2.1. Also there are three equations of equilibrium and they are used to determine the unknown reactions. The total numbers of unknowns that can be determined by the equations of equilibrium are three. The determination of more three unknowns requires *additional equations* or methods to be used.

The additional equations can be obtained from the additional conditions. These equations are considered if the structures are constructed with *intermediate hinge and with link element*. The intermediate hinge is constructed to make the internal moments at it equal zero. Fig. 2.2 showed intermediate hinge in the beam of steel bridge. The beam is modeled by straight line and the intermediate hinge is modeled by *circle* in its position. The intermediate hinge gives one additional equation; the moment at it equal zero.

Link element is a member of structure (component). It likes beam with two intermediate hinges at its ends; start and the end. It does not carry external loads perpendicular to the directions of the link or bending moments. Also the link has the ability of carrying the force parallel to the direction of the link.



Sec. 1-1

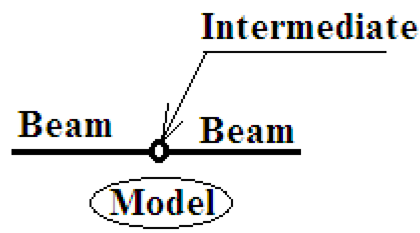


Fig. 2.2: The intermediate hinge

The total number of unknown and the number of the available equations (equilibrium equations and the additional equations) were used to determine the type of the structures. It is customary to divide structures into <u>*statically determinate*</u> and <u>*statically indeterminate*</u>.

1. <u>Statically determinate structure</u>

It is a system for which all reactions of supports can be determined by means of available equations (equations of equilibrium and the additional equations) and the internal forces also can be found by method of sections. The internal forces are the forces created in structures under the external forces and they will be explained in the next chapter. This means that the unknown in statically determinate structure is same as the available equations.

<u>*Example (1)*</u>

Fig. 2. 3. a. represents a simple beam with one span; L length. This beam is considered to be subjected to two loads P and this beam support on two columns at a&b. The two columns are simulated by two supports; one is hinged and the other is roller support. Find the type of the structure: <u>statically determinate structure or statically indeterminate structure.</u>

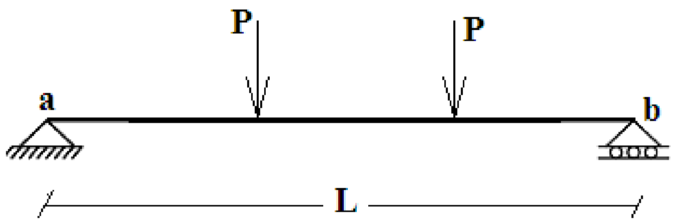


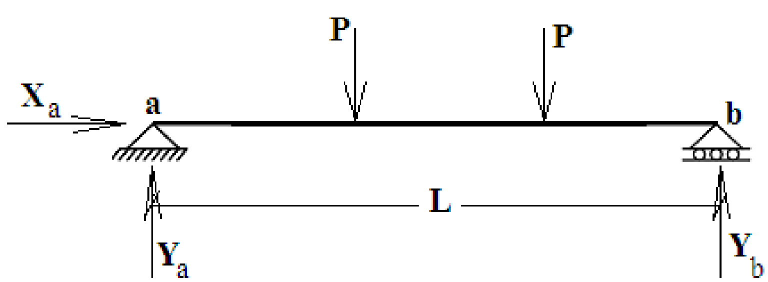
Fig. 2.3.a

Solution

- The numbers of unknown at the point (a) are two reactions X_a&Y_a
- The number of unknowns at the point (b) is one reaction Y_b
  - ➢ So that the total numbers of the unknown are three unknowns.

✓ The numbers of available equations are three equations: the equilibrium equations. $\sum F_x = 0$ $\quad \sum F_y = 0$ $\quad \sum M = 0$

The number of unknowns = the numbers of available equations so that the structure is statically determinate.



Example (2)

Fig. 2. 4. a. represents a continuous beam. This beam is considered to be supported on three columns at a, b &c. The three columns are simulated by three supports; one is hinged and the other is roller support. Also the beam has an intermediate hinge at joint#d. Find the type of the structure: statically determinate structure or statically indeterminate structure.

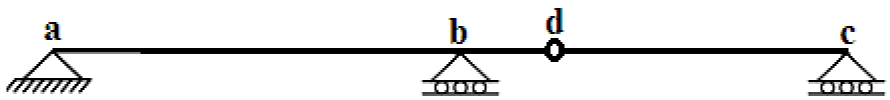


Fig. 2.4.a

Solution

- The numbers of unknown at the point (a) are two reactions X_a&Y_a
- The number of unknowns at the point (b) is one reaction Y_b
- The number of unknowns at the point (c) is one reaction Y_c
  - ➢ So that the total numbers of the unknown are four unknowns.

✓ The numbers of available equations are four equations: the equilibrium equations. $\sum F_x = 0$ $\sum F_y = 0$ $\sum M = 0$

$$and\ additional\ equation \sum M_d = 0$$

The number of unknowns = The numbers of available equations so that the structure is statically determinate.

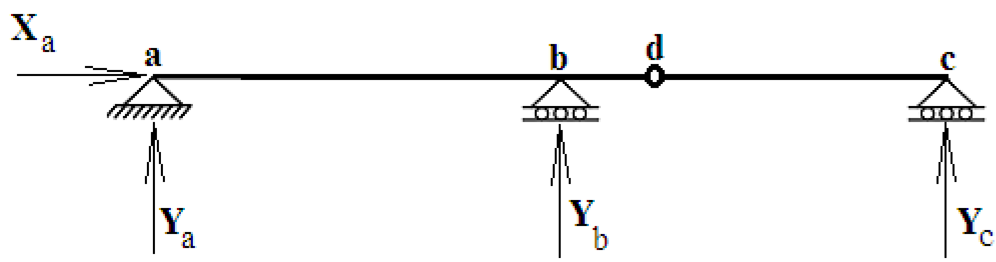


Fig. 2.4.b

Example (3)

A frame structure is presented in Fig. 2. 5. a. The frame is different than beam in construction. In the beam, the reinforcements of the columns do not insert in the beam. On the other hand the reinforcements of the columns in the frame structure must be inserted in the beams. The connection between the beam and column is considered hinged in the beam structure but in the frame structure the connection is not considered as hinged support. Find the type of the structure: <u>statically determinate structure or statically indeterminate structure.</u>

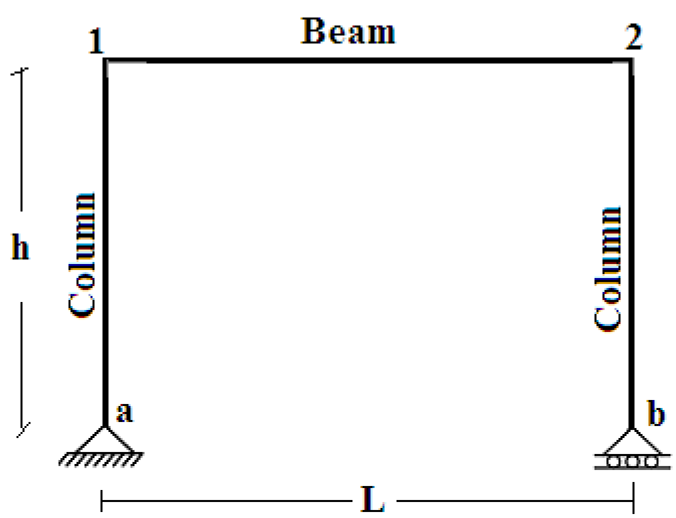


Fig. 2.5.a

Solution

- The numbers of unknown at the point (a) are two reactions X_a&Y_a
- The number of unknowns at the point (b) is one reaction Y_b
  - ➢ So that the total numbers of the unknown are three unknowns.

✓ The numbers of available equations are four equations: the equilibrium equations. $\sum F_x = 0 \quad \sum F_y = 0 \quad \sum M = 0$

The number of unknowns = The numbers of available equations so that the structure is statically determinate.

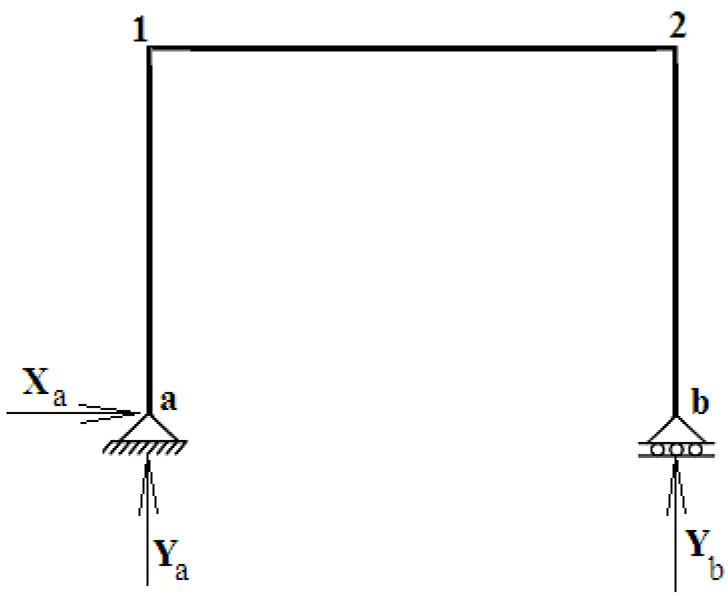


Fig. 2.5.b

<u>Example (4)</u>

A frame structure is presented in Fig. 2. 6. Find the type of the structure: <u>statically determinate structure or statically indeterminate structure.</u>

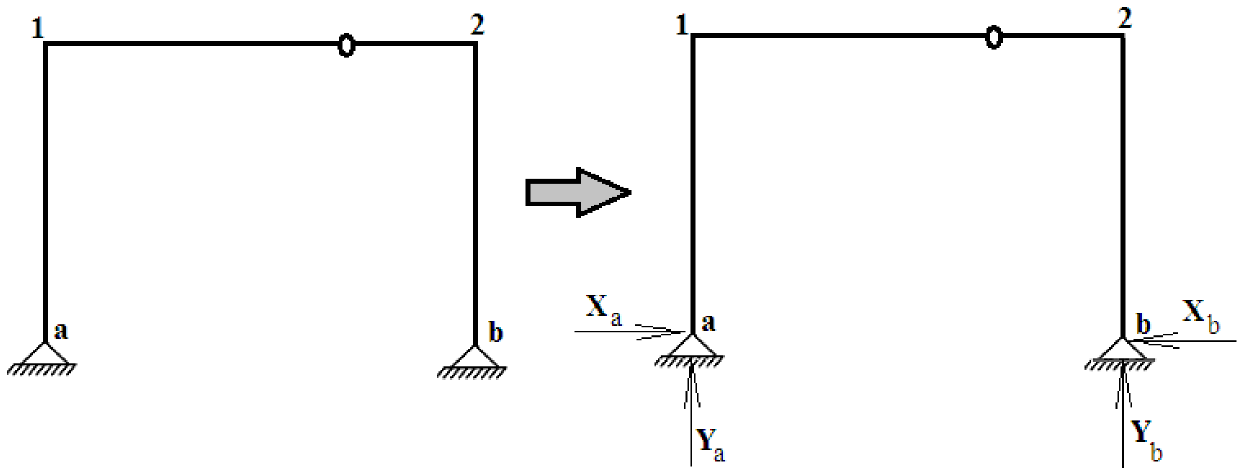


Fig. 2.6

Solution

- The numbers of unknown at the point (a) are two reactions X_a&Y_a
- The number of unknowns at the point (b) is one reaction X_b&Y_b
  - ➢ So that the total numbers of the unknown are three unknowns.

✓ The numbers of available equations are four equations: the equilibrium equations. $\sum F_x = 0$ $\sum F_y = 0$ $\sum M = 0$ $and\ additional\ equation\ \sum M_d = 0$

The number of unknowns = The numbers of available equations so that the structure is statically determinate.

Example (5)

A frame structure is presented in Fig. 2. 7. Find the type of the structure: statically determinate structure or statically indeterminate structure.

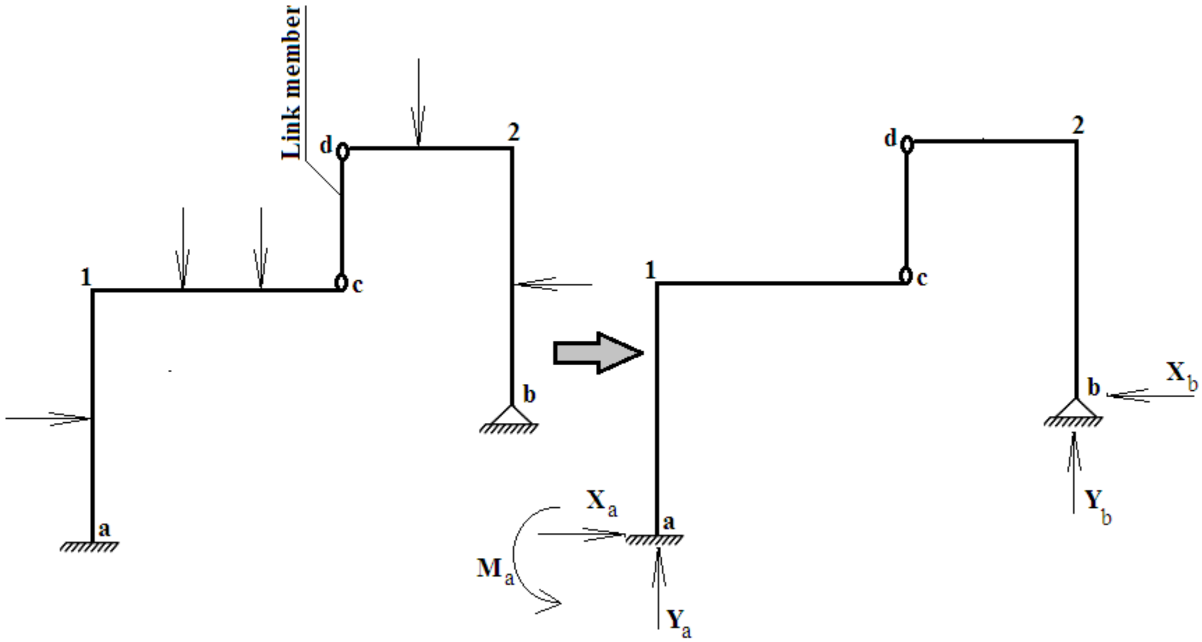


Fig. 2.7

Solution

- The numbers of unknown at the point (a) are two reactions X_a&Y_a
- The number of unknowns at the point (b) is one reaction X_b&Y_b
  - ➢ So that the total numbers of the unknown are five unknowns.

✓ The numbers of available equations are four equations: the equilibrium equations. $\sum F_x = 0$ $\sum F_y = 0$ $\sum M = 0$ *and two additional equations*

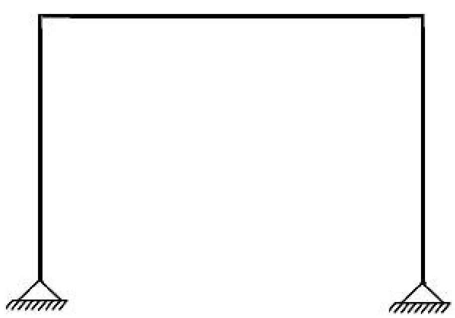
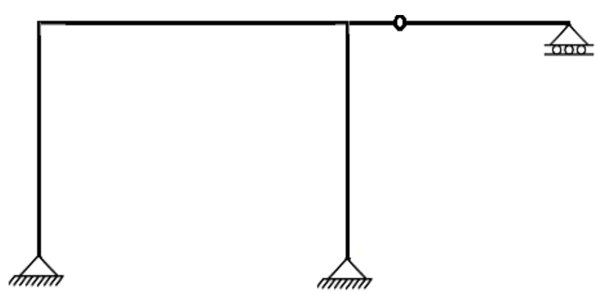
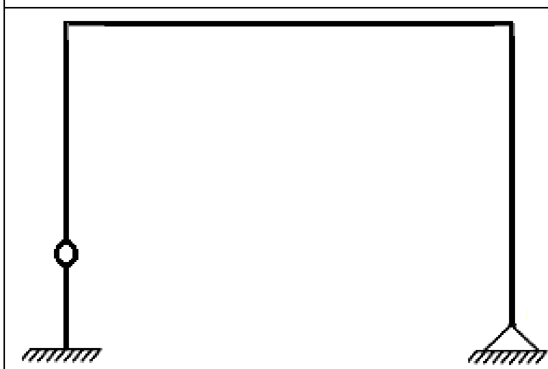
The internal moment and shear force in the link element =0

The number of unknowns (5) = The numbers of available equations (5) so that the structure is statically determinate.

2. <u>Statically indeterminate structure</u>.

It a system of structures in which the reactions of supports can not be determined by means of available equations (equations of equilibrium and the additional equations). The indeterminacy of the structure may be either external, internal, or both. Table 2.2 illustrates some type of statically determinate structures.

Table 2.2: Classification of the structures

Structure	No. Unknown	No. Equation	Classification
	4	3	Statically indeterminate (1)
	5	4	Statically indeterminate (1)
	6	4	Statically indeterminate (2)

2.5. Calculation the Reactions at Supports

To determine the reactions at the support, the available equations are used. The following examples will serve to illustrate the procedure of calculation of the reactions by applying available equations.

2.6. Solved Examples

Example (6)

For simple beam (a-b) presented in Fig. 2.8, calculate the reactions at the two supports under the applied load.

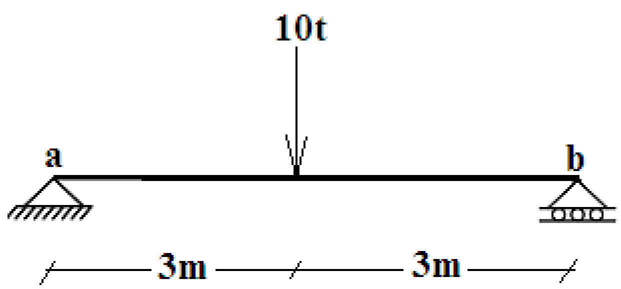


Fig. 2.8.a

Solution

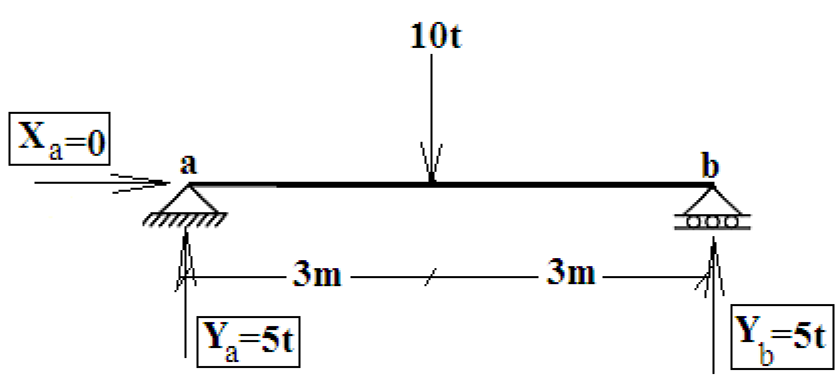


Fig. 2.8.b

- *Finding the reactions*

a) $\sum F_x = 0 \gg\gg\gg\gg X_a = 0$

$b) \sum M_a = 0 \ggg\ggg 10x3 - Y_b x6 = 0$

$$\therefore Y_b = \frac{30}{6} = 5t \uparrow$$

$c) \sum M_b = 0 \ggg\ggg 10x3 - Y_a x6 = 0$

$$\therefore Y_a = \frac{30}{6} = 5\, t \uparrow$$

d) <u>*Check*</u> $\sum \mathrm{F_y} = 10 - 5 - 5 = 0 \ggg\ggg \mathrm{o\,k}$

Example (7)

For simple beam (a-b) presented in Fig. 2.9, calculate the reactions at the two supports under the applied load.

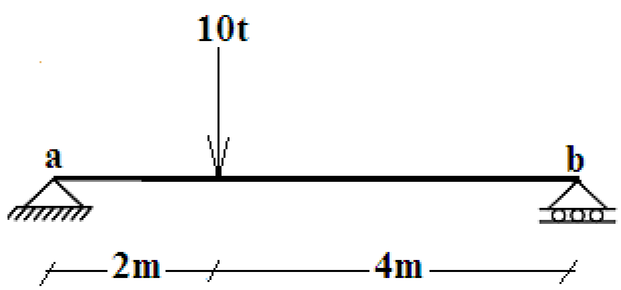


Fig. 2.9.a

Solution

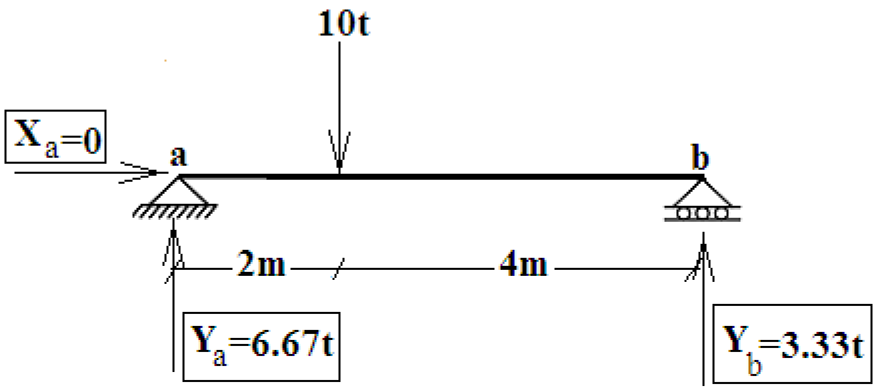


Fig. 2.9.b

- *Finding the reactions*

a) $\sum \mathrm{F_x} = 0 \ggg\ggg \mathrm{X_a} = 0$

b) $\sum M_a = 0 \ggg\ggg 10x2 - Y_b x6 = 0$

$$\therefore Y_b = \frac{20}{6} = 3.33t \uparrow$$

c) $\sum M_b = 0 \ggg\ggg 10x4 - Y_a x6 = 0$

$$\therefore Y_a = \frac{40}{6} = 6.67\ t \uparrow$$

d) <u>*Check*</u> $\sum \mathrm{F_y} = 10 - 6.67 - 3.33 = 0 \ggg\ggg \mathrm{o\,k}$

<u>*Example (8)*</u>

For simple beam (a-b) presented in Fig. 2.10, calculate the reactions at the two supports under the applied load.

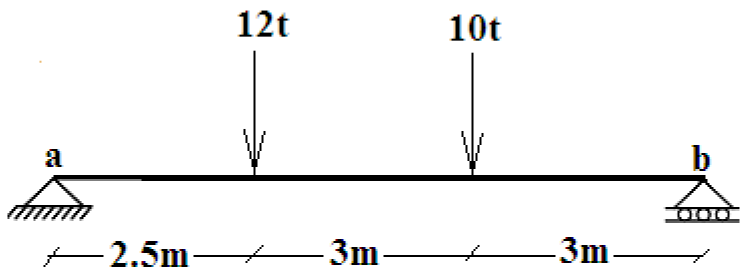


Fig. 2.10.a

Solution

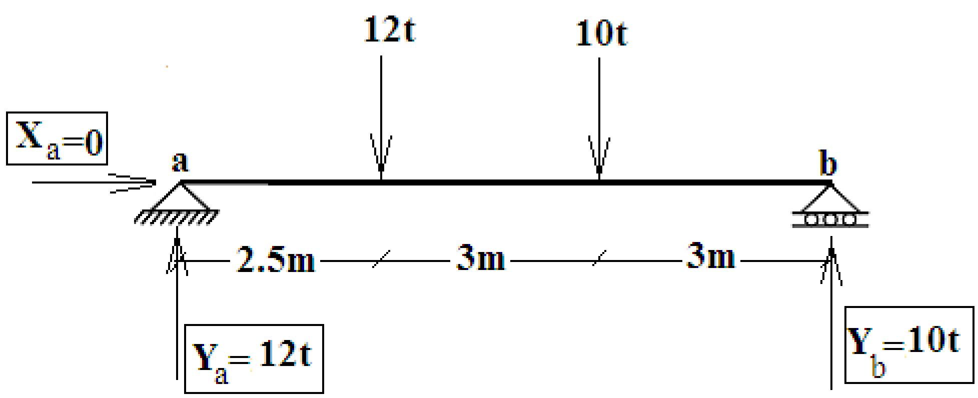


Fig. 2.10.b

➢ *Finding the reactions*

a) $\sum F_x = 0 \ggg\ggg X_a = 0$

b) $\sum M_a = 0 \ggg\ggg 12x2.5 + 10x5.5 - Y_b x8.5 = 0$

$$\therefore Y_b = \frac{85}{8.5} = 10t \uparrow$$

c) $\sum M_b = 0 \ggg\ggg 10x3 + 12x6 - Y_a x8.5 = 0$

$$\therefore Y_a = \frac{102}{8.5} = 12\ t \uparrow$$

d) *Check* $\sum F_y = 10 + 12 - 10 - 12 = 0 \ggg\ggg$ o k

Example (9)

For simple beam (a-b) presented in Fig. 2.11, calculate the reactions at the two supports under the applied load.

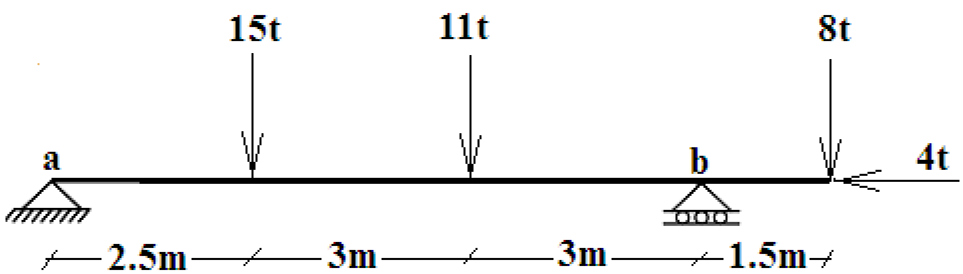


Fig. 2.11.a

Solution

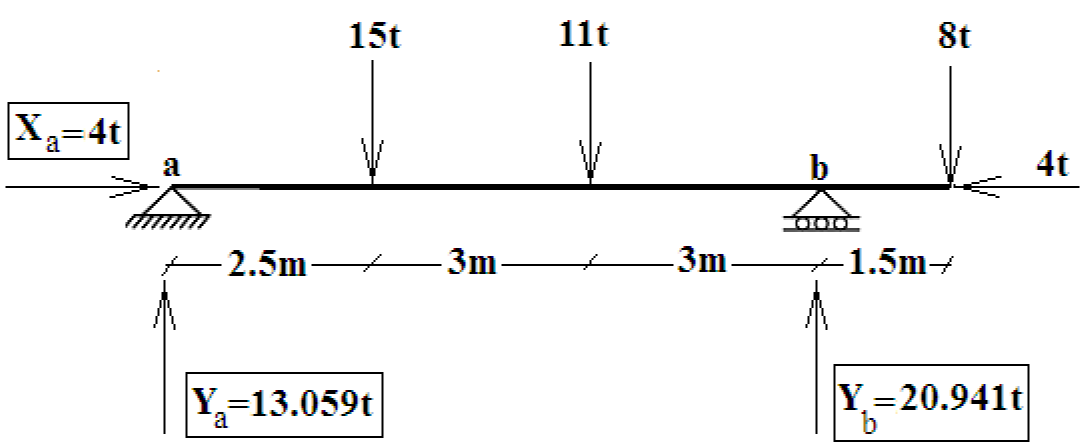


Fig. 2.11.b

- *Finding the reactions*

a) $\sum F_x = 0$ ⋙⋙⋙ $X_a = 4t \rightarrow$

b) $\sum M_a = 0$ ⋙⋙⋙ $15x2.5 + 11x5.5 + 8x10 - Y_b x8.5 = 0$

$$\therefore Y_b = \frac{178}{8.5} = 20.941t \uparrow$$

c) $\sum M_b = 0$ ⋙⋙⋙ $15x6 + 11x3 - 8x1.5 - Y_a x8.5 = 0$

$$\therefore Y_a = \frac{111}{8.5} = 13.059\ t \uparrow$$

d) *Check* $\sum \mathrm{F_y} = 15 + 11 + 8 - 20.941 - 13.059 = 0 \gg\gg\gg\gg$ o k

Example (10)

For simple beam (a-b) presented in Fig. 2.12, calculate the reactions at the two supports under the applied load.

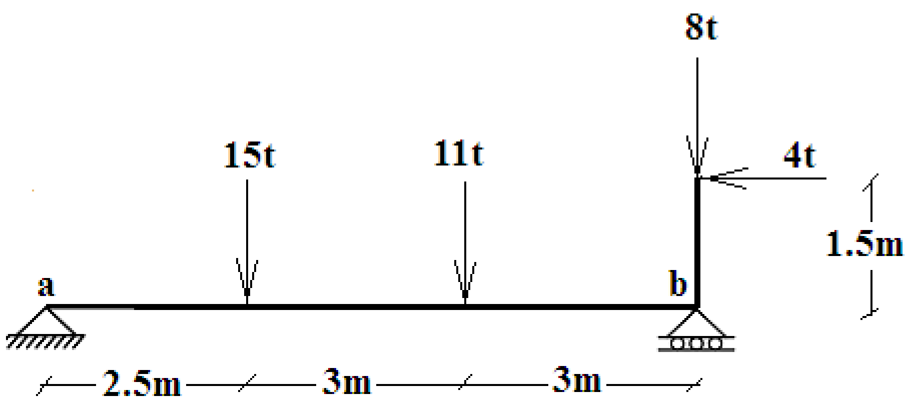


Fig. 2.12.a

Solution

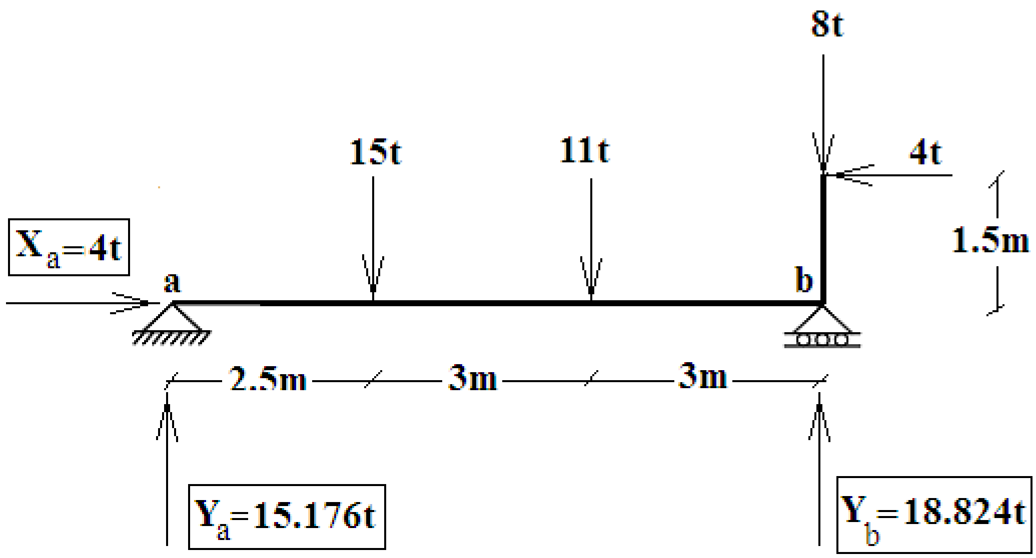


Fig. 2.12.b

➢ *Finding the reactions*

a) $\sum F_x = 0 \ggg\ggg X_a = 4t \rightarrow$

b) $\sum M_a = 0 \ggg\ggg 15x2.5 + 11x5.5 + 8x8.5 - 4x1.5 - Y_b x8.5 = 0$

$$\therefore Y_b = \frac{160}{8.5} = 18.824t \uparrow$$

c) $\sum M_b = 0 \ggg\ggg 15x6 + 11x3 + 4x1.5 - Y_a x8.5 = 0$

$$\therefore Y_a = \frac{129}{8.5} = 15.176\ t \uparrow$$

d) <u>*Check*</u> $\sum F_y = 15 + 11 + 8 - 18.824 - 15.176 = 0 \ggg\ggg$ o k

---

<u>Example (11)</u>

For simple beam (a-b) presented in Fig. 2.13, calculate the reactions at the two supports under the applied load.

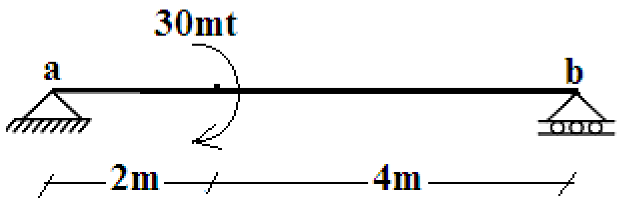


Fig. 2.13.a

Solution

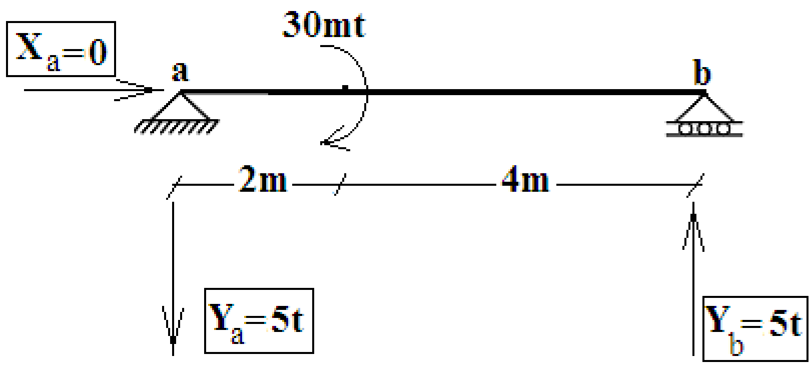


Fig. 2.13.b

- *Finding the reactions*

a) $\sum \mathrm{F_x} = 0 \ggg\ggg \mathrm{X_a} = 0$

b) $\sum M_a = 0 \ggg\ggg 30 - Y_b x6 = 0$

$$\therefore Y_b = \frac{30}{6} = 5t \uparrow$$

c) $\sum M_b = 0 \ggg\ggg -30 - Y_a x6 = 0$

$$\therefore Y_a = \frac{-30}{6} = 5\ t \downarrow$$

d) <u>*Check*</u> $\sum \mathrm{F_y} = 5 - 5 = 0 \ggg\ggg$ o k

---

<u>*Example (12)*</u>

For simple beam (a-b) presented in Fig. 2.14, calculate the reactions at the two supports under the applied load.

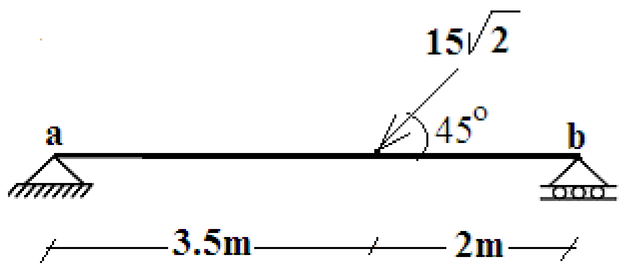


Fig. 2.14.a

Solution

- At the first, the inclined load is analyzed into two components; the vertical component P_v= P sinα =15t and the horizontal components P_h= P cosα = 15t as indicated in Fig. 2.14.a.

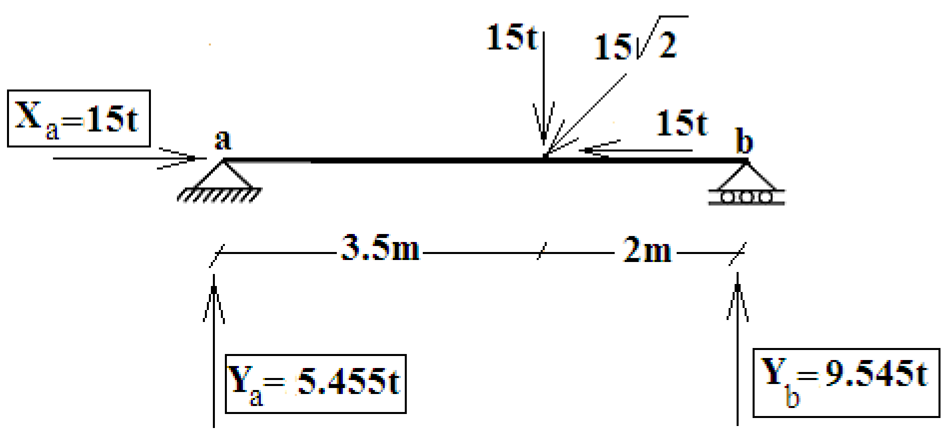


Fig. 2.14.b

- *Finding the reactions*

a) $\sum F_x = 0 \gg\gg\gg X_a = 15t \rightarrow$

b) $\sum M_a = 0 \gg\gg\gg 15x3.5 - Y_b x5.5 = 0$

$$\therefore Y_b = \frac{52.5}{5.5} = 9.545t \uparrow$$

c) $\sum M_b = 0 \gg\gg\gg\gg 15x2 - Y_a x5.5 = 0$

$$\therefore Y_a = \frac{30}{5.5} = 5.455\ t \downarrow$$

d) <u>*Check*</u> $\sum \mathrm{F_y} = 15 - 5.455 - 9.545 = 0 \gg\gg\gg\gg$ o k

---

<u>Example (13)</u>

For simple beam (a-b) presented in Fig. 2.15, calculate the reactions at the two supports under the applied load.

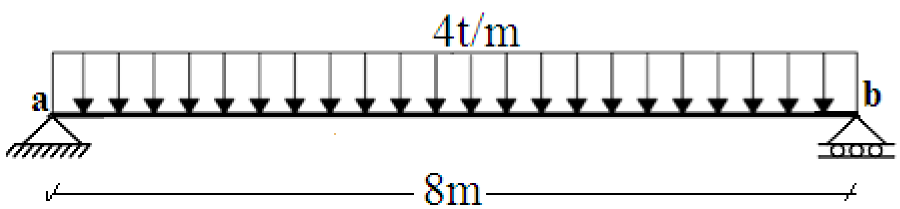


Fig. 2.15.a

<u>Solution</u>

- *At the first, the uniform load is concentrated according to its type as indicated in table 2.3(see Fig. 2.15.b).*
- *At the second, the reaction will be calculated*

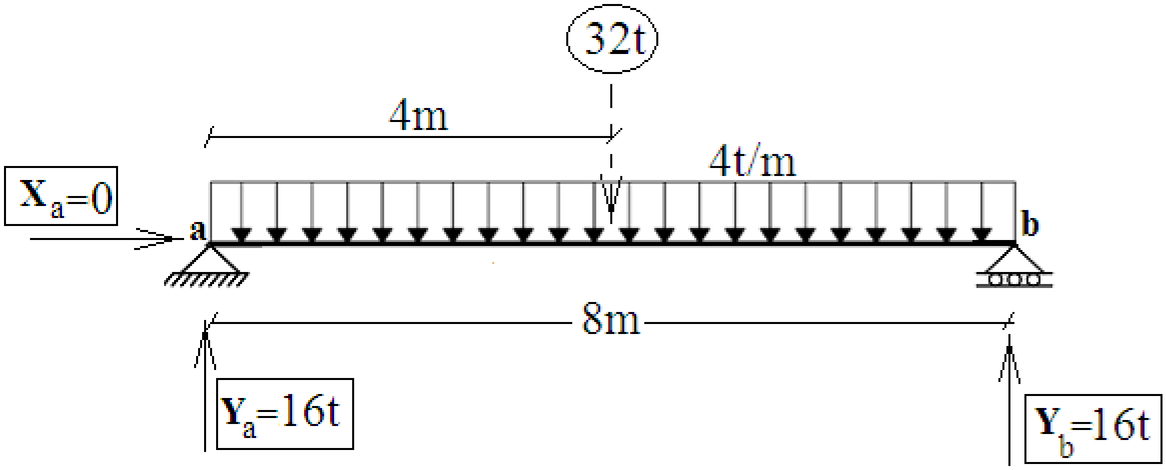


Fig. 2.15.b

a) $\sum F_x = 0 \ggg\ggg X_a = 0$

b) $\sum M_a = 0 \ggg\ggg 32x4 - Y_b x8 = 0$

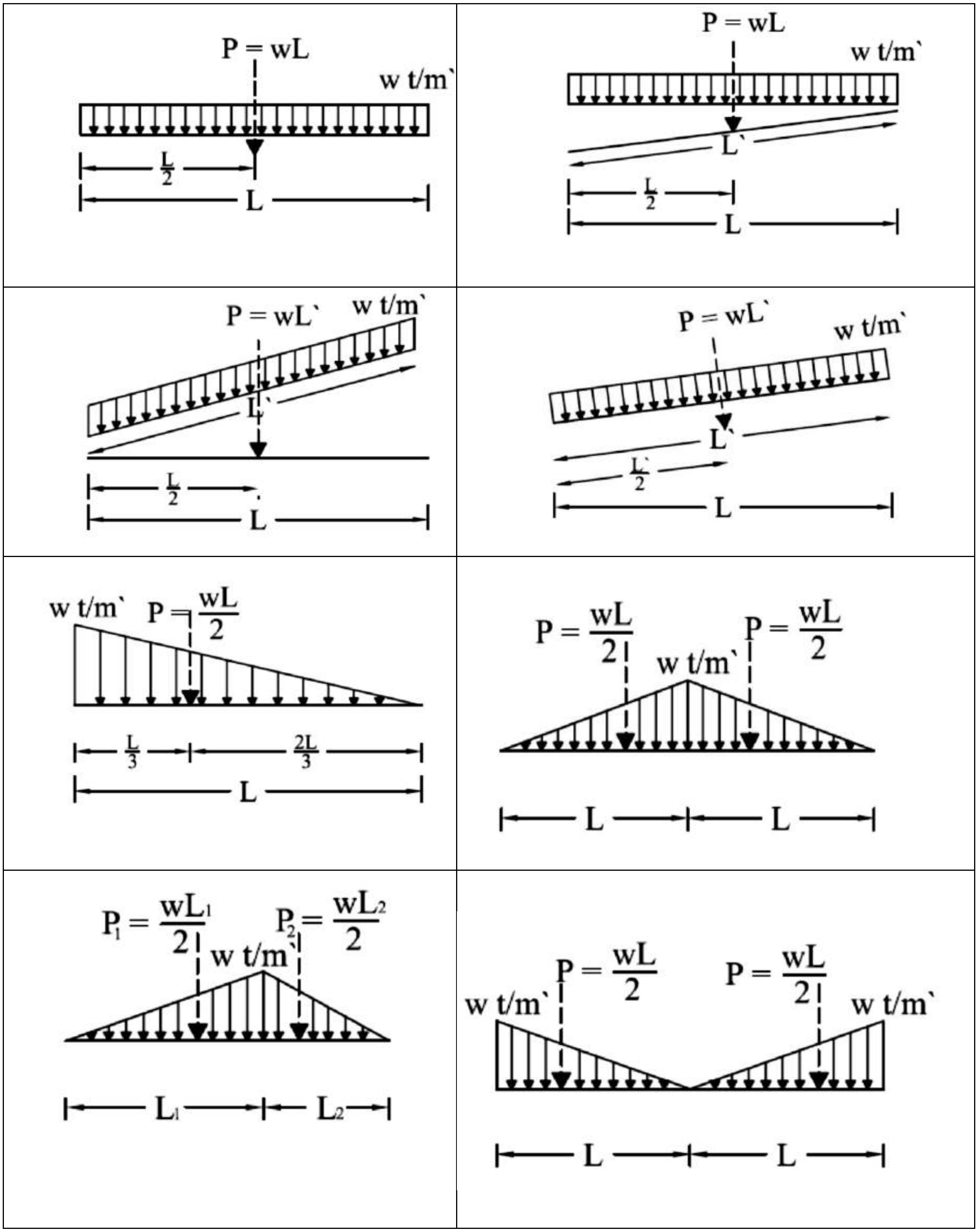
$$\therefore Y_b = \frac{128}{8} = 16t \uparrow$$

c) $\sum M_b = 0 \ggg\ggg 32x4 - Y_a x8 = 0$

$$\therefore Y_a = \frac{128}{8} = 16\ t \uparrow$$

d) *<u>Check</u>* $\sum F_y = 32 - 16 - 16 = 0 \ggg\ggg$ o k

Table 2.3: Type of uniform load



Example (14)

For cantilever beam presented in Fig. 2.16, calculate the reactions at the fixed support under the applied load.

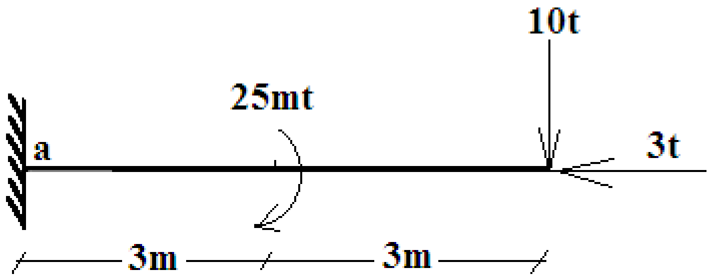


Fig. 2.16.a

Solution

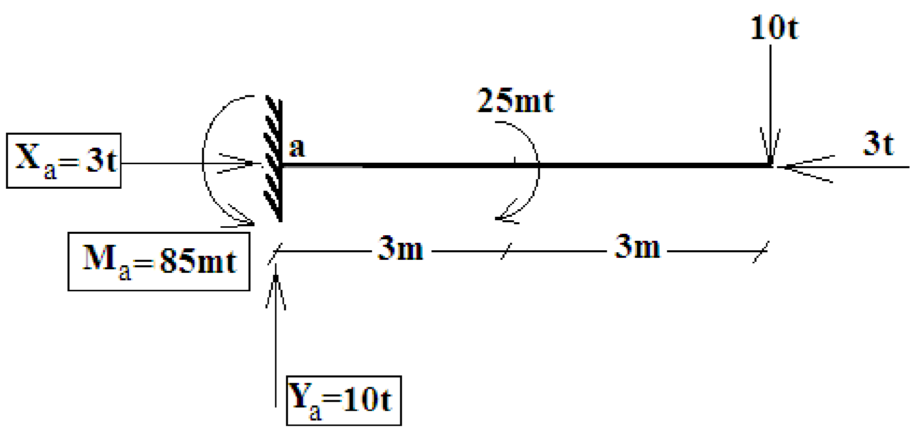


Fig. 2.16.b

a) *The fixed support has three reactions X_a, Y_a and M_a*

b) $\sum F_x = 0 \gg\gg\gg\gg\gg\gg X_a = 3t \rightarrow$

c) $\sum M_a = 0 \gg\gg\gg\gg\gg\gg 25 + 10x6 - M_a = 0$

$\therefore M_a = 85mt$

d) $\sum F_y = 0 \gg\gg\gg\gg\gg\gg 10 - Y_a = 0 \qquad \therefore Y_a = 10\ t \uparrow$

Example (15)

For cantilever beam presented in Fig. 2.17, calculate the reactions at the fixed support under the applied load.

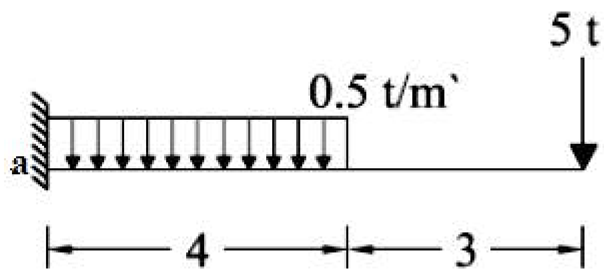


Fig. 2.17.a

<u>Solution</u>

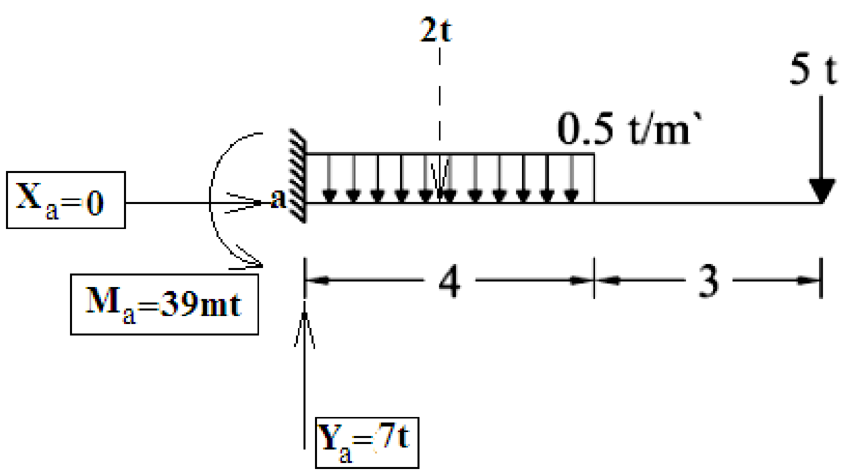


Fig. 2.17.b

- *The fixed support has three reactions X_a, Y_a and M_a*

a) $\sum F_x = 0 \gg\gg\gg X_a = 0$

b) $\sum M_a = 0 \gg\gg\gg 2x2 + 5x7 - M_a = 0$

$\therefore M_a = 39mt$

c) $\sum F_y = 0 \gg\gg\gg 7 - Y_a = 0 \qquad \therefore Y_a = 7\ t \uparrow$

Example (16)

For frame presented in Fig. 2.18, calculate the reactions at the two supports under the applied load.

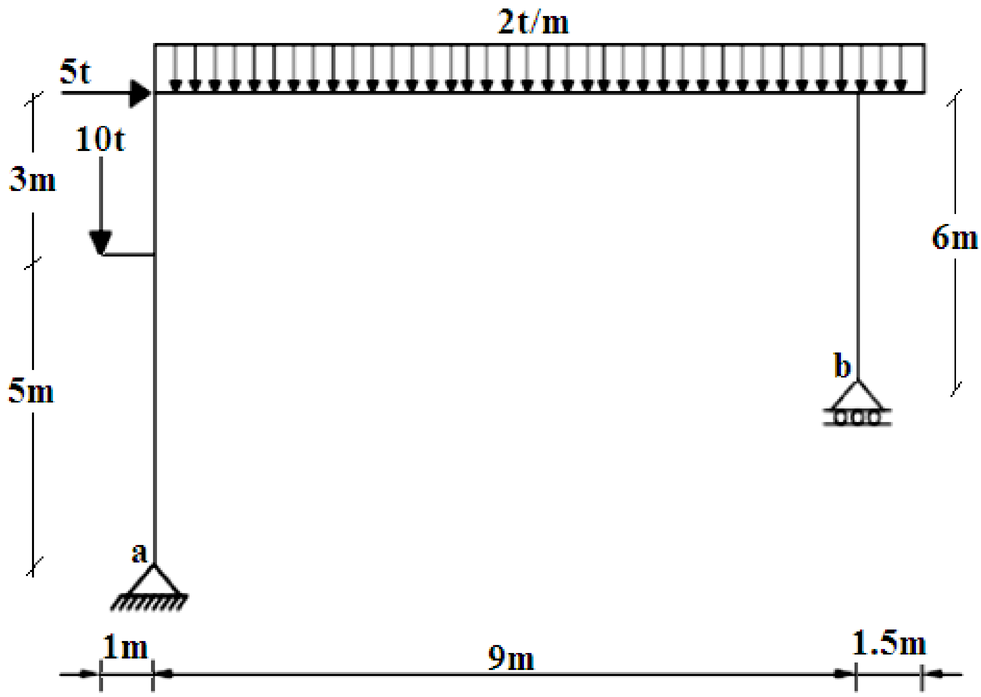


Fig. 2.18.a

Solution

$\sum F_x = 0 \ggg\ggg X_a = 5t \leftarrow$

$\sum M_a = 0 \ggg\ggg 18x4.5 + 3x9.75 + 5x8 - 10x1 - Y_b x9 = 0$

$$\therefore Y_b = 15.58t \uparrow$$

$\sum M_b = 0 \ggg\ggg -18x4.5 + 3x0.75 + 5x6 - 10x10 + 5x2 + Y_a x9 = 0$

$$\therefore Y_a = 15.42\ t \uparrow$$

Check $\sum F_y = 18 + 3 + 10 - 15.58 - 15.42 = 0 \ggg\ggg$ o k

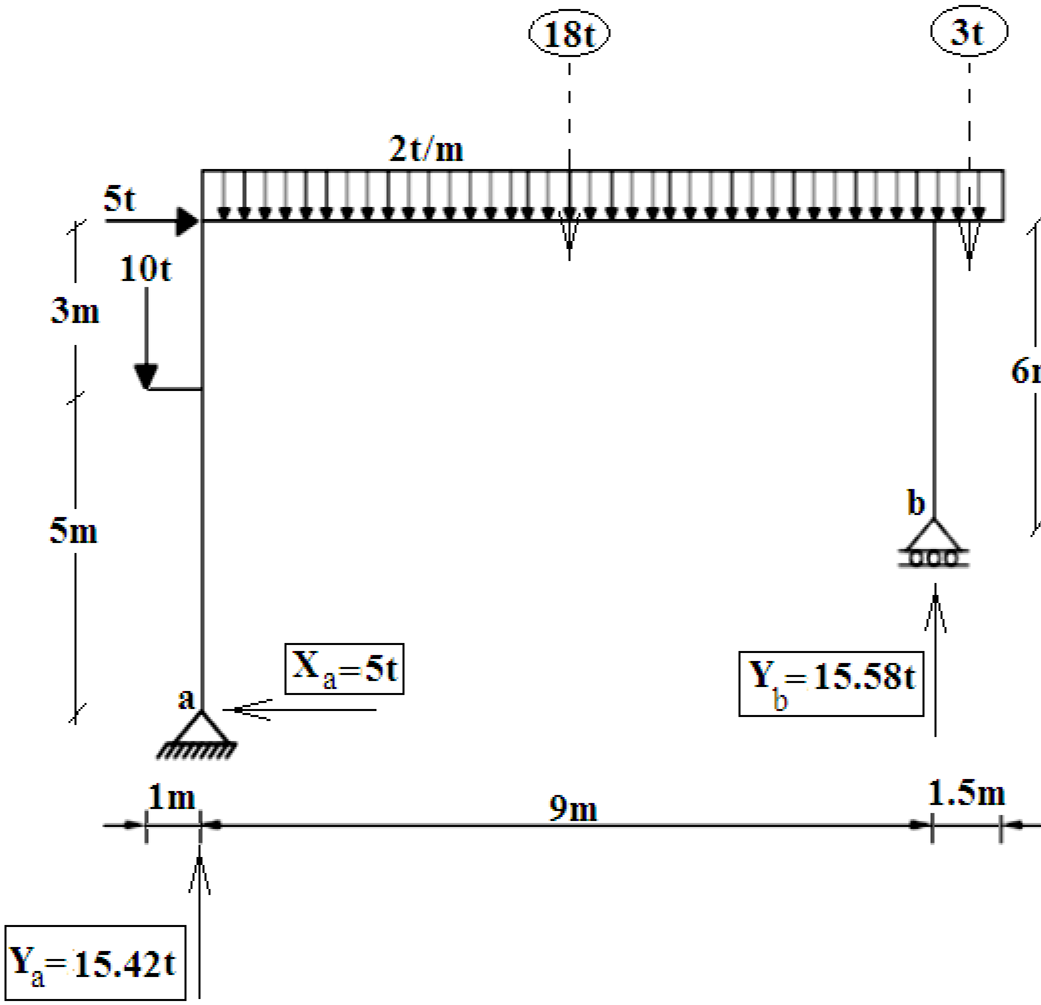


Fig. 2.18.b

Example (17)

For frame presented in Fig. 2.19, calculate the reactions at the two supports under the applied load.

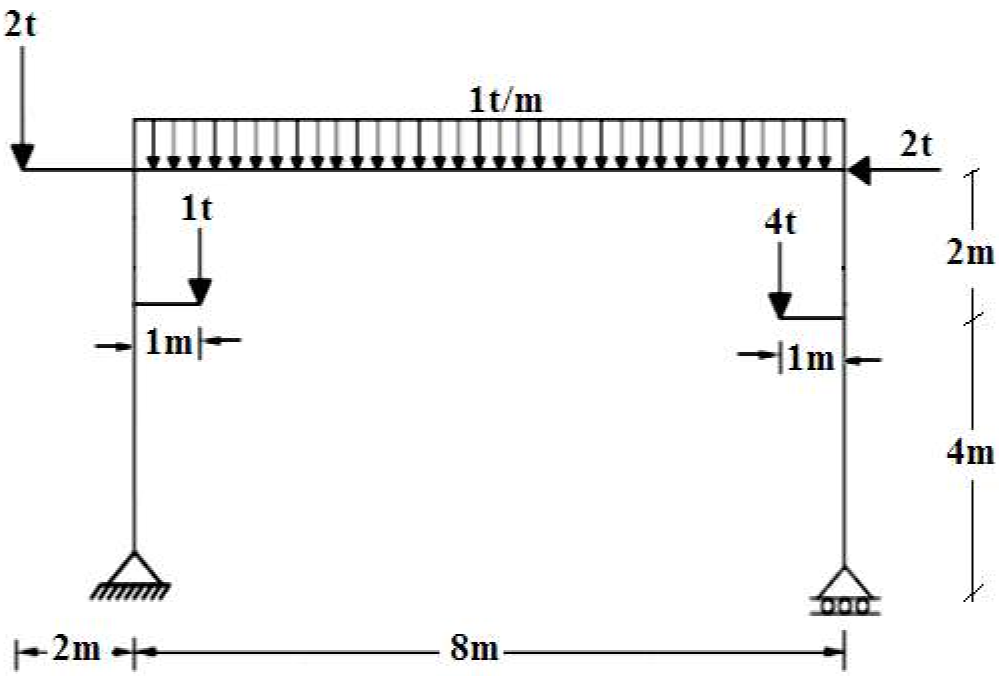


Fig. 2.19.a

Solution

$\sum F_x = 0 \ggg\ggg\ggg X_a = 2t \leftarrow$

$\sum M_a = 0 \gg 1x1 + 8x4 + 4x7 - 2x6 - 2x2 - Y_b x8 = 0$

$$\therefore Y_b = 5.625t \uparrow$$

$\sum M_b = 0 \ggg\ggg\ggg 8x4 + 2x10 + 4x1 + 1x7 + 2x6 - Y_a x8 = 0$

$$\therefore Y_a = 9.375\ t \uparrow$$

Check $\sum F_y = 8 + 2 + 4 + 1 - 9.375 - 5.625 = 0 \ggg\ggg\ggg$ ok

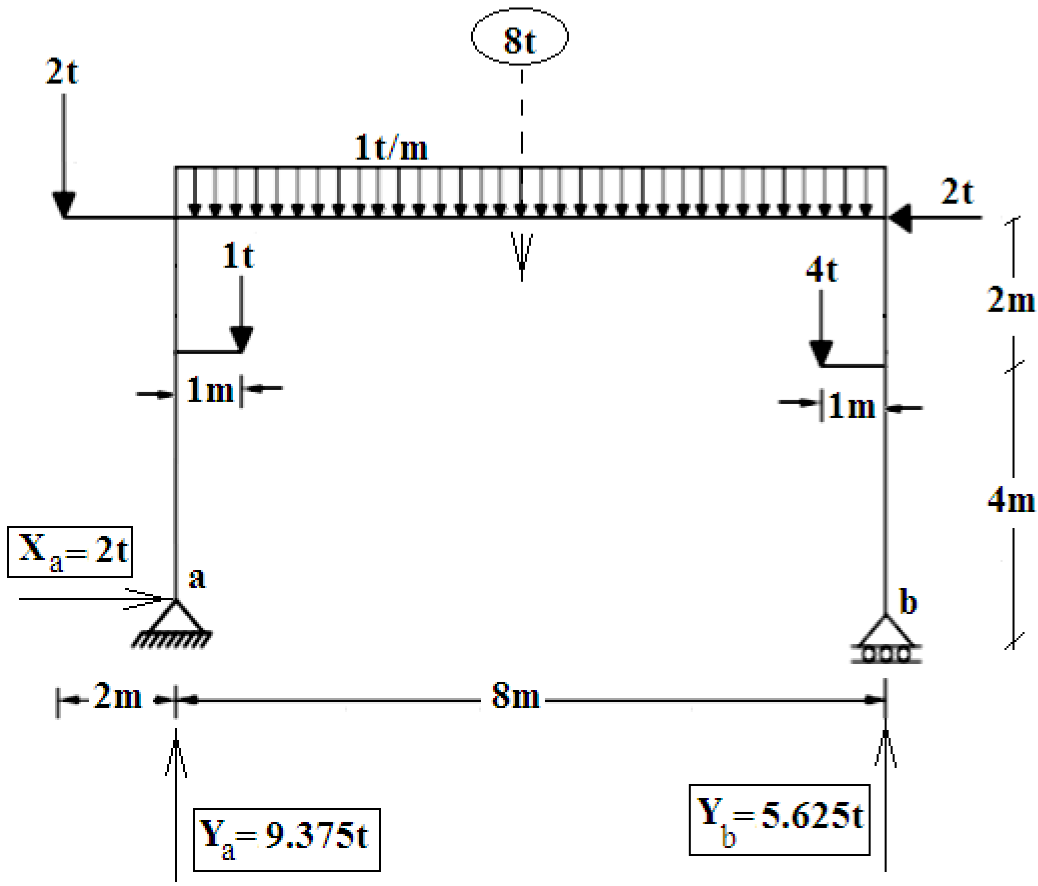


Fig. 2.19.b

Example (18)

For beam presented in Fig. 2.20, calculate the reactions at the supports under the applied load.

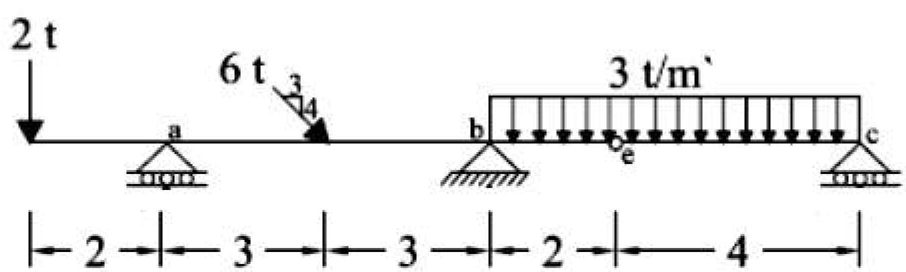


Fig. 2.20.a

Solution

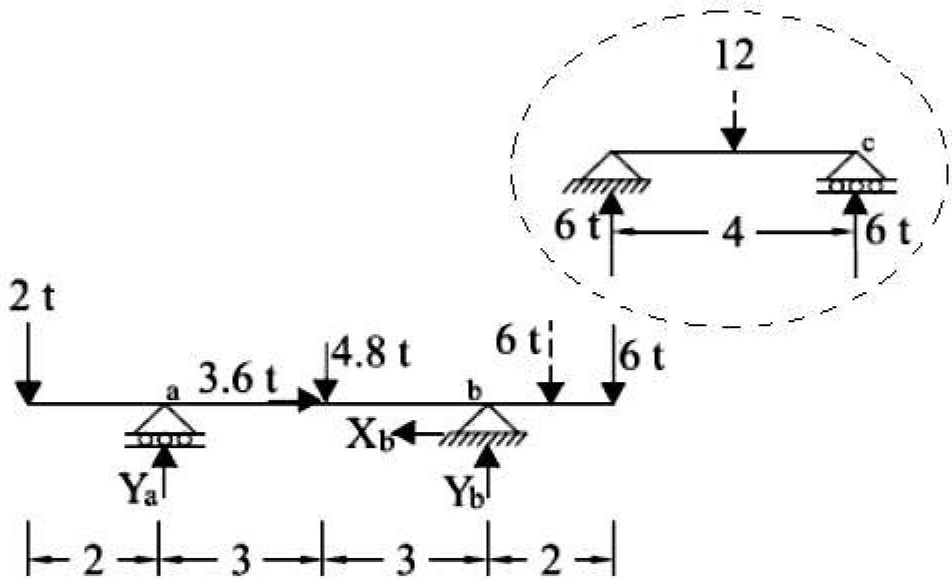


Fig. 2.20.b

- For part ab

$\sum M_b = 0.0 \qquad 6x2 + 6x1 - 4.8x3 - 2x8 + Y_a x6 = 0 \qquad Y_a = 2.07 \uparrow$

$\sum M_a = 0.0 \qquad 6x8 + 6x7 + 4.8x3 - 2x2 + Y_b x6 = 0 \qquad Y_b = 16.73 \uparrow$

$Check\ \sum F_y = 2 + 4.8 + 6 + 6 - 2.07 - 16.73 = 0 \ldots$ Ok

$\sum F_x = 0 \quad X_b = 3.6t \leftarrow$

Check For all structure

$\sum F_y = 2 + 4.8 + 6 + 12 - 2.07 - 16.73 - 6 = 0 \ldots$ Ok

---

Example (19)

For beam presented in Fig. 2.21, calculate the reactions at the supports under the applied load.

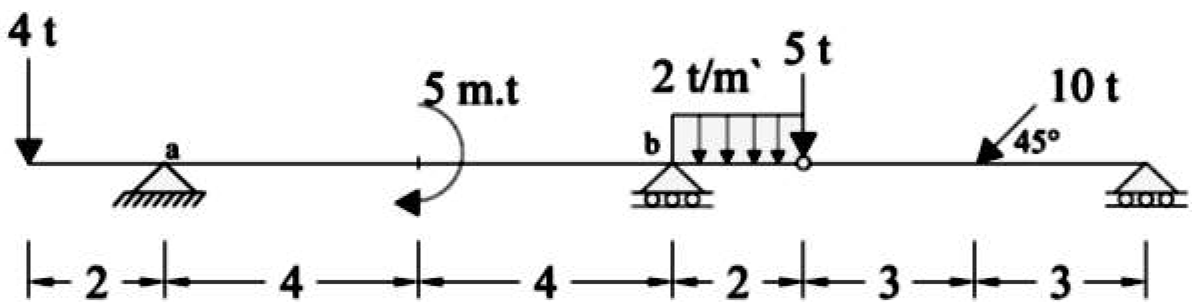


Fig. 2.21.a

Solution

- For part ab

$\sum M_b = 0.0 \qquad 5 + 4x1 + 8.53x2 - 4x10 + Y_a x8 = 0 \qquad Y_a = 1.74 \uparrow$

$\sum M_a = 0.0 \qquad 5 - 4x2 + 4x9 + 8.53x10 - Y_b x8 = 0 \qquad Y_b = 14.79 \uparrow$

$Check\ \sum F_y = 4 + 4 + 8.53 - 1.74 - 14.79 = 0 \ldots$ Ok

$\sum F_x = 0 \quad X_b = 7.07t \rightarrow$

Check For all structure

$\sum F_y = 4 + 4 + 7.07 + 5 - 14.79 - 1.74 - 3.54 = 0 \ldots$ Ok

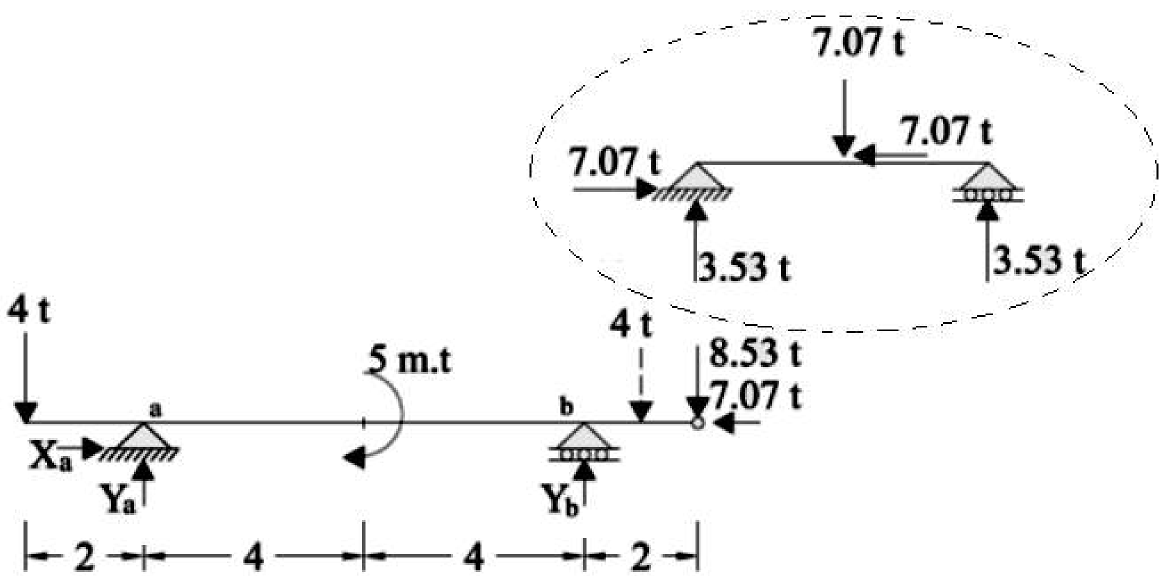


Fig. 2.21.b

Example (20)

For beam presented in Fig. 2.22, calculate the reactions at the two supports under the applied load.

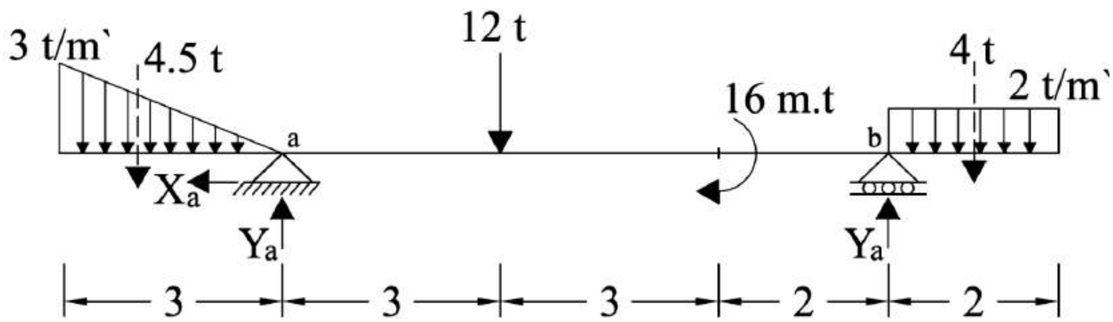


Fig. 2.22

$\sum F_x = 0 \quad X_a = 0$

$\sum M_b = 0.0 \quad 4x1 + 16 - 12x5 - 4.5x10 + Y_a x8 = 0 \quad Y_a = 10.625 \uparrow$

$\sum M_a = 0.0 \quad 12x3 + 16 + 4x9 - 4.5x2 - Y_b x8 = 0 \quad Y_b = 9.875 \uparrow$

$Check \sum F_y = 4.5 + 12 + 4 - 10.625 - 9.875 = 0 \ldots$ Ok

Example (21)

For beam presented in Fig. 2.23, calculate the reactions at the two supports under the applied load.

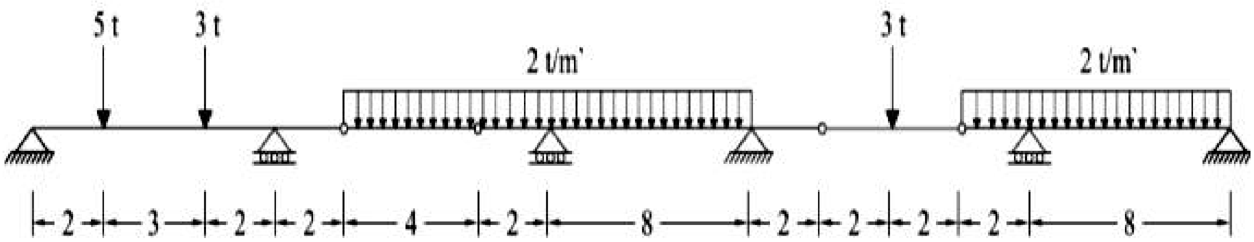
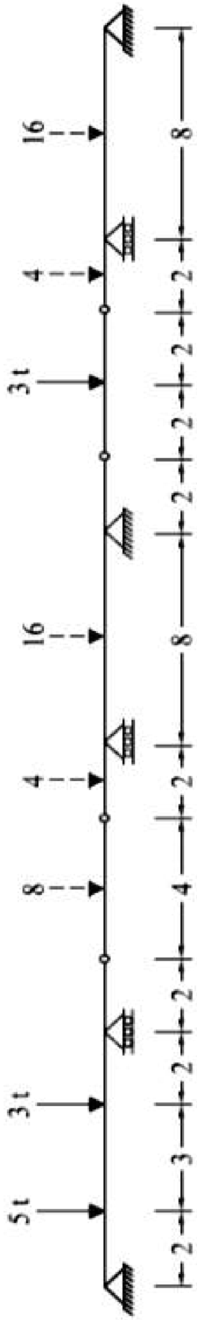
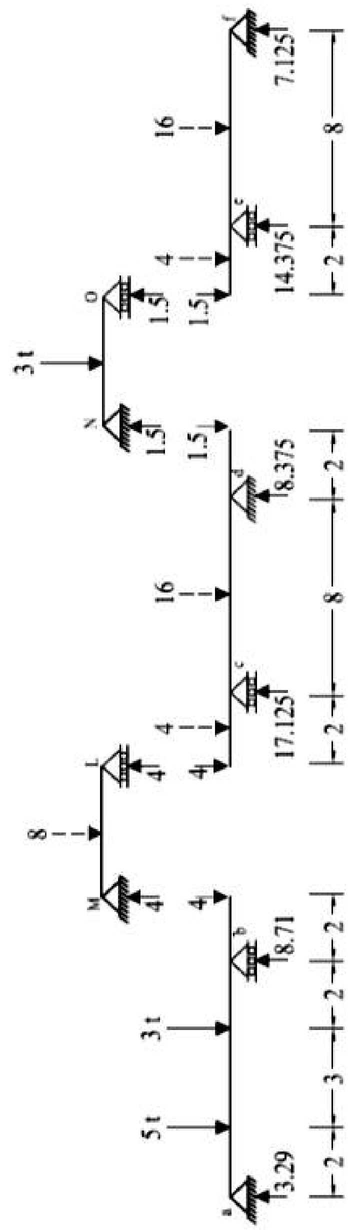


Fig. 2.23




3.29
8.71
17.125
8.375
14.375
7.125

Example (22)

For frame presented in Fig. 2.24, calculate the reactions at the two supports under the applied load.

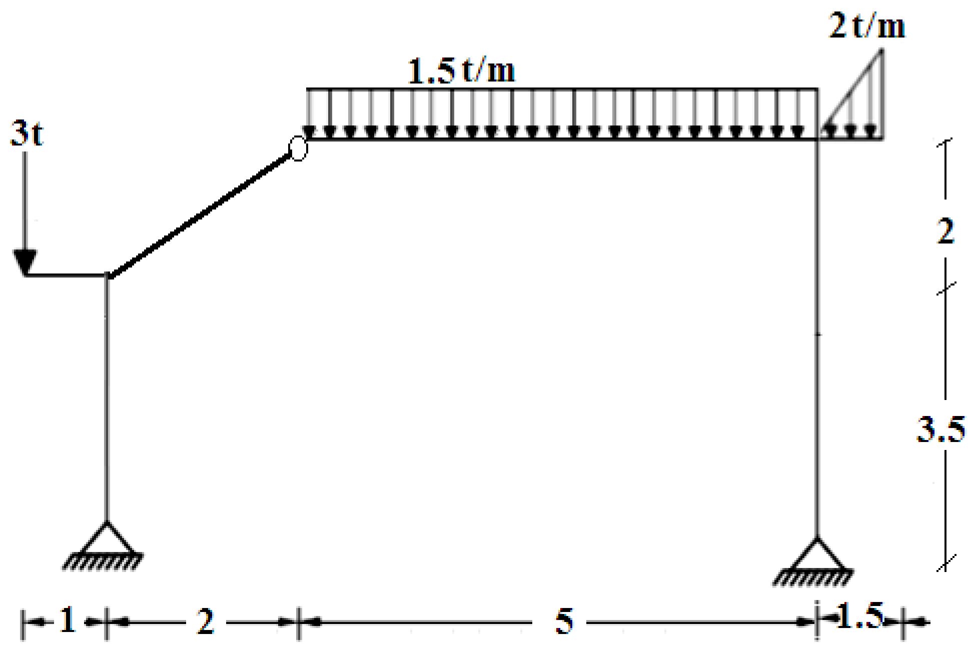


Fig. 2.24.a

$\sum M_b = 0.0 \qquad -7.5x2.5 - 3x8 + 1.5x1 + Y_a x7 = 0 \quad Y_a = 5.89t \uparrow$

$\sum M_a = 0.0 \qquad 7.5x4.5 + 1.5x8 - 3x1 - Y_b x7 = 0 \quad Y_b = 6.11t \uparrow$

$Check \sum F_y = 3 + 7.5 + 1.5 - 5.89 - 6.11 = 0 \ldots$ Ok

$\sum M_c = 0 \quad for\ left\ part$

$$-1(3x3 + X_a x5.5 + 5.89x2 = 0.0 \quad X_a = 0.505t \rightarrow$$

$\sum M_c = 0 \quad for\ right\ part$

$$7.5x2.5 + 1.5x6 - 6.11x5 + X_b x5.5 = 0.0 \quad X_b = 0.505t \leftarrow$$

Check… $\sum F_x = 0.0$

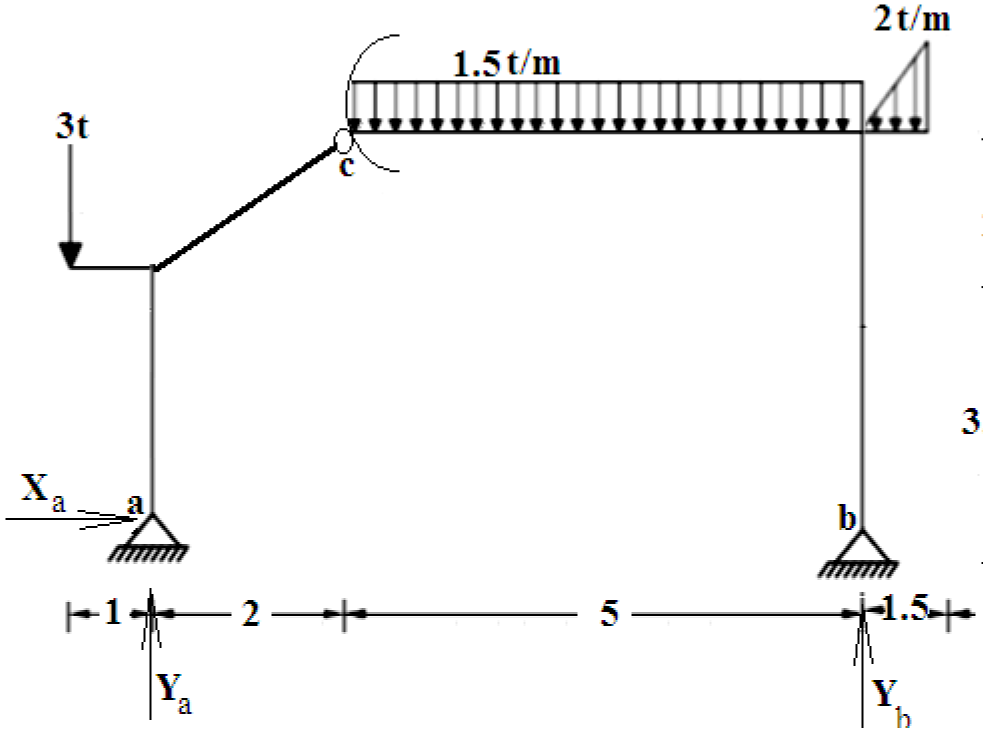


Fig. 2.24.b

Example (23)

For frame presented in Fig. 2.25, calculate the reactions at the two supports under the applied load.

Solution

$$\sum M_b = 0.0 \quad -20x5 - 7.07x14 + 7.07x5 - 20x20 + Y_a x18 = 0$$
$$Y_a = 31.31t \uparrow$$

$$\sum M_a = 0.0 \quad 7.07x4 + 7.07x5 + 20x13 - 20x2 - Y_b x18 = 0$$
$$Y_b = 15.76t \uparrow$$

$Check \sum F_y = 20 + 7.07 + 20 - 15.76 - 31.31 = 0 \ldots$ Ok

$\sum M_c = 0 \quad for\ left\ part$

$$-7.07x4 - 20x10 + 31.31x8 - X_a x5 = 0.0 \quad X_a = 4.44t \rightarrow$$

$\sum M_c = 0 \quad for\ right\ part$

$$20x5 - 15.76x10 + X_b x5 = 0.0 \quad X_b = 11.51t \leftarrow$$

$$Check \sum F_x = 4.44 + 7.07 - 11.51 = 0 \ldots \text{ Ok}$$

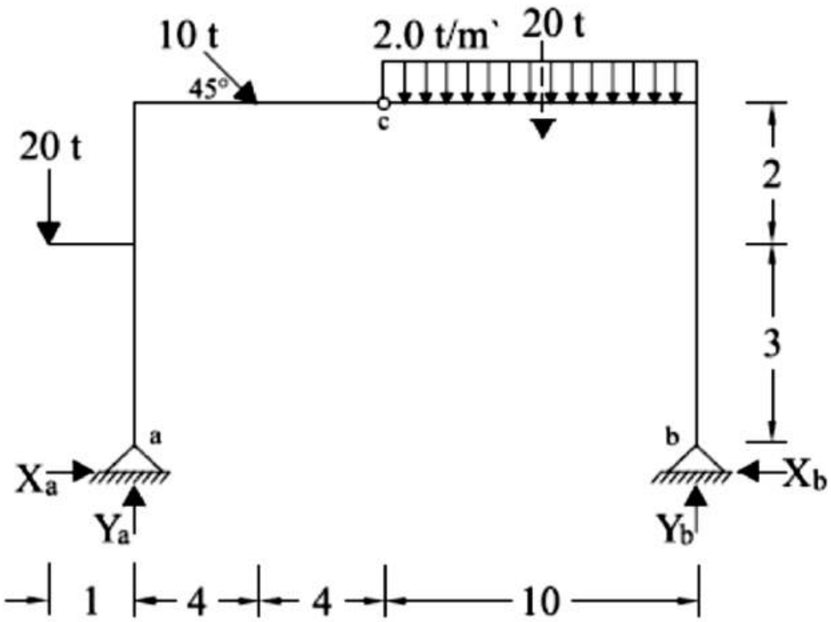


Fig. 2.25

Example (24)

For frame presented in Fig. 2.26, calculate the reactions at the two supports under the applied load.

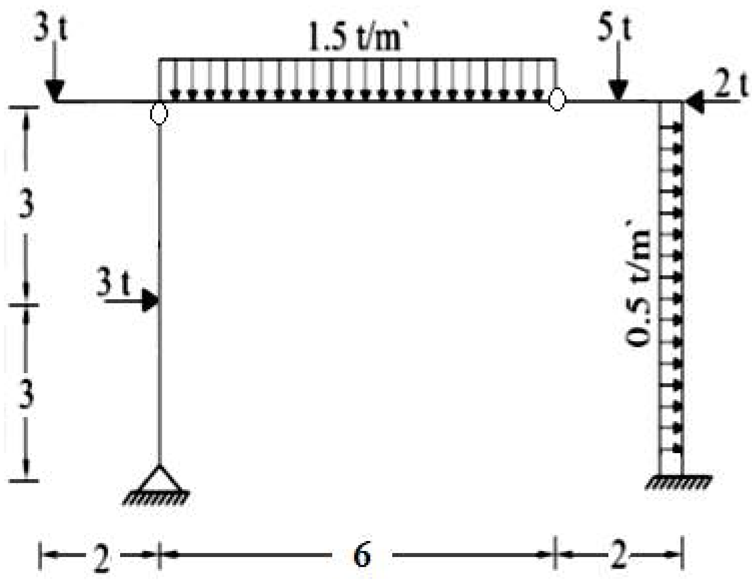


Fig. 2.26.a

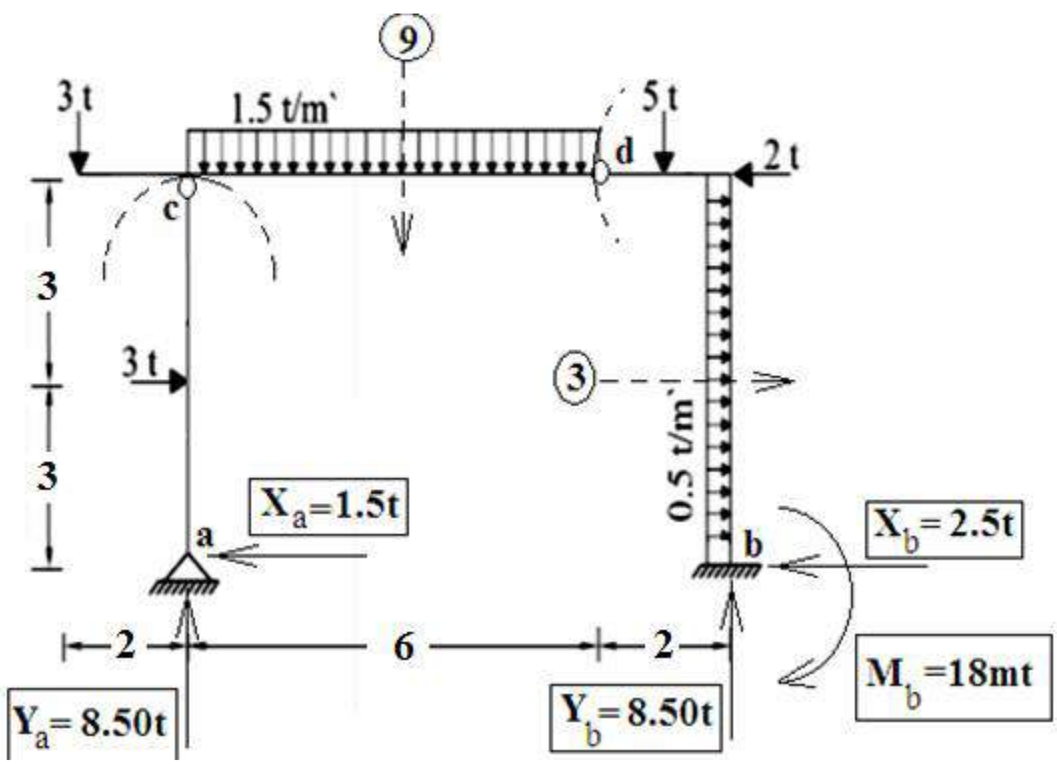


Fig. 2.26.b

$\sum M_c = 0 \quad for\ left\ part$

$$-3x3 + X_a x6 = 0.0 \quad X_a = 1.5t \leftarrow$$

$\sum M_d = 0 \quad for\ left\ part$

$$-(9x3 + 3x6 + 3x3 - 1.5x6) + y_a x6 = 0.0 \quad y_a = 8.50t \uparrow$$

$\sum F_x = 0 \qquad X_b = 2.50 \leftarrow$

$$\sum M_b = \sum M_b \ \ Fixed\ support$$

$$-(5x1 + 9x5 + 3x10 + 4x6 - 6x3 - 8.5x8) = M_b\ Fixed \quad M_b = 18mt$$

$\sum M_d = 0.0 \quad For\ right\ part$

$$5x1 - 3x3 + 0.5x6 + 18 - y_b x2 = 0.0 \quad y_b = 8.50t \uparrow$$

$Check\ \sum F_y = 5 + 3 + 12 - 8.5 - 8.5 = 0 \ldots$ Ok

Check $\sum F_c = 0.0$ for right part

$$= -3x2 + 9x3 + 5x7 - 3x3 - 8.5x8 + 0.5x6 + 18 = 0.0 \quad Ok$$

Example (25)

For frame presented in Fig. 2.27, calculate the reactions at the two supports under the applied load.

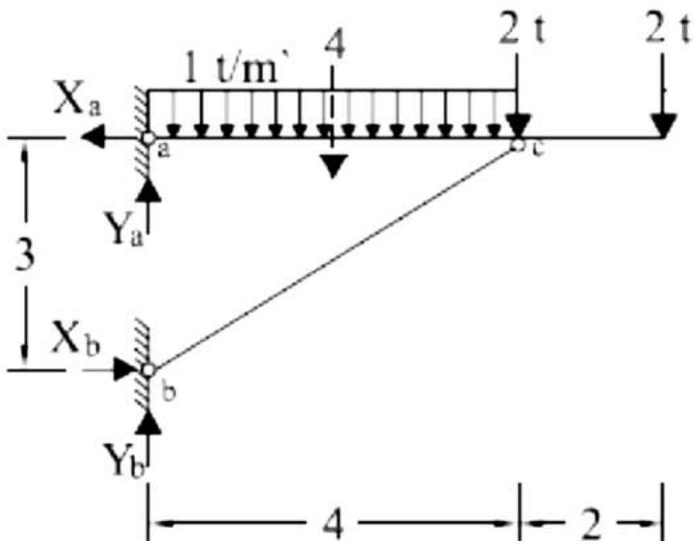


Fig. 2.27

Solution

$\sum M_b = 0.0 \quad 4x2 + 2x4 + 2x6 - X_a x3 = 0 \quad Y_a = 9.33t \leftarrow$

$\sum M_a = 0.0 \quad 4x2 + 2x4 + 2x6 - X_b x3 = 0 \quad X_b = 9.33t \rightarrow$

$Check\ \sum F_x = 0 \ldots$ Ok

$$\frac{9.33}{Y_b} = \frac{4}{3} \qquad Y_b = 6.9997 \cong 7t \uparrow$$

$F_y = 0.0 \quad 4 + 2 + 2 - 7 - Y_a = 0.0 \quad Y_a = 1.0t \uparrow$

Example (26)

For frame presented in Fig. 2.28, calculate the reactions at the two supports under the applied load.

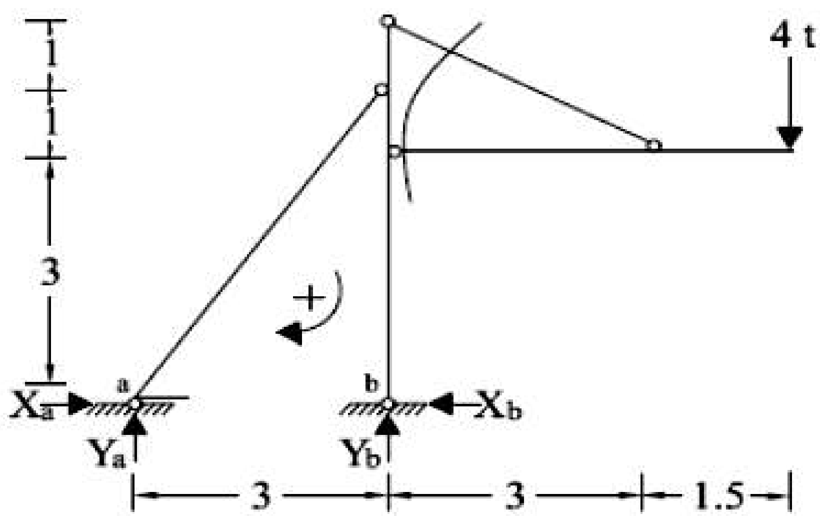


Fig. 2. 28

Solution

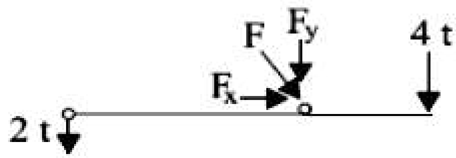
$\sum M_b = 0.0 \quad 4x4.5 - Y_a x3 = 0 \quad Y_a = 6t \downarrow$

$\sum M_a = 0.0 \quad 4x7.5 - Y_b x3 = 0 \quad Y_b = 10t \uparrow$

$Check\ \sum F_y = 4 + 6 - 10 = 0 \ldots$ Ok

$$\frac{X_a}{Y_a} = \frac{3}{4} \qquad X_a = 4.5t \leftarrow$$

$$\sum F_x = 0.0 \qquad X_b = 4.5t \rightarrow$$



$$\sum F_y = 0 \qquad F_y = 6\,\text{t} \uparrow \qquad \frac{F_x}{F_y} = \frac{3}{2} \qquad F_x = 9\,\text{t} \leftarrow$$

<u>Example (27)</u>

For frame presented in Fig. 2.29, calculate the reactions at the two supports under the applied load.

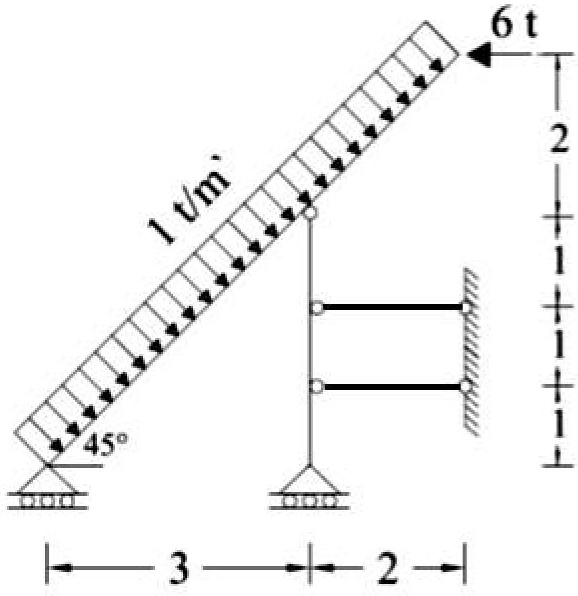


Fig. 2.29

$\sum M_f = 0 \quad for\ left\ part$

$6x1.5 + 6x1.5 - y_a x3 + 4x1 - 4x1 + 15x2 = 0.0 \quad y_a = 13.33t \uparrow$

$\sum F_y = 0 \qquad\qquad y_b = 3.33t \downarrow$

$\sum M_f = 0 \quad for\ right\ part$

$X_d x1 + X_c x2 = 0.0 \quad \gg\gg \quad X_d = -2X_c \qquad\qquad (1)$

$\sum F_x = 0 \quad \gg\gg\gg \quad -X_d - X_c - 5 = 0 \qquad\qquad (2)$

$By\ solving\ Eq.1\ and\ Eq.2 \qquad\qquad X_c = 5t \leftarrow \qquad X_d = 10t \rightarrow$

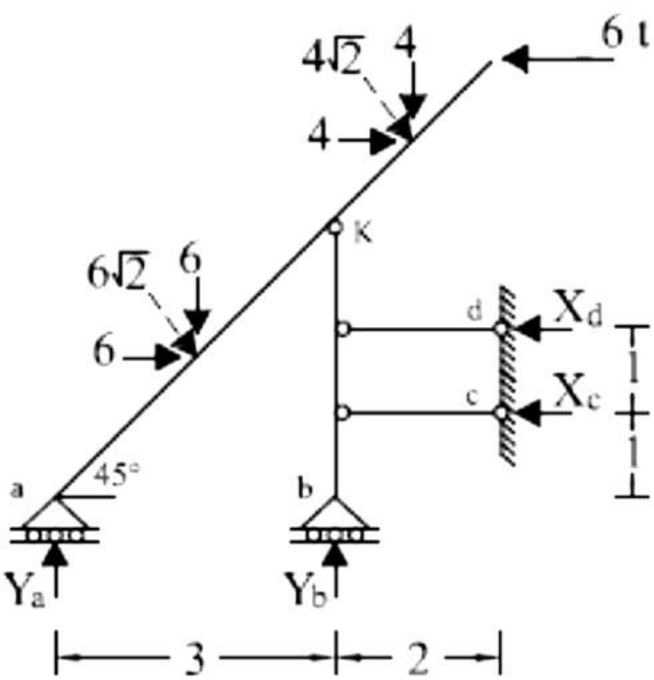


Fig. 2. 29.b

Example (28)

For frame presented in Fig. 2.29, calculate the reactions at the two supports under the applied load.

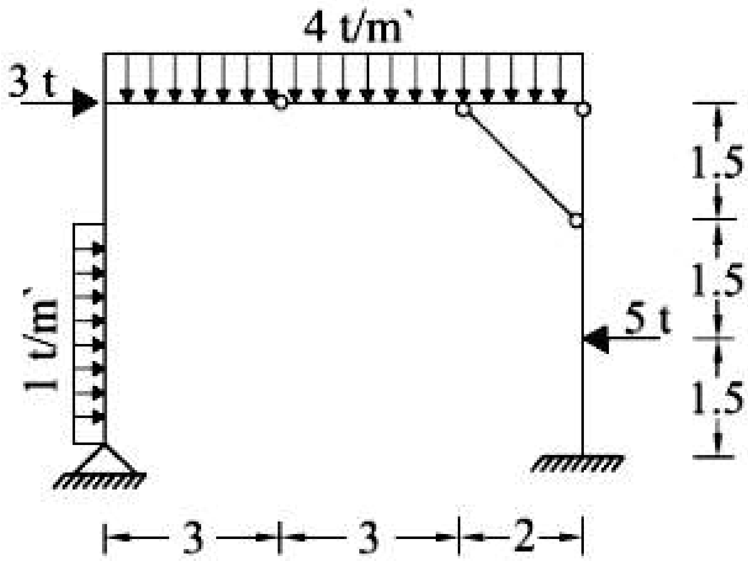


Fig. 2.29

Solution

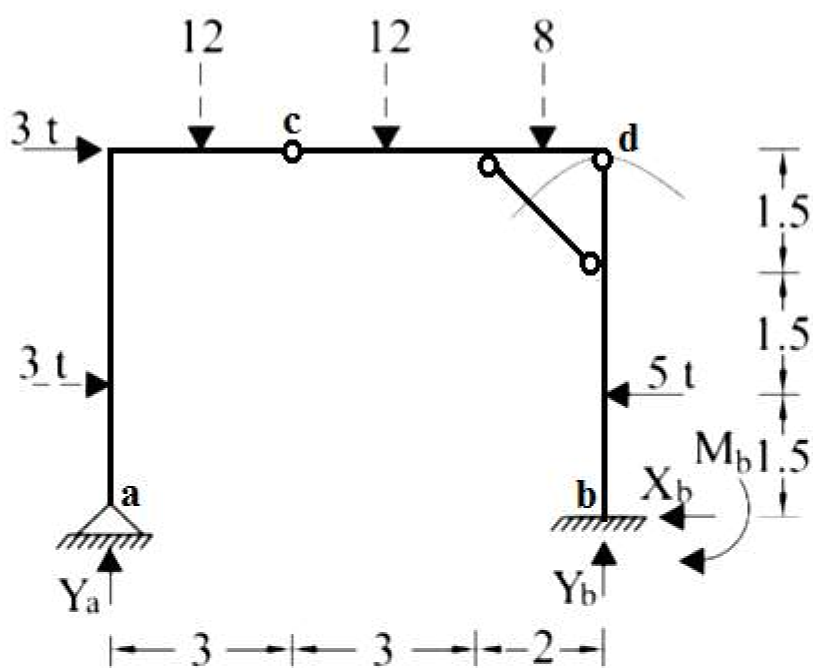


Fig. 2.29.b

$F_x = 0.0 \quad X_b = 1.0t \leftarrow$

$\sum M_c = 0.0 \quad For\ left\ part$

$-12x1.5 - 3x3 + Y_a x3 = 0.0 \qquad Y_a = 9.0t \uparrow$

$\sum F_y = 0.0 \qquad Y_b = 23t \uparrow$

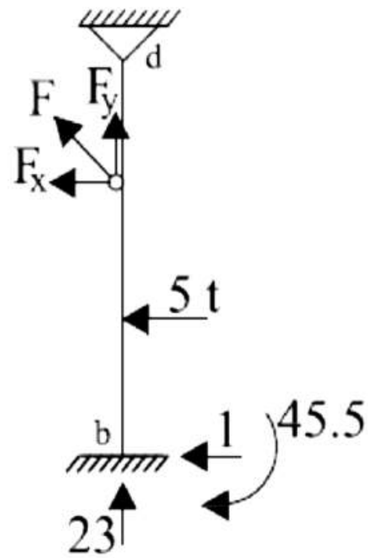
$M_a = 0.0\ \ for\ all\ structure$

$$12x1.5 + 12x4.5 + 8x7 + 3x4.5 + 3x4.5 + 3x1.5 - 5x1.5 - 23x8 + M_b = 0.0 \qquad M_b = 45.5mt$$

$Check \qquad \sum M_c = 0.0\ for\ right\ part \quad \ldots\ldots\ldots\ldots\ldots .Ok$

$\sum M_d = 0.0 \qquad F_x = 43.33t \rightarrow$

$\frac{F_x}{F_y} = \frac{2}{1.5} \qquad F_y = 32.5t \qquad F = 72.22t\ Comp$



Example (29)

For frame presented in Fig. 2.30, calculate the reactions at the two supports under the applied load.

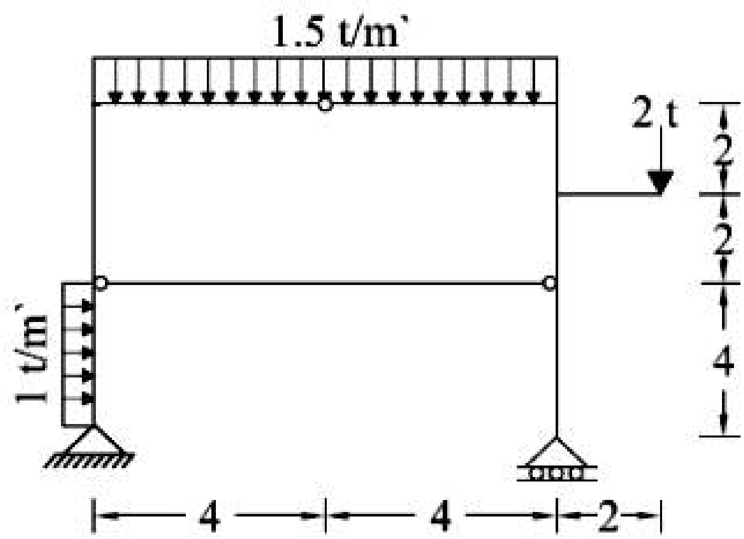


Fig. 2.30.a

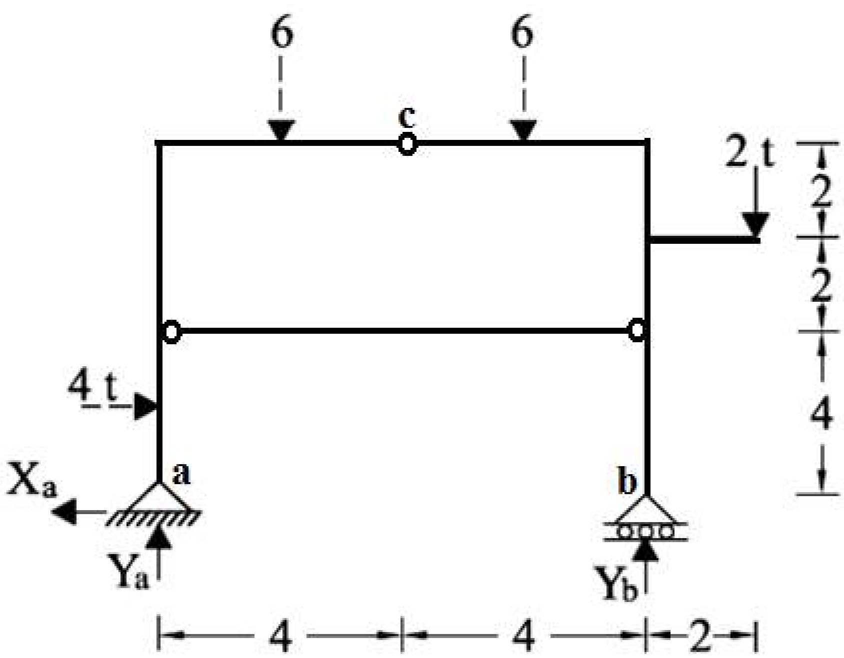
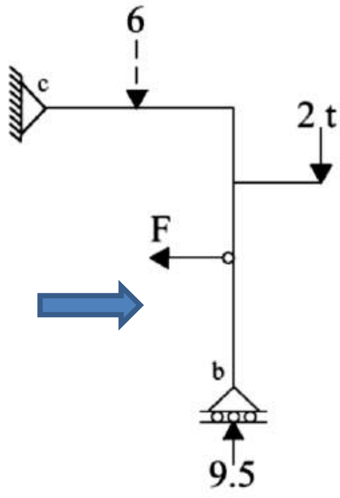


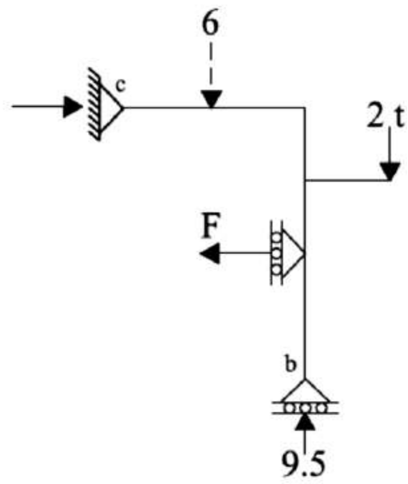
Fig. 2.30.b

$\sum F_x = 0.0 \qquad X_a = 4.0t \leftarrow$

$\sum \mathrm{M_a} = 0.0 \quad 6\mathrm{x}2 + 6\mathrm{x}6 + 2\mathrm{x}10| + 4\mathrm{x}2 - \mathrm{Y_b}\mathrm{x}8 = 0 \quad \mathrm{Y_b} = 9.5\mathrm{t} \uparrow$

$\sum M_b = 0.0 \quad 6x2 + 6x6 - 2x2 - 4x2 - Y_a x8 = 0.0 \quad Y_a = 4.5t \uparrow$





$\sum M_c = 0 \qquad 6x2 + 2x6 - 9.5x4 + Fx4 = 0.0 \qquad F = 3.5t \leftarrow \quad (tension)$

Example (30)

For beam presented in Fig. 2.31, calculate the reactions at the supports under the applied load.

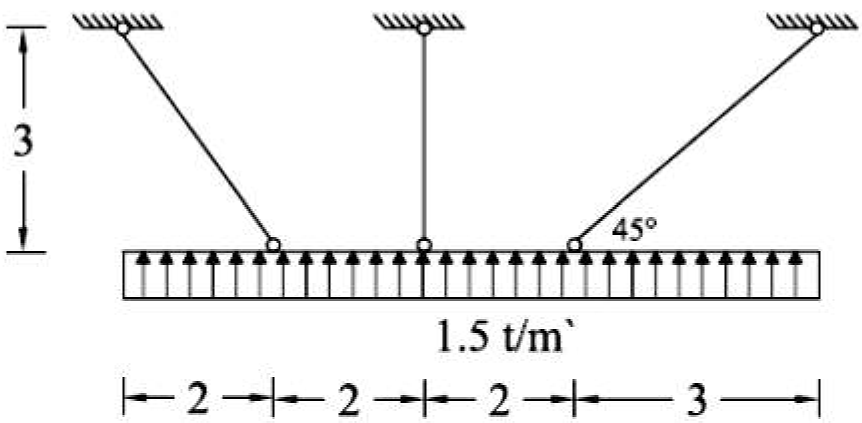


Fig. 2.31.a

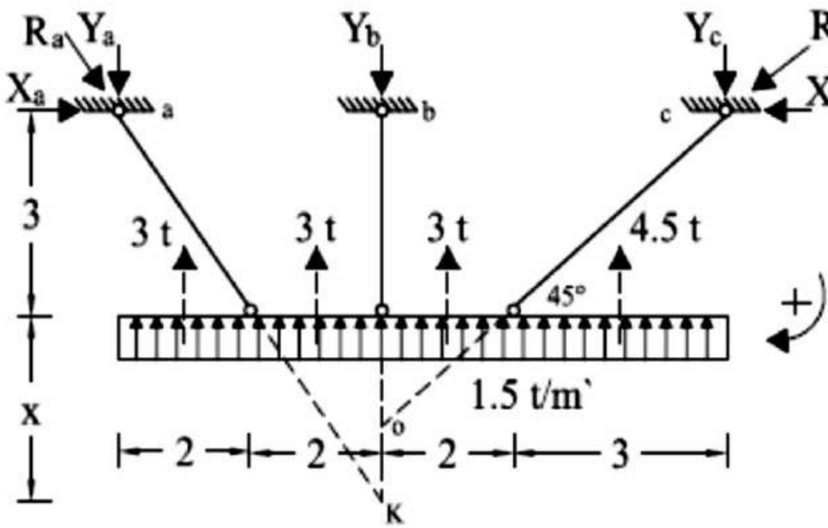


Fig. 2.31.b

$\sum M_o = 0.0$ $\quad 3x1 + 3x3 - 3x1 - 4.5x3.5 - Y_a x4 + X_a x5 = 0.0$

$-6.75 - 4xY_a + X_a x5 = 0.0$ $\quad Eq.1$

$\frac{X_a}{Y_a} = \frac{2}{3}$ $\quad X_a = \frac{2}{3} Y_a$ $\quad Eq.\ 2$

$By\ solving\ Eq.1\&2 \quad Y_a = 10.125t \uparrow \quad X_a = 6.75t \leftarrow$

$\sum M_c = 0.0$ $\quad -6.75 + 5xY_c - X_c 6 = 0.0 \quad Y_c = X_c$

$$-6.75 + (5 - 6)Y_c = 0.0$$

$$Y_c = 6.75t \uparrow \qquad X_c = 6.75t \rightarrow$$

$\sum M_a = 0.0$ $\quad -(3x1 + 3x3 + 3x5 + 4.5x7.5) + 4Y_b - 6.75x9 =$
0.0 $\quad Y_b = 30.375t \downarrow$

$Check$ $\quad \sum F_y = 0.0 \quad Ok$

Chapter (3)

INTERNAL FORCES IN THE BEAMS

3.1. Introduction

The internal forces are the forces that are created in the structures during transferring loads to the supports. Also these terms indicate the behaviour of the structures under the applied load. In the current chapter, the internal forces created in the beams are studied. As explained in the previous chapters, the beam is a component of the structures supports on the columns and carries the loads from the slab. The beam may be simple beam, cantilever, beam with overhang, compound and continuous beams. All the types of beams are indicated in Fig. 3.1.

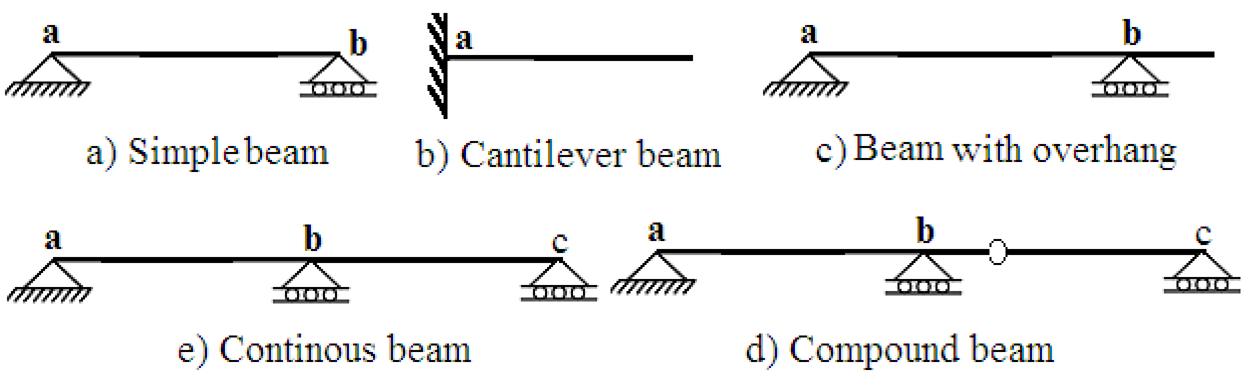


Fig. 3.1: Types of beam

From this figure, it can be seen that the simple beam is beam supported on two supports and the cantilever beam is supports on one support; fixed support. The beam with overhang is a beam with one or two cantilever. The compound beam is beam has intermediate hinges. The continuous beam has two or more span.

3.2. Type of the Internal Force

The loads on the beam transfer to the supports through the beam by three internal forces in the two dimensional analysis (2D). These internal forces are normal force, shear force and bending moment and these can be proved by considering a free body diagram of the beam as shown in Fig. 3.2. We take a section in the beam as shown in Fig. 3.2.c. This section must be in equilibrium so that horizontal force, shear force and bending moment must be created at the cutting section to maintain equilibrium as shown in Fig. 3.2.d. These forces are called the internal forces at this section.

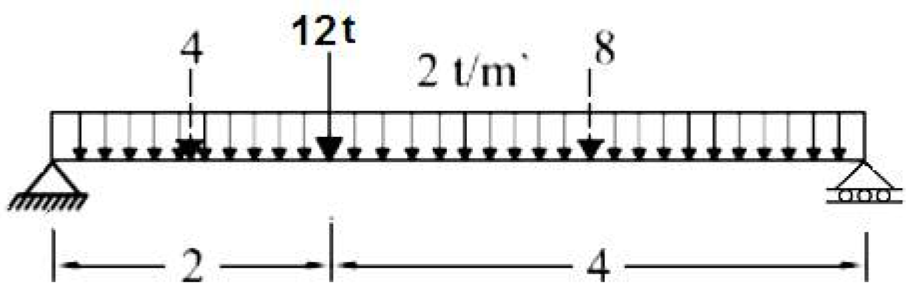


Fig. 3.2. a: Beam

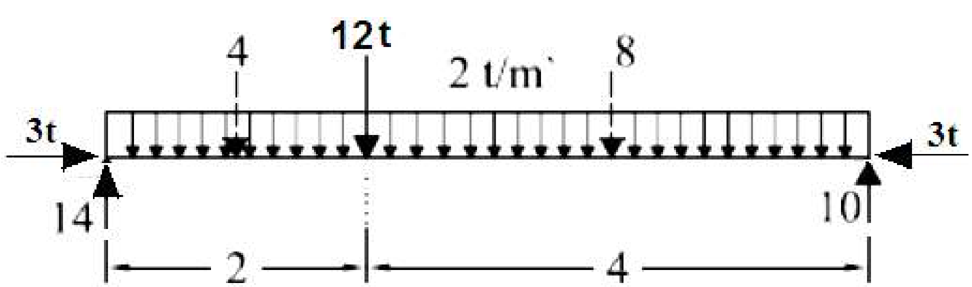


Fig. 3.2. b: FBD

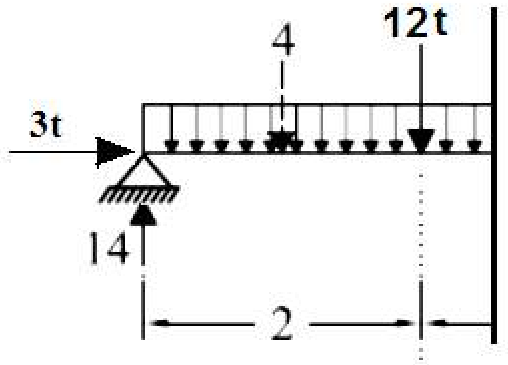


Fig. 3.2. c: Section

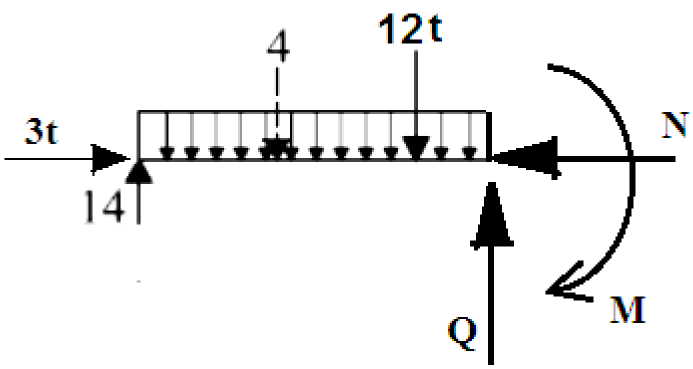
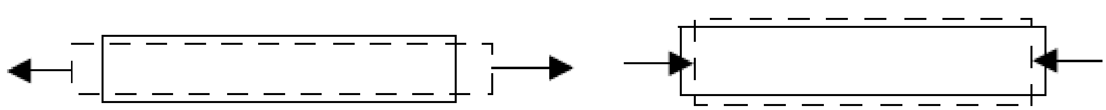


Fig. 3.2. d: Internal forces

The internal forces in the beam and the frame can be explained as below:-

1- Normal Force or Thrust (N)

It is the summation of all forces to the left or the right of the section parallel to the center line of the beam. It may be tension (+) or compression (-) as illustrated in Fig. 3.3.



aa) Tension force (+)

b) Compression force (-)

Fig. 3.3: The normal force

2- Shear Force

Shear force is created where two opposite forces try to cut tear or rip something in two (see Fig. 3.4). The shear force is numerically equal to the algebraic sum of all the vertical forces acting on the free body taken from either sides of the section. Also it can be defined as the summation of all the forces perpendicular to the center line of the beam.

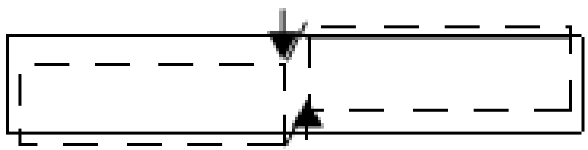


Fig. 3.4: Deformation of the shear force

3- Bending Moment

The bending moment is a term used to describe the force or torque. It is exerted on a material and leads to the event of bending or flexure within that material. The bending is measured in terms of force and distance. The member is being bent under the bending moment. Its deformation is characterized by a bent shape stretched and squashed at the same time as shown in Fig. 3.5.

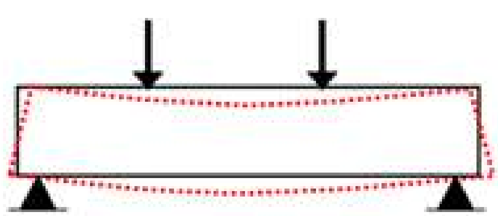


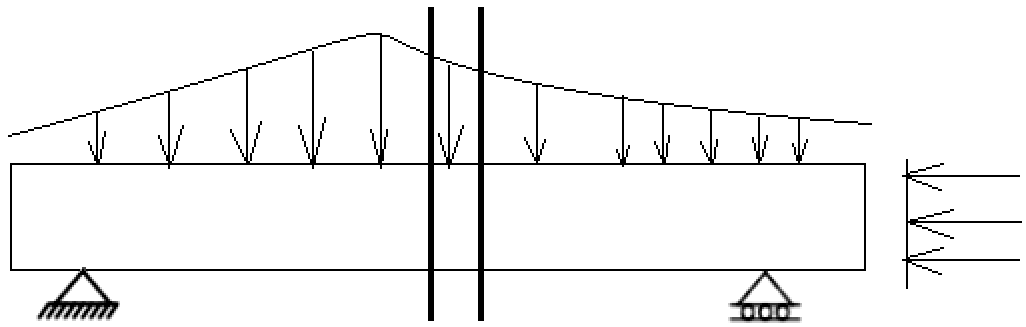
Fig. 3.5: Flexure of the beam

- *Normal Force Diagram* (NFD) indicates how a force applied parallel to the axis of a beam is transmitted along the length of that beam.
- *Shear Force Diagram* (SFD) indicates how a force applied perpendicular to the axis (parallel to cross section) of a beam is transmitted along the length of that beam.
- *Bending Moment Diagram* (BMD) shows how the applied loads to a beam create a moment variation along the length of the beam.

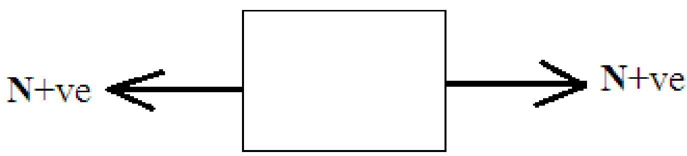
3.3. Sign Convention

Sin conventions of the internal forces that have been mostly commonly adopted in structural analysis are shown in Fig. 3.6 The normal force is considered tension force and is taken positive sign if the summation of the forces parallel the beam center line from left or right interred the cross section.

The shear force is considered to be positive if the sum of the force from the left is upward. Also the shear force is considered to be negative if the sum of the force from the left is downward or the sum of the force from the right is upward.



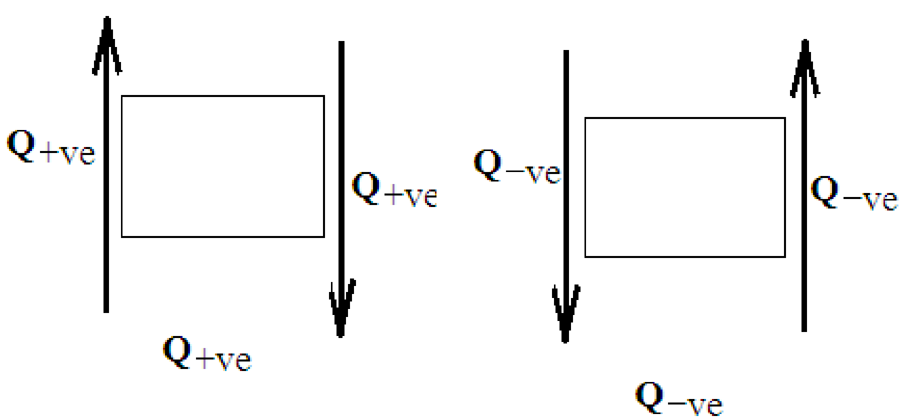
Section



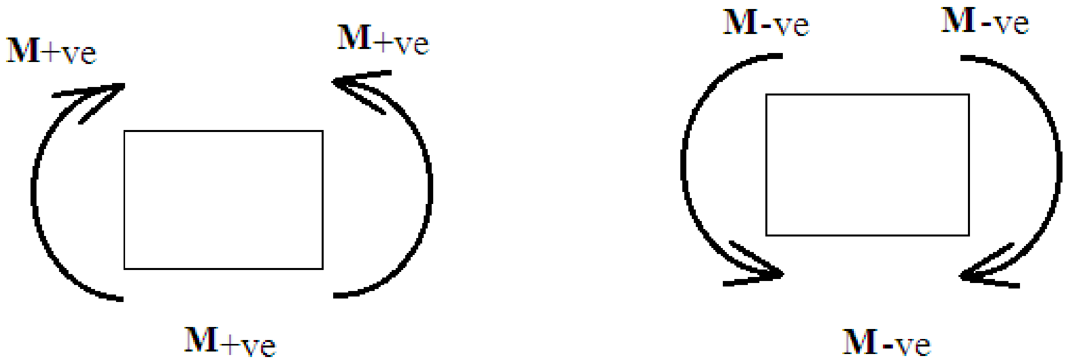
Tension

Compression

1) Normal Force



2) Shear Force



3) Bending Moment

Fig. 3.6: Sign convention of the internal forces

3.4. Relationships between the Applied Forces, Shear Force and Bending Moment

To improve these relationships, take a portion with distance dx from the simple beam shown in Fig. 3.7. By applying the equations of static equilibrium on this section, it can be concluded that:-

1- $\sum F_y = 0 \gg Q - pdx - (Q - Qdx) = 0 \therefore \frac{dQ}{dx} = -p \ldots\ldots\ldots\ldots\ldots\ldots (1)$

This means that the slope of shearing force diagram at any point is equal to the intensity of the load at that point (p t/m). From equation (1), it can be noted that

- ❖ For the portion of the beam with no loads, the slop of shear force diagram is zero.
- ❖ For the portion of the beam in which the applied load is uniform load, the slop of shear force diagram is constant.

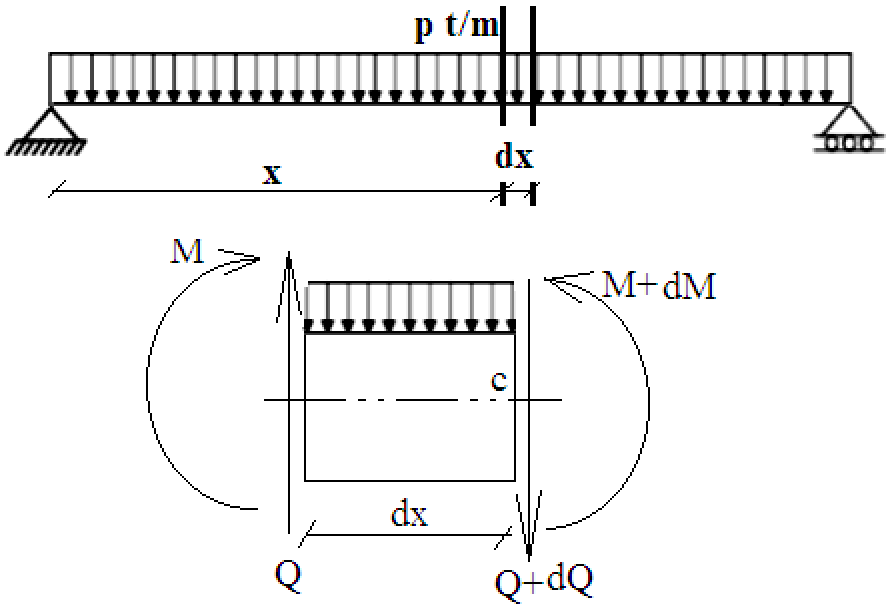


Fig. 3.7

$$2\text{-}\ \sum M_c = 0 \ \ \ggg M + Qdx - pdx\frac{dx}{2} - (M + dM) = 0 \ldots\ldots\ldots\ldots (2)$$

$$By\ neglecting\ the\ tearm\ pdx\frac{dx}{2}\ becouse\ it\ is\ very\ small,$$

$$it\ can\ be\ found\ that$$

$$\frac{dM}{dx} = Q \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

This means that the slope of the bending moment diagram at any point equals to the shearing force at that point. From equation (3), it can be noted that

- The bending moment diagram will be a straight line in the portion of the beam in which the shear force diagram is constant.
- The bending moment diagram will be curved in the portion of the beam in which the shear force diagram varies in any manner.
- At a point of zero shears, the maximum bending moment occurs.

3.5. Solved Examples

Example (1)

For the given simple beam in Fig. 3.8, draw the normal force, shear force and bending moment diagram due to the applied load.

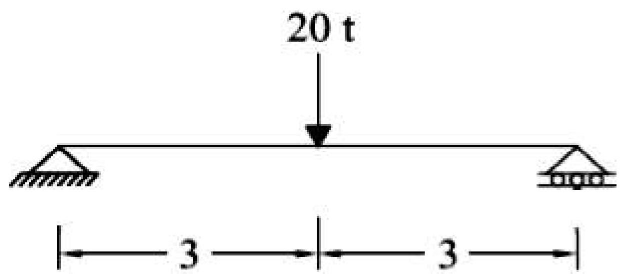


Fig. 3.8.a

Solution

- *At the first, find the reactions at the two supports*

The horizontal reaction at a =0 and the vertical reactions at a&b are 10t

- *At the second, draw normal force.*

The normal force is the summation of all the forces parallel to the axis of the centre line of the beam and it is zero.

- *At the third, draw shear force as the bellow steps:-*

1- Start the Shear Force Diagram at the first value of the force acting on the beam point#a. In this case it is a +10 t due to the reaction at point a.
2- Keep moving across the beam, stopping at every load that acts on the beam.
3- Under the concentrated load, a negative 20t force comes. We will minus 20 t from the existing 10t (10t – 20t = -10t).

4- Moving across the beam again, we come to another force; a positive 10 t (reaction at support b). Again, add this +10t to the shear force diagram (-10t). This force makes zero shear force at the end of the beam. It is the final Shear Force Diagram (SFD).

➢ *At the fourth, draw bending moment diagram as the bellow steps:-*

1- Start the bending moment diagram at the support a. At this joint, the bending moment equals zero.

2- Calculate the moment under the applied load. The moment $=Y_a x3=30mt$.

3- Join the calculated moment with the moment at a.

4- The end moment (at point#b) is zero.

5- Join the moment at joint#b with the moment under the concentrated load.

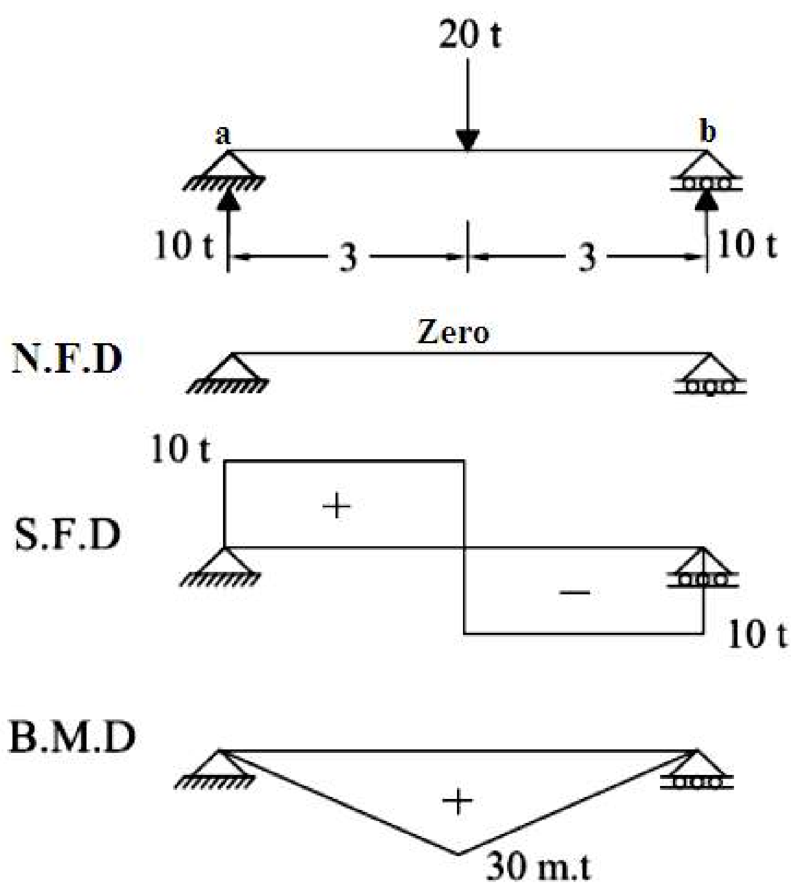


Fig. 3.8.b

Example (2)

For the given simple beam in Fig. 3.9, draw the normal force, shear force and bending moment diagram due to the applied load.

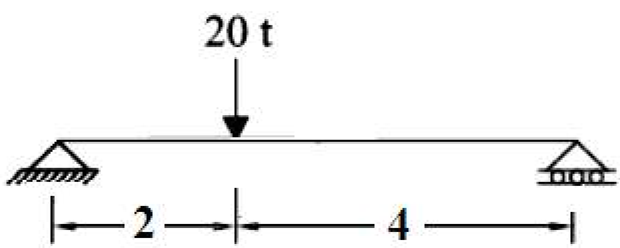


Fig. 3.9.a

Solution

- *Finding the reactions*

a) $\sum F_x = 0 \ggg\ggg X_a = 0$

b) $\sum M_a = 0 \ggg\ggg 20x2 - Y_b x6 = 0 \quad \therefore Y_b = \frac{40}{6} = 6.67t \uparrow$

c) $\sum M_b = 0 \ggg\ggg 20x4 - Y_a x6 = 0 \quad \therefore Y_a = \frac{80}{6} = 13.33\ t \uparrow$

d) *Check* $\sum F_y = 20 - 6.67 - 13.33 = 0 \ggg\ggg$ o k

- *For drawing B.M.D*

M_1 from left side = 13.333×2= 26.67 m.t

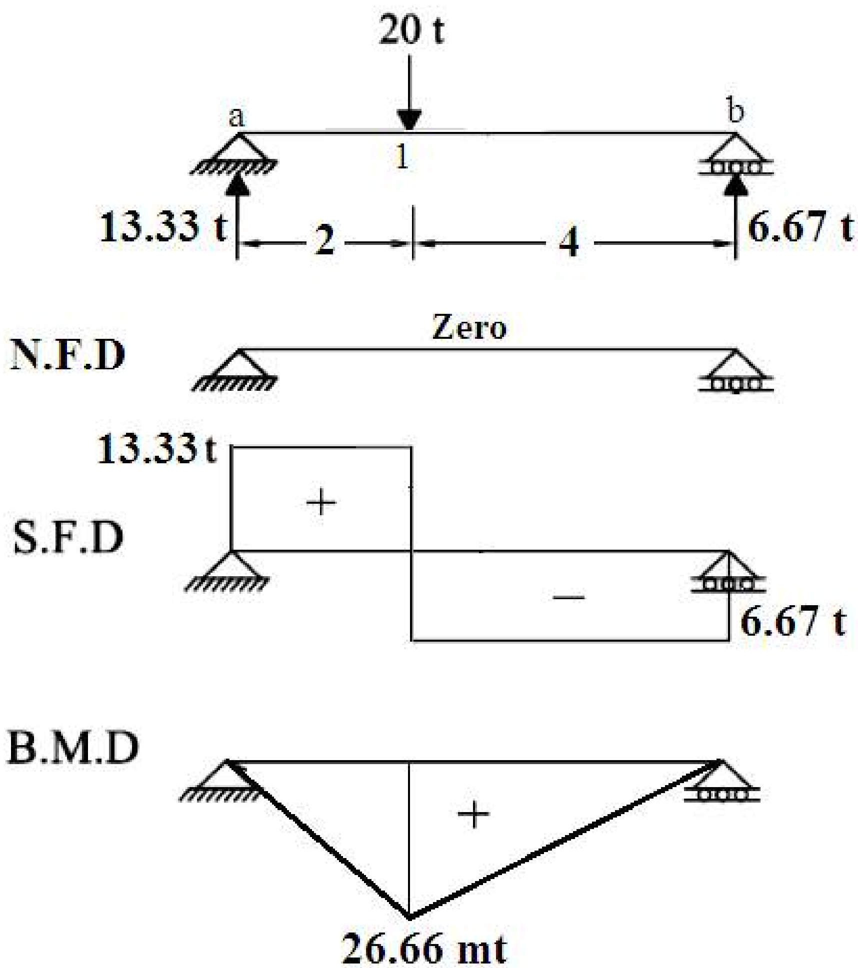


Fig. 3.9.b

<u>Example (3)</u>

Draw the normal force, shear force and bending moment diagrams due to the applied load for the given simple beam in Fig. 3.10.

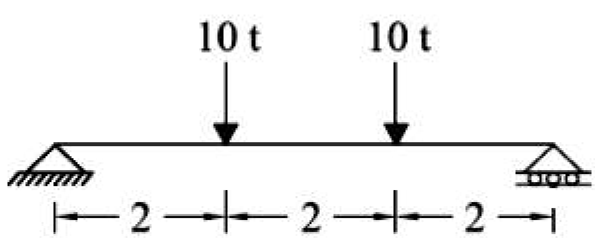


Fig. 3.10.b

<u>Solution</u>

- *Finding the reactions*

a) $\sum \mathrm{F_x} = 0 \ggg\ggg \mathrm{X_a} = 0$

b) $\sum M_a = 0 \ggg\ggg 10x2 + 10x4 - Y_b x6 = 0 \therefore Y_b = \frac{60}{6} = 10\ t \uparrow$

c) $\sum M_b = 0 \ggg\ggg 10x2 + 10x4 - Y_a x6 = 0 \therefore Y_a = \frac{60}{6} = 10\ t \uparrow$

d) <u>*Check*</u> $\sum \mathrm{F_y} = 10 + 10 - 10 - 10 = 0 \ggg\ggg$ ok

- *For drawing B.M.D*

M_1 from left side = 10×2= 20 m.t

M_2 from left side = 10×4-10x2= 20 m.t

or M_2 from right side = 10×2= 20 m.t

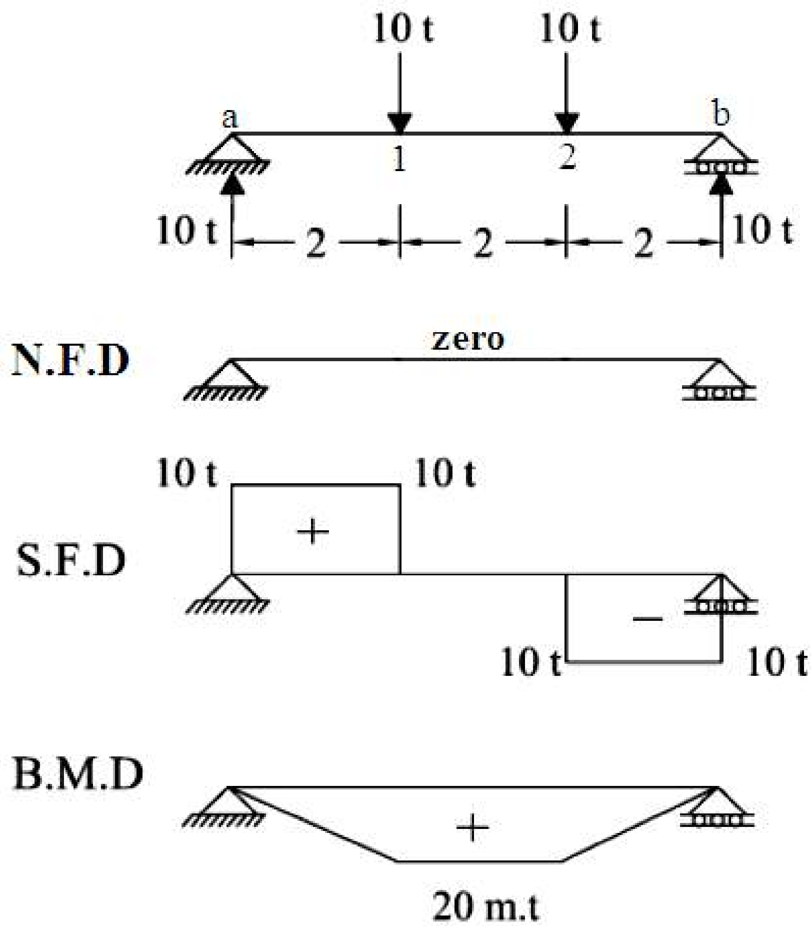


Fig. 3.10.b

Example (4)

Draw the normal force, shear force and bending moment diagrams due to the applied load for the given cantilever in Fig. 3.11.

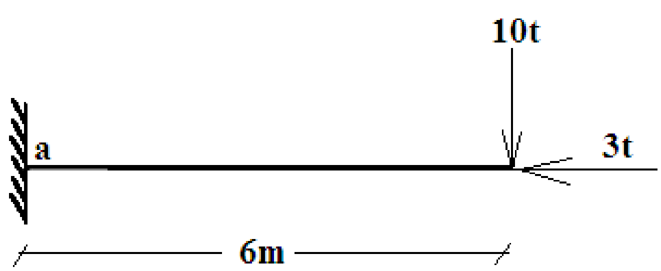


Fig. 3.11.a

Solution

➤ *Finding the reactions*

The fixed support has three reactions Y_a, X_a and M_a

a) $\sum F_x = 0 \ggg\ggg X_a = 3 \rightarrow$

b) $\sum M_{at\ a} = 0 \ggg\ggg 10x6 - M_a = 0 \therefore M_a = 60mt$ ↺

c) $\sum F_y = 10 - Y_a = 0 \gg Y_a = 10t \uparrow$

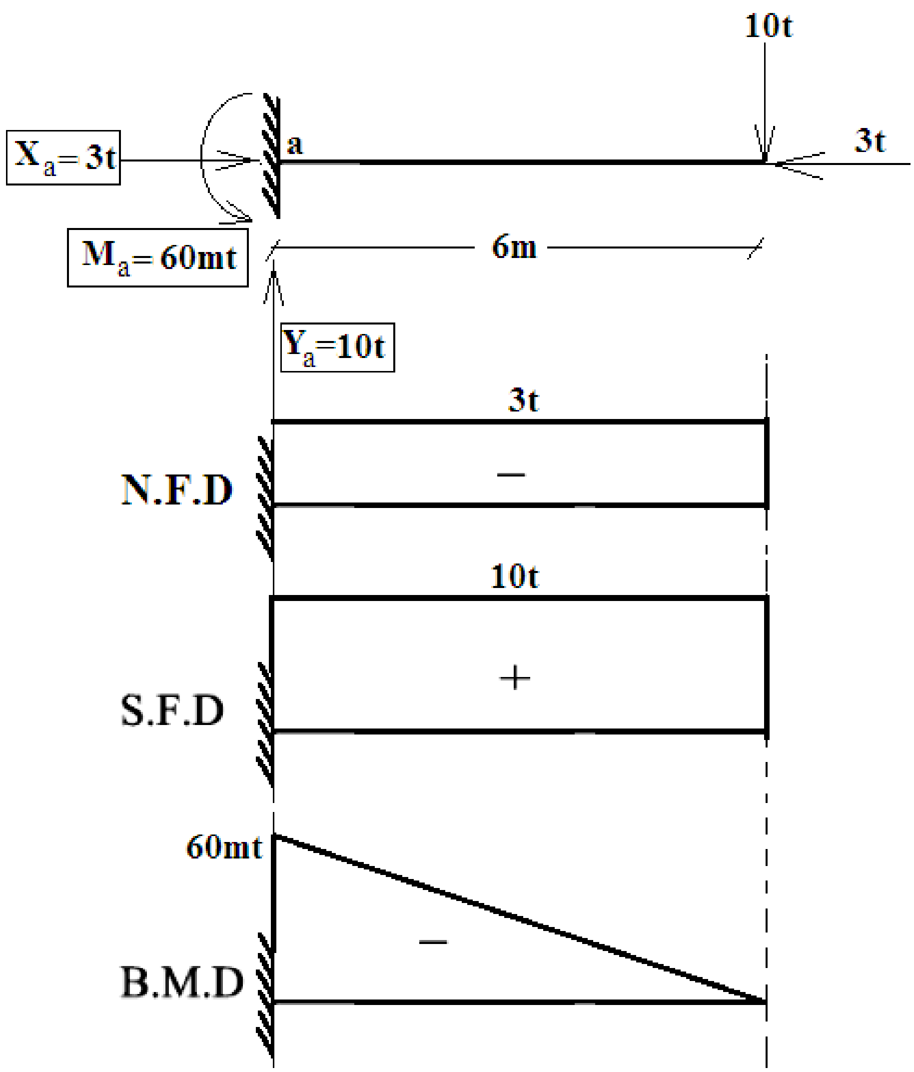


Fig. 3.11.b

Example (5)

Draw the normal force, shear force and bending moment diagrams due to the applied load for the given cantilever in Fig. 3.12.

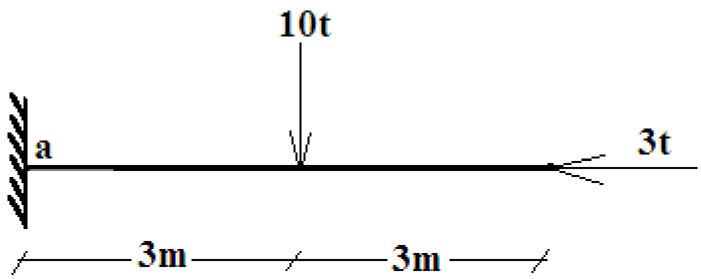


Fig. 3. 12.a

Solution

- *Finding the reactions*

The fixed support has three reactions Y_a, X_a and M_a

a) $\sum F_x = 0 \gg\gg\gg X_a = 3 \rightarrow$

b) $\sum M_{at\ a} = 0 \gg\gg\gg 10x3 - M_a = 0$

$$\therefore M_a = 30mt$$

c) $\sum F_y = 10 - Y_a = 0 \gg Y_a = 10t\ \uparrow$

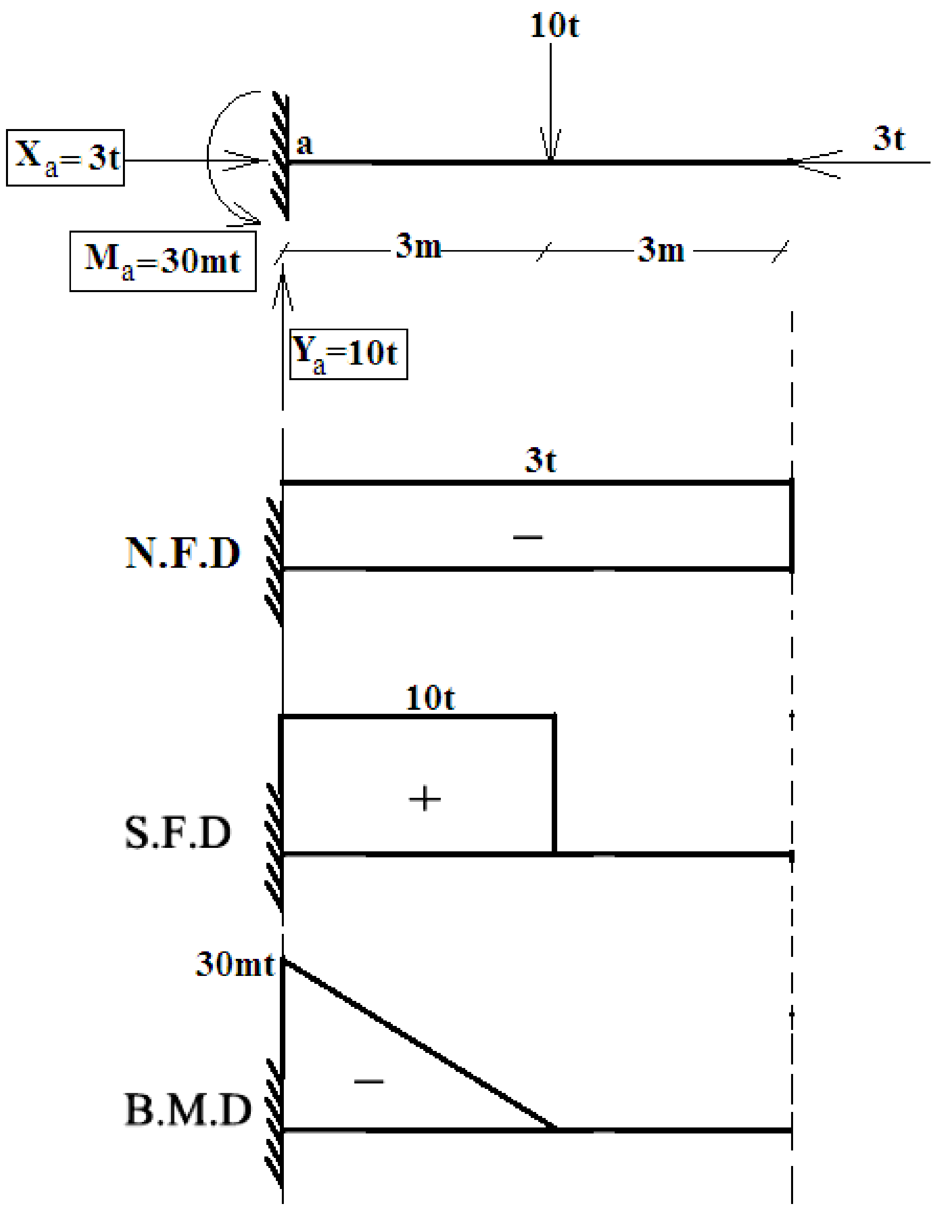


Fig. 3.12.b

Example (6)

The given simple beam in Fig. 3.13 subjected to uniform load w, draw the normal force, shear force and bending moment diagram due to the applied load.

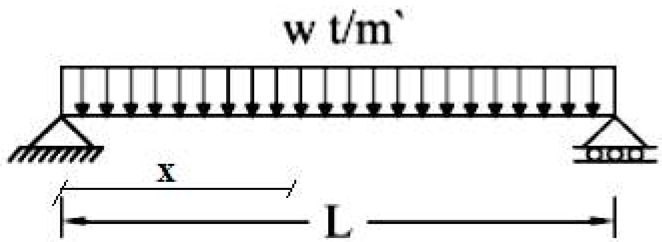


Fig. 3.13.a

Solution

1- Calculate the reactions at the two supports due to the uniform loads

$$Y_a = Y_b = \frac{wL}{2} \text{ due to symetry}$$

2- Normal forces equal zero at any point of the beam due to the absent of the forces that are parallel to the center line of the beam.

3- Shear force diagram

- At point a, the shear force equals the reaction Y_a
- At any joint of the beam with distance x from point a , the shear force can be calculated from below equation:-

$$Q_x = \frac{wL}{2} - wx \ldots \ldots (a)$$

4- Bending moment diagram

- At point a and b, the bending moment equal zero
- At any joint of the beam with distance x from point a, the shear force can be calculated from below equation:-

$$M_x = Y_a x - \frac{wx^2}{2} \ldots \ldots (b)$$

It can be noted that the maximum bending moment at shear force=0 at the mid-span and it equals $\frac{wL^2}{8}$

- From Eq. b, it can be observed that the bending moment diagram on the beam under uniform load is second order parabola.
- To draw this diagram, flow the below steps:-
  a) Draw a line from the beam mid-span perpendicular to the center line of the beam with $wL^2/4$ length to give point c.
  b) From the half of the drawn line in step (a), draw a line parallel to the beam center line. This line is called tangent#1.
  c) Draw a line passes through the start point (a) and the given point (c) in the first step and other line passes through the end point (b) and point (c). The first line is called tangent#2 and the second is called tangent#3.
  d) Draw parabola tangent the three tangents.

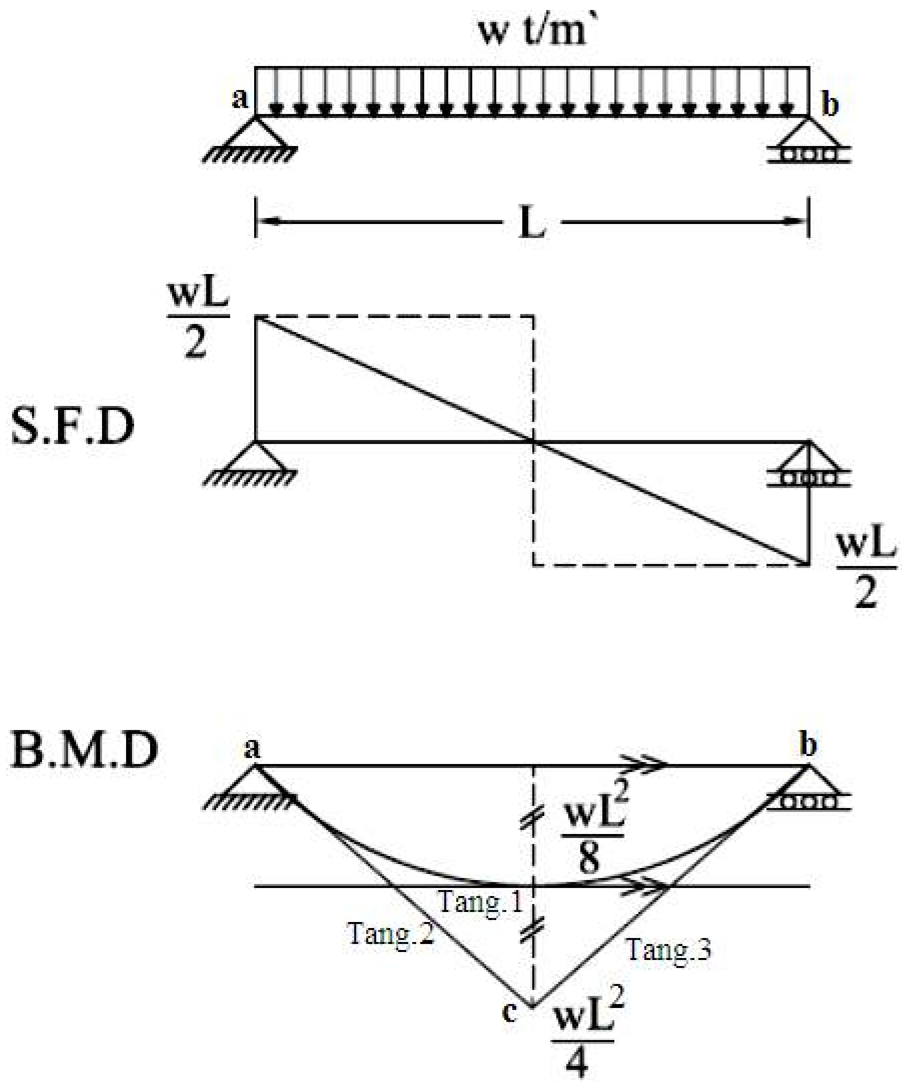


Fig. 3.13.b

Example (7)

The given simple beam in Fig. 3.14 subjected to triangle load w, draw shear force and bending moment diagram due to the applied load.

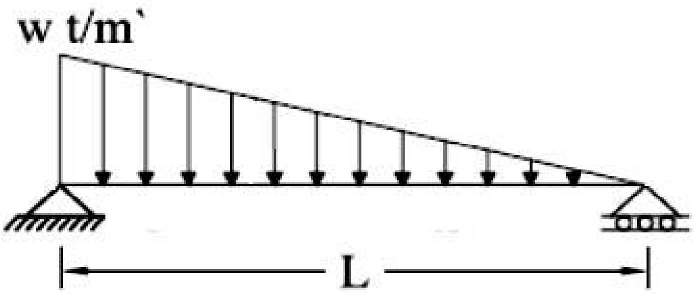


Fig. 3.14.a

<u>*Solution*</u>

- Calculate the reactions at the two supports

a) $\sum \mathrm{F_x} = 0 \ggg\ggg\ggg \mathrm{X_a} = 0$

b) $\sum M_a = 0 \ggg\ggg\ggg \frac{WL}{2} x \frac{L}{3} - Y_b x L = 0 \gg \mathrm{Y_b} = \frac{\mathrm{wL}}{6}$

c) $\sum M_b = 0 \ggg\ggg\ggg \frac{WL}{2} x \frac{2L}{3} - Y_a x L = 0 \gg \mathrm{Y_a} = \frac{\mathrm{wL}}{3}$

d) <u>*Check*</u> $\sum \mathrm{F_y} = \frac{\mathrm{wL}}{2} - \frac{\mathrm{wL}}{6} - \frac{\mathrm{wL}}{3} = 0 \ggg\ggg\ggg$ ok

- *To draw the Shear Force Diagram, flow the below steps*:-

a) Draw a line at point (a) perpendicular to the center line of the beam with $\mathrm{Y_a}$ length (upward or downward) to get point (y). Point (a) is at the base of the triangle load.

b) Draw a dashed line parallel to the beam center line from point (y). This dashed line continues till the concentrated point of the triangle load.

c) Upward or downward with value of wL/2. Then draw a dash line#2 till the support b (head of triangle load).

d) Divide the dashed line#2 to get point x.

e) Draw inclined line pass through point x and point y.

f) Draw parabola tangent the inclined line and the dashed line#2.

- *To draw the bending moment diagram, flow the below steps:-*

a) Draw a vertical line from the concentrated point of the triangle load perpendicular to the center line of the beam with $wL^2/9$ length to give point c.

b) Draw a line parallel to the beam center line with $wL^2/9\sqrt{2}$ distance on the vertical line. This line is called tangent#1.

c) Draw one line passes through the start point (a) and the given point (c) in the first step and other line passes through the end point (b) and point (c). The first line is called tangent#2 and the second is called tangent#3.

d) Draw parabola tangents the three tangents.

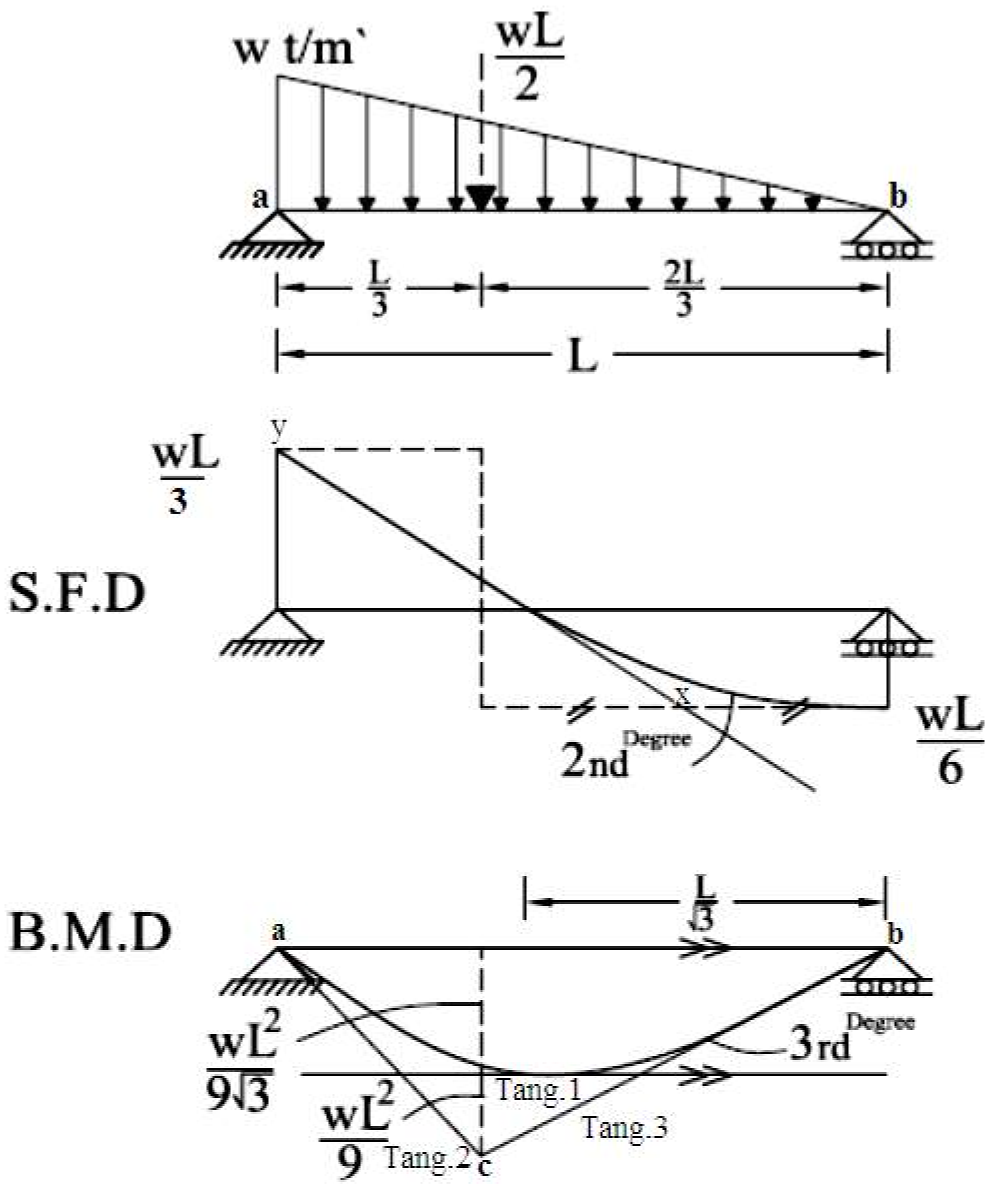


Fig. 3.14.b

Example (8)

The given simple beam in Fig. 3.15 subjected to triangle load w, draw shear force and bending moment diagram due to the applied load.

Solution

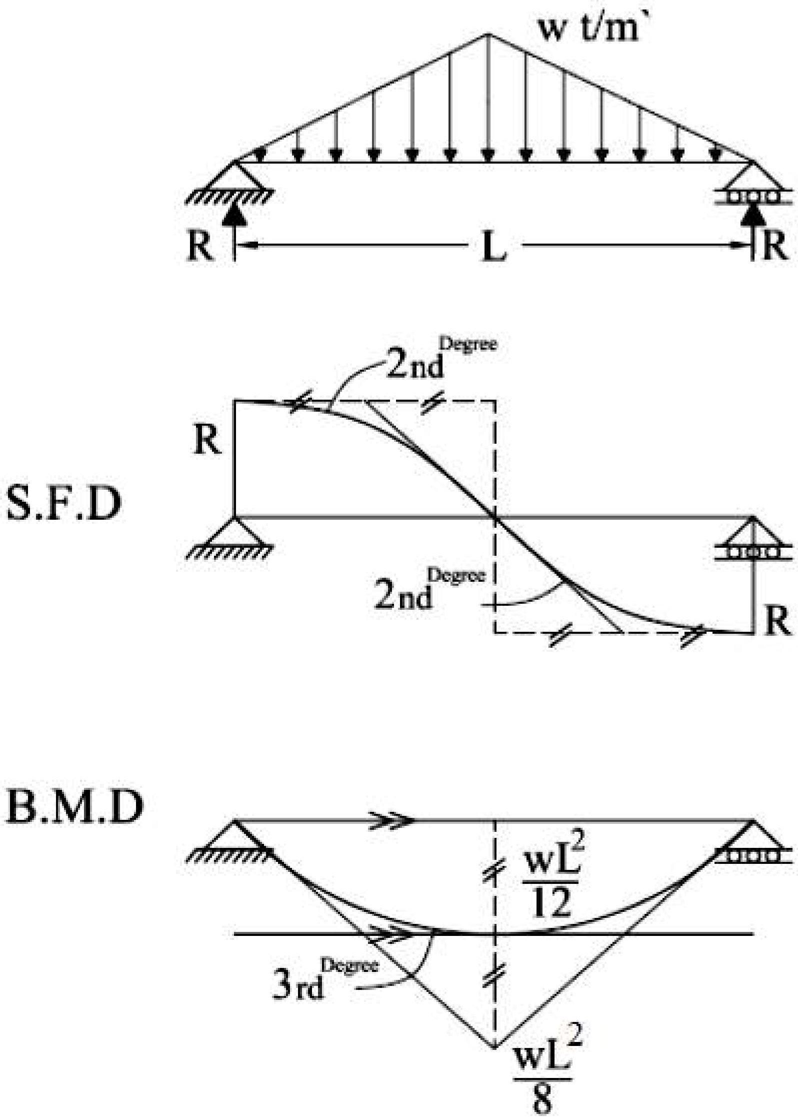


Fig. 3.15

Example (9)

The given simple beam in Fig. 3.16 subjected to triangle load w, draw shear force and bending moment diagram due to the applied load.

Solution

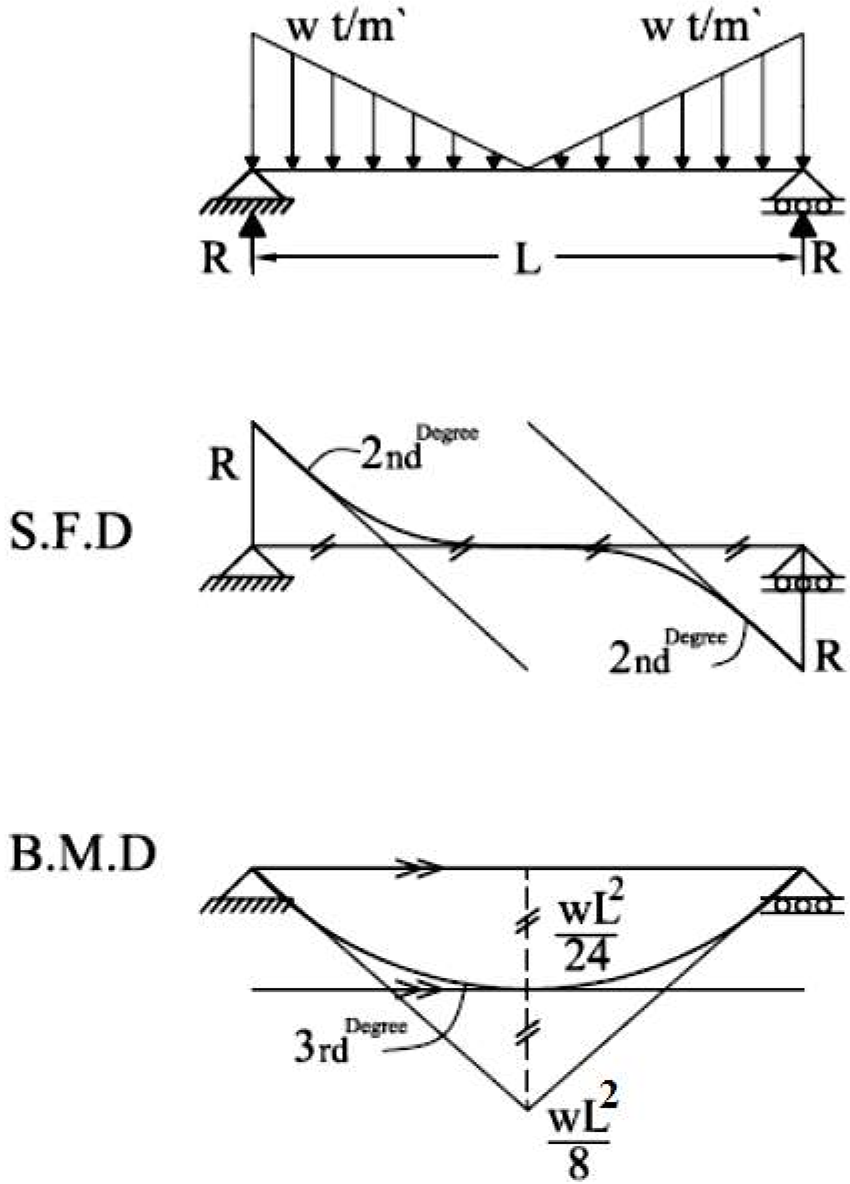


Fig. 3.17

Example (10)

For the shown cantilever in Fig. 3.18 subjected to uniform load w, draw shear force and bending moment diagram due to the applied load.

Solution

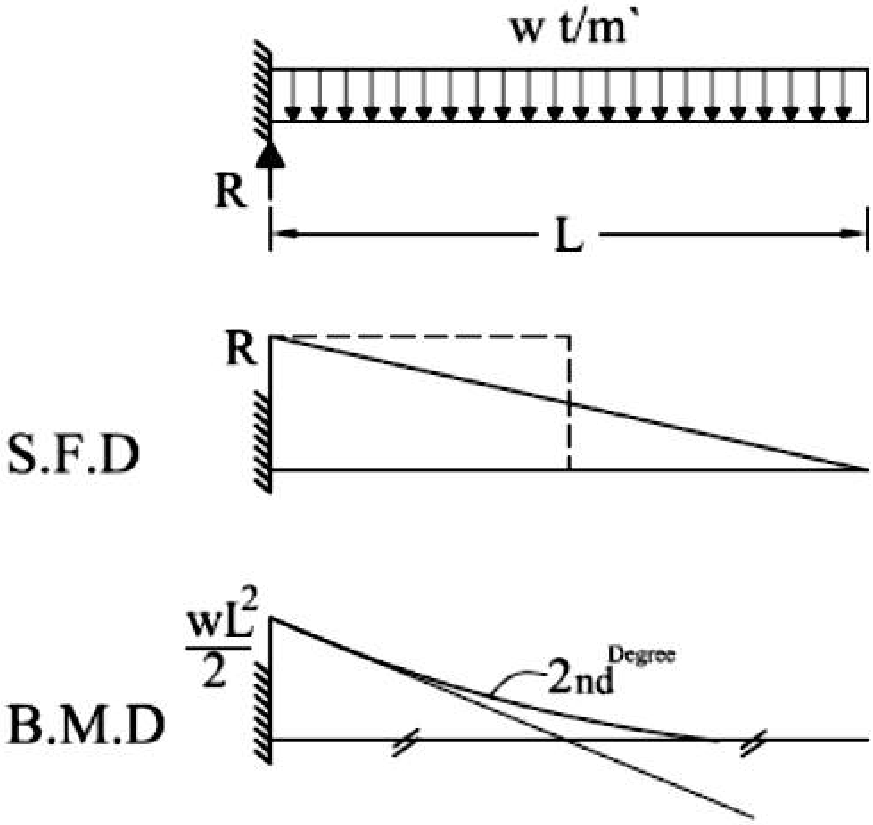


Fig. 3.18

Example (11)

The given cantilever in Fig. 3.19 subjected to triangle load w, draw shear force and bending moment diagram due to the applied load.

Solution

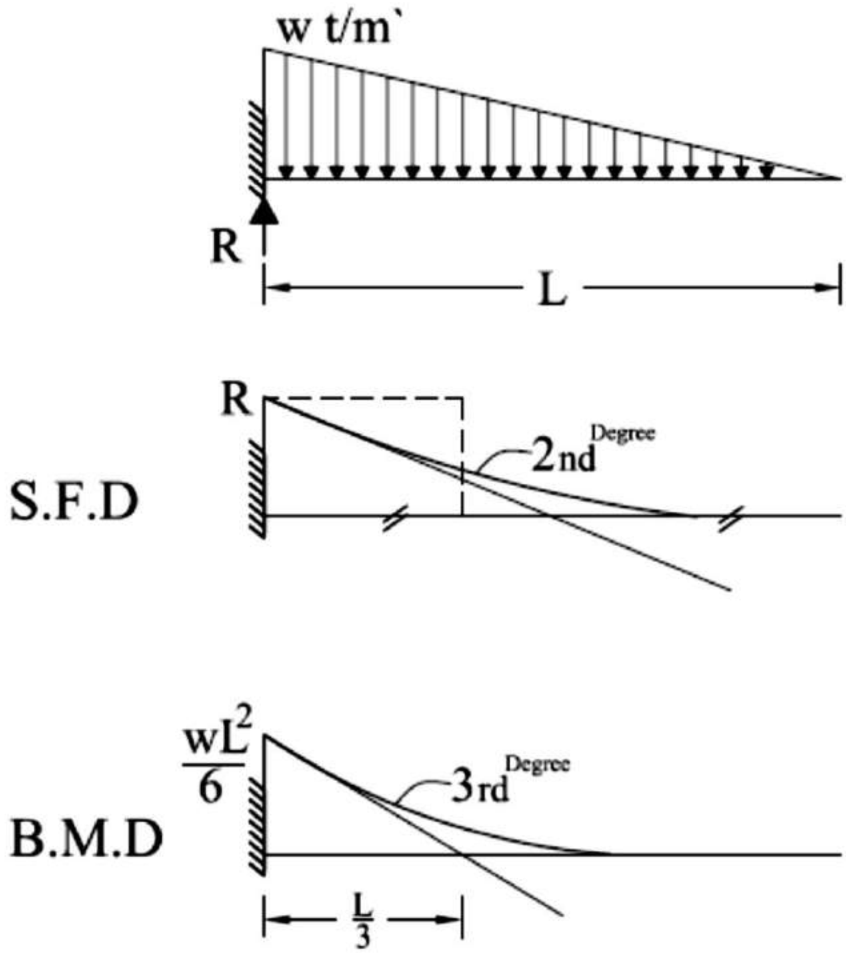


Fig. 3.19

Example (12)

The given cantilever in Fig. 3.20 subjected to triangle load w, draw shear force and bending moment diagram due to the applied load.

Solution

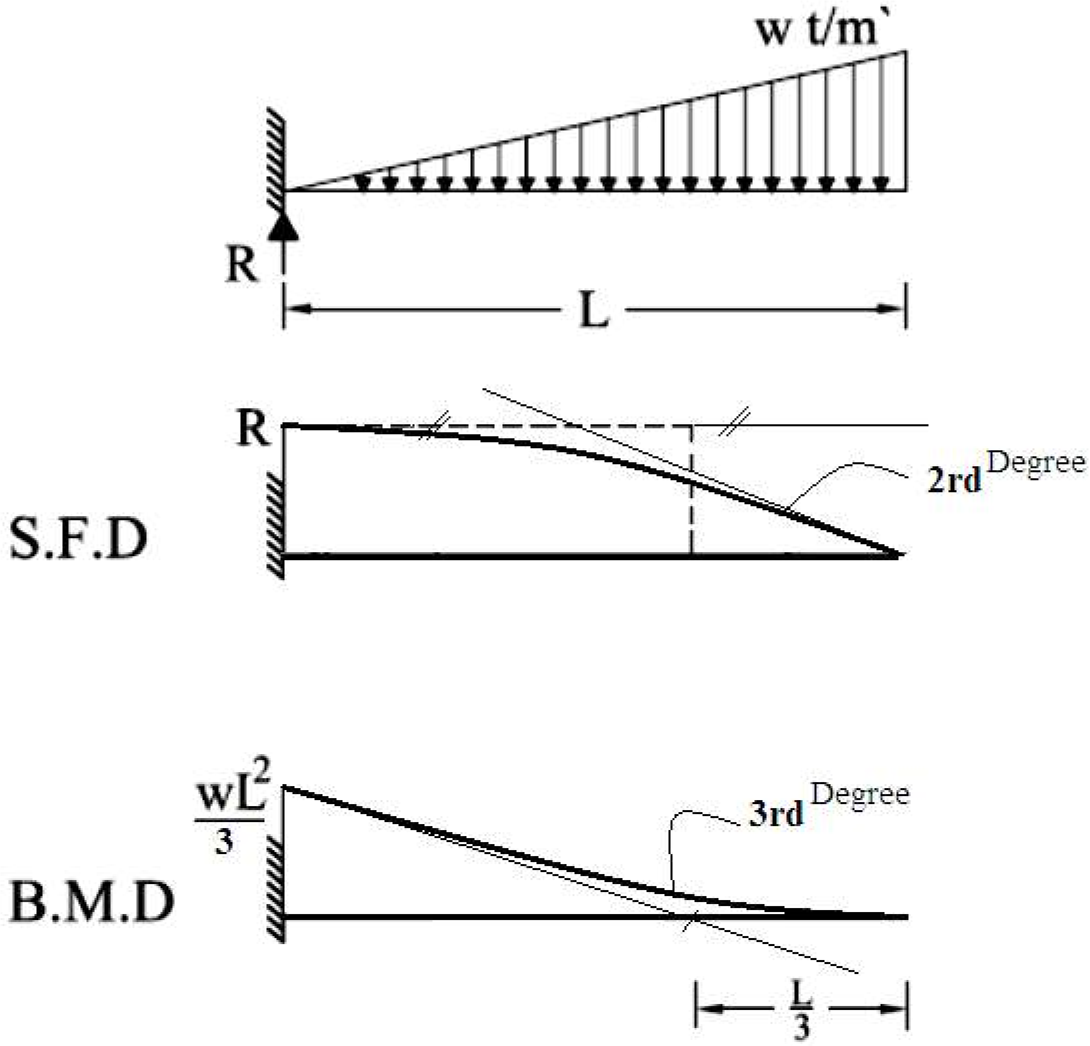


Fig. 3.20

Example (13)

For the given simple beam in Fig. 3.21, draw the normal force, shear force and bending moment diagram due to the applied load.

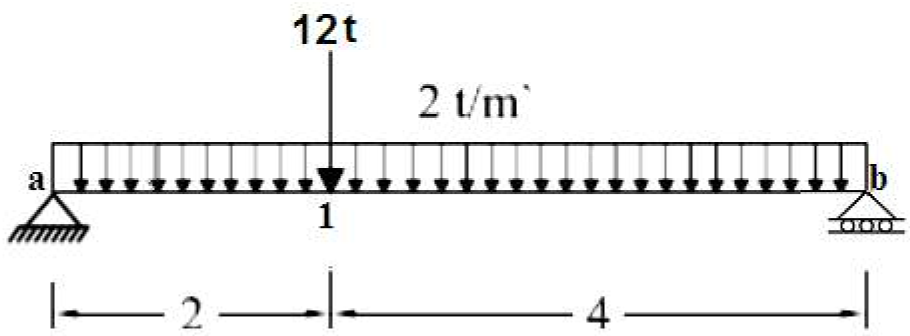


Fig. 3.21

<u>*Solution*</u>

- *Finding the reactions*

a) $\sum F_x = 0 \ggg\ggg X_a = 0$

b) $\sum M_a = 0 \ggg\ggg 12x2 + 4x1 + 8x4 - Y_b x6 = 0$

$$\therefore Y_b = \frac{60}{6} = 10t \uparrow$$

c) $\sum M_b = 0 \ggg\ggg 8x2 + 12x4 + 4x5 - Y_a x6 = 0$

$$\therefore Y_a = \frac{84}{6} = 14t \uparrow$$

d) <u>*Check*</u> $\sum F_y = 12 + 4 + 8 - 14 - 10 = 0 \ggg\ggg$ ok

- *For drawing B.M.D*

M_1 from left side = 14×2-4x1 = 24 m.t

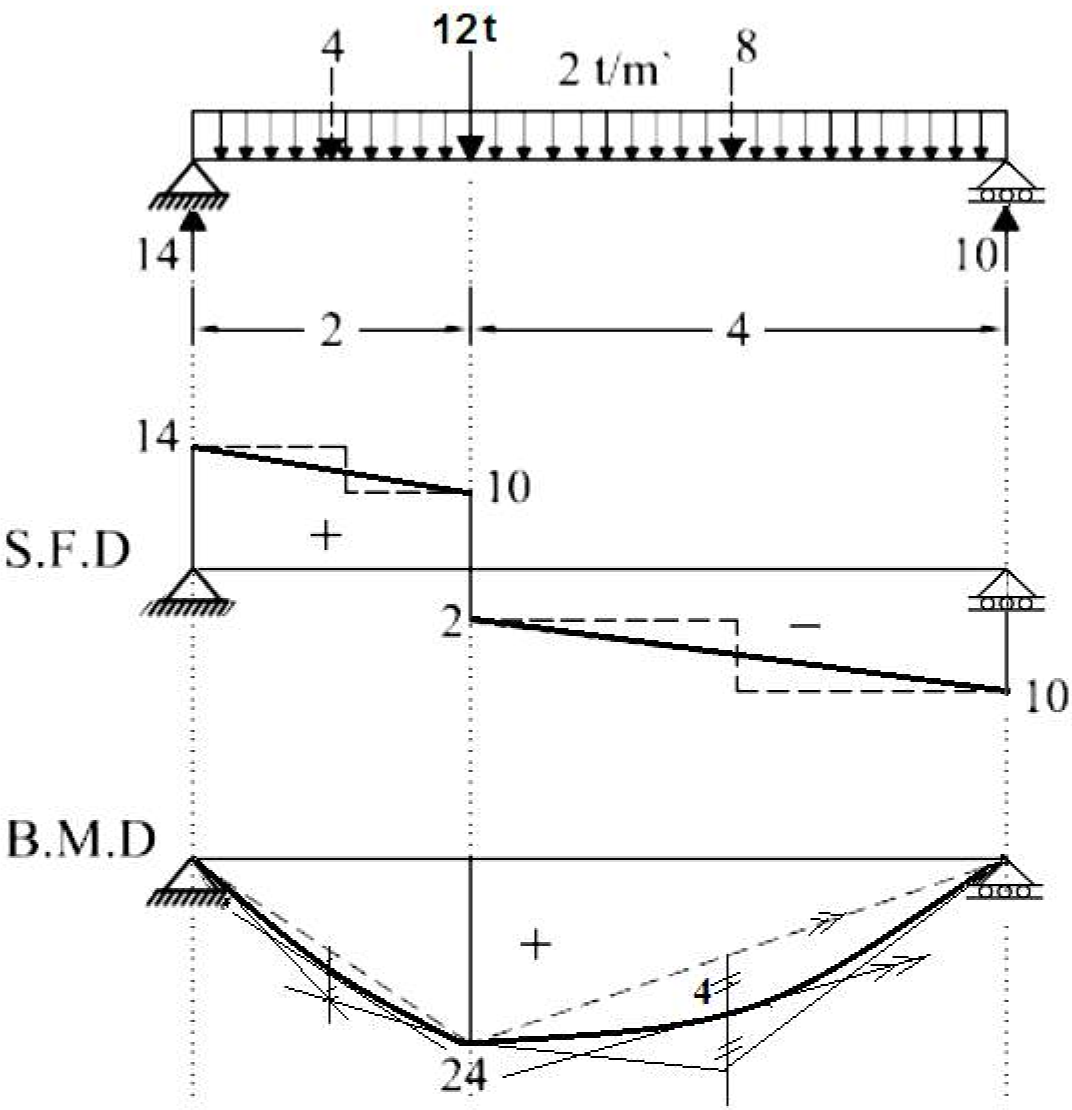


Fig. 3.21

Example (14)

Draw the shear force and bending moment diagram due to the applied load for the given simple beam in Fig. 3.22.

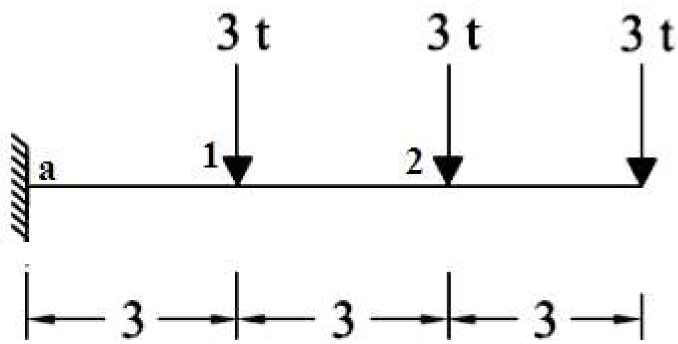


Fig. 3.22.a

<u>*Solution*</u>

- *Finding the reactions*

The fixed support has three reactions Y_a, X_a and M_a

d) $\sum F_x = 0 \ggg\ggg X_a = 0$

e) $\sum M_{at\ a} = 0 \ggg\ggg 3x3 + 3x6 + 3x9 - M_a = 0$

$$\therefore M_a = 54mt$$

f) $\sum F_y = 3 + 3 + 3 - Y_a = 0 \gg Y_a = 9t \uparrow$

- *For drawing B.M.D*

M_2 from right side = -3×3= -9 m.t

M_1 from right side = -3×3-3x6= -27 m.t

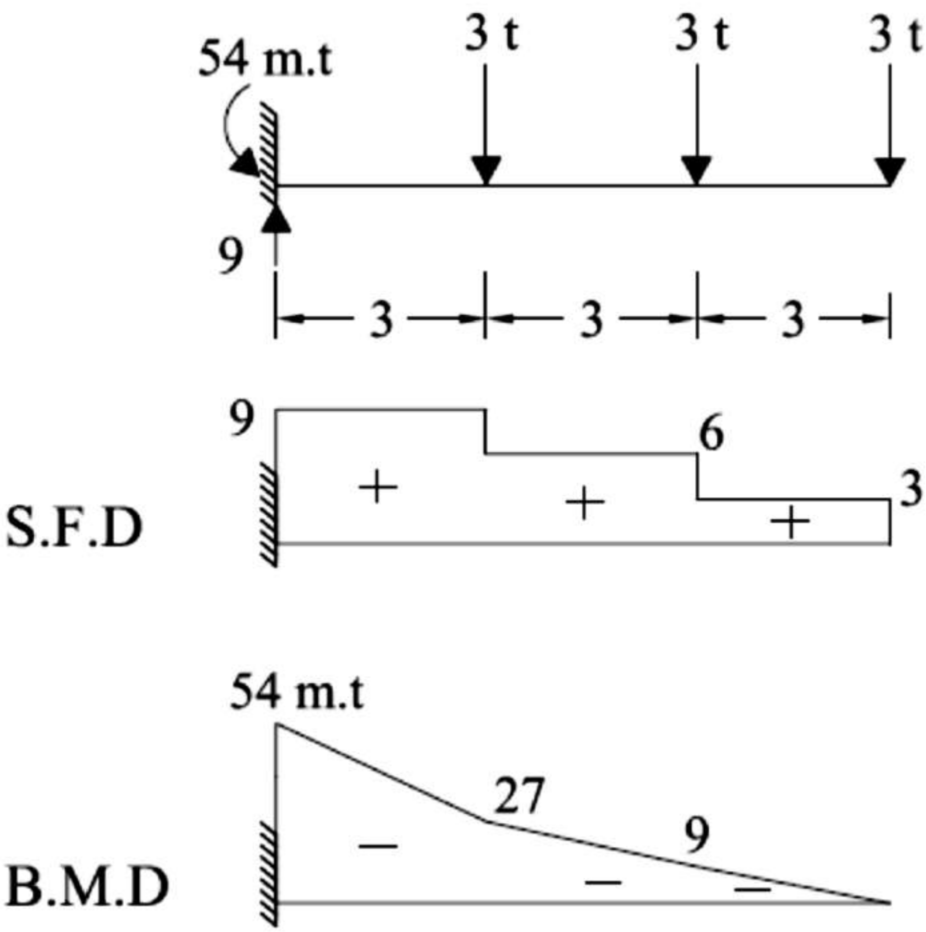


Fig. 3.22.b

Example (15)

Draw the normal force, shear force and bending moment diagram due to the applied load for the given simple beam in Fig. 3.23.

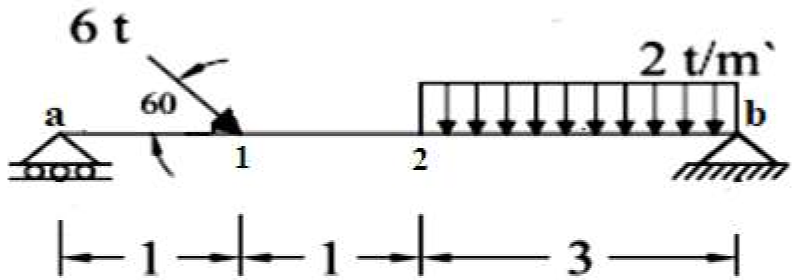


Fig. 3.23.a

Solution

➢ *Finding the reactions*

a) $\sum F_x = 0 \ggg\ggg X_a = 0$

b) $\sum M_a = 0 \ggg\ggg 5.19x1 + 6x3.5 - Y_b x5 = 0$

$$\therefore Y_b = \frac{26.190}{5} = 5.238t \uparrow$$

c) $\sum M_b = 0 \ggg\ggg 6x1.5 + 5.19x4 - Y_a x5 = 0$

$$\therefore Y_a = \frac{29.76}{5} = 5.952t \uparrow$$

d) *Check* $\sum \int F_y = 6 + 5.19 - 5.952 - 5.238 = 0 \ggg\ggg$ ok

➢ *For drawing B.M.D*

M_1 from left side = 5.95×1 = 5.59 m.t

M_2 from left side = 5.95×2 -5.19×1= 6.72 m.t

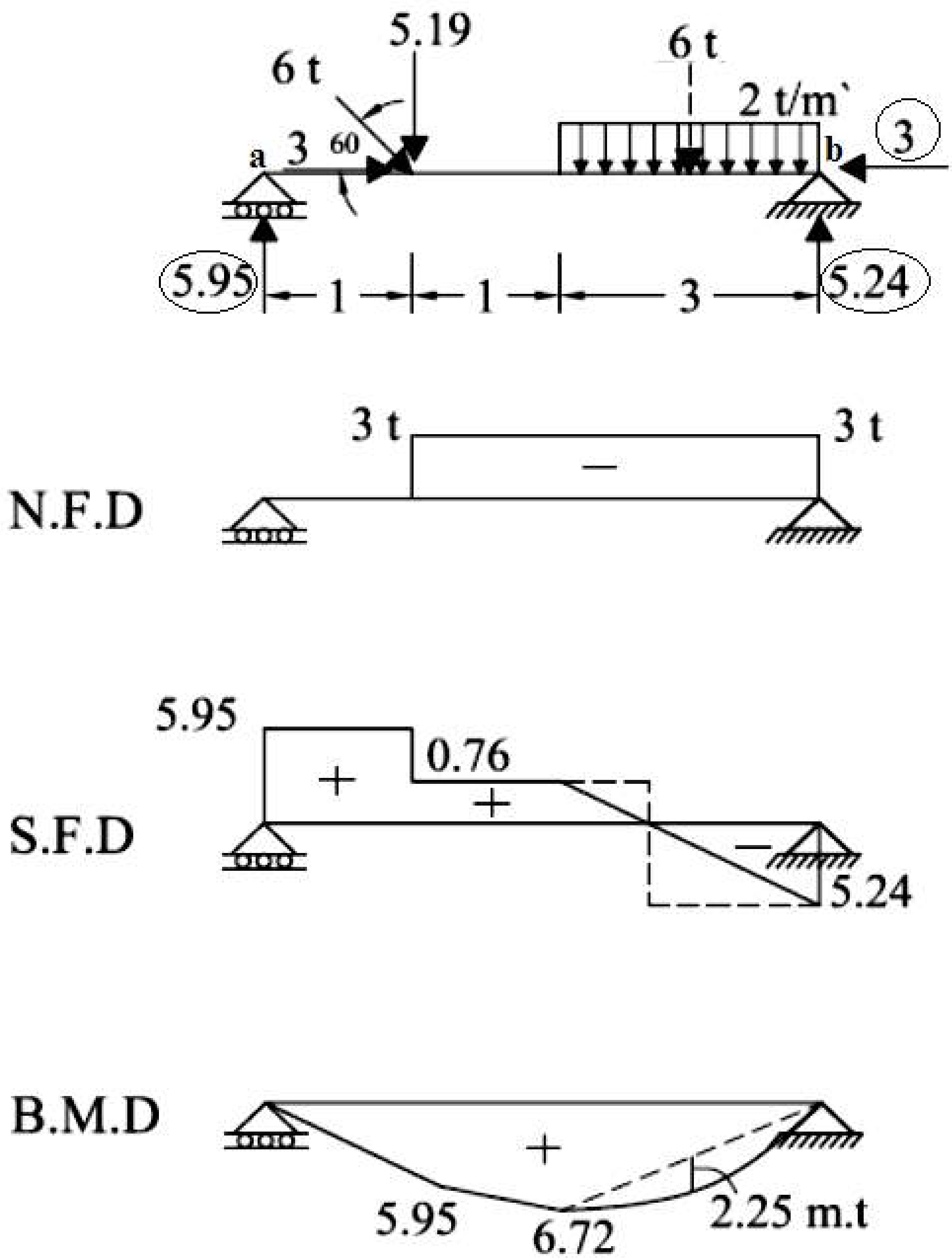


Fig. 3.23.b

Example (16)

Draw the shear force and bending moment diagrams due to the applied bending moment for the given simple beam in Fig. 3.24.

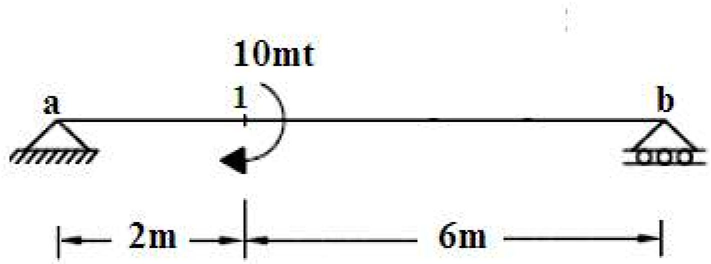


Fig. 3.24.a

<u>*Solution*</u>

- *Finding the reactions*

a) $\sum F_x = 0 \gg\gg\gg\gg X_a = 0$

b) $\sum M_a = 0 \gg\gg\gg\gg 10 - Y_b x8 = 0$

$$\therefore Y_b = \frac{10}{8} = 1.25t \uparrow$$

c) $\sum M_b = 0 \gg\gg\gg\gg 10 + Y_a x8 = 0$

$$\therefore Y_a = \frac{-10}{8} = 1.25t \downarrow$$

<u>*Check* $\sum F_y = 1.25 - 1.25 = 0 \gg\gg\gg\gg$ ok</u>

- *For drawing B.M.D*

M_1 from left side = -1.25×2 = -2.5 m.t

M_1 from right side=1.25×6=7.5

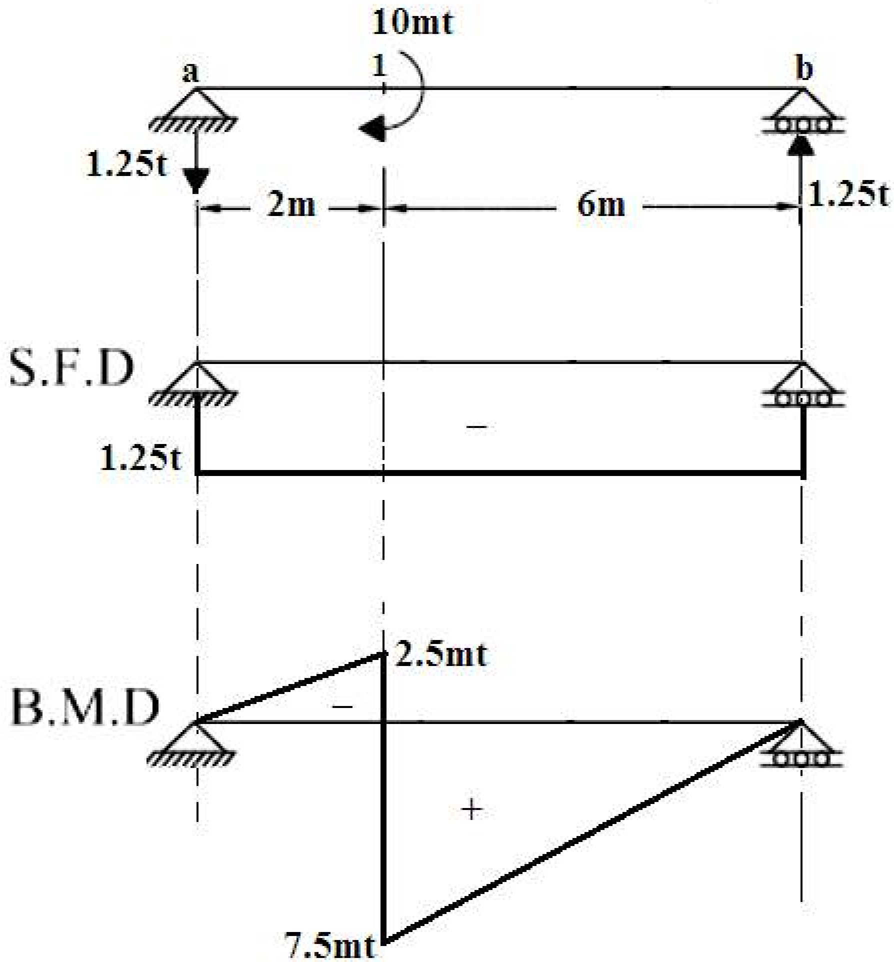


Fig. 3.24.b

Example (17)

Draw the shear force and bending moment diagrams due to the applied bending moment for the given simple beam in Fig. 3.25.

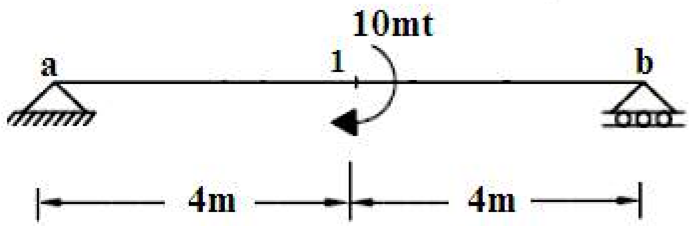


Fig. 3.25.a

Solution

- *Finding the reactions*

a) $\sum F_x = 0 \ggg\ggg X_a = 0$

b) $\sum M_a = 0 \ggg\ggg 10 - Y_b x8 = 0$

$$\therefore Y_b = \frac{10}{8} = 1.25t \uparrow$$

c) $\sum M_b = 0 \ggg\ggg 10 + Y_a x8 = 0$

$$\therefore Y_a = \frac{-10}{8} = 1.25t \downarrow$$

Check $\sum F_y = 1.25 - 1.25 = 0 \ggg\ggg$ ok

- *For drawing B.M.D*

M_1 from left side = -1.25×4 = -5 m.t

M_1 from right side=1.25×4=5

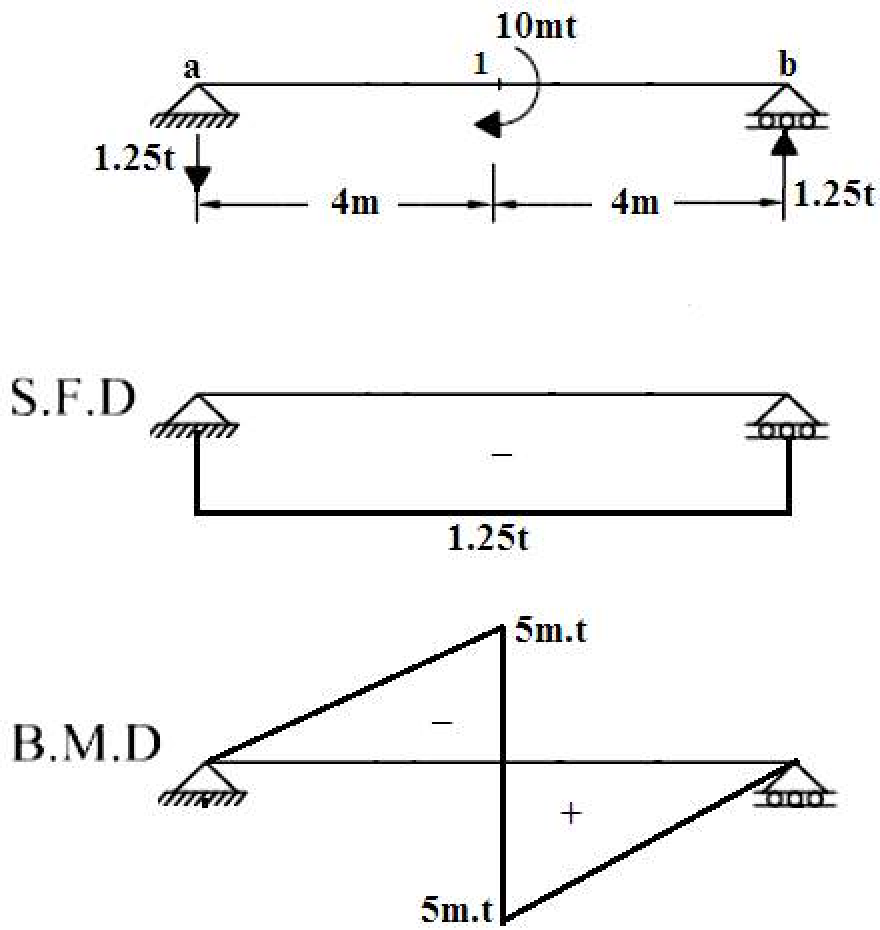


Fig. 3.25.b

<u>Example (18)</u>

Draw the shear force and bending moment diagrams due to the applied bending moment for the given simple beam in Fig. 3.26.

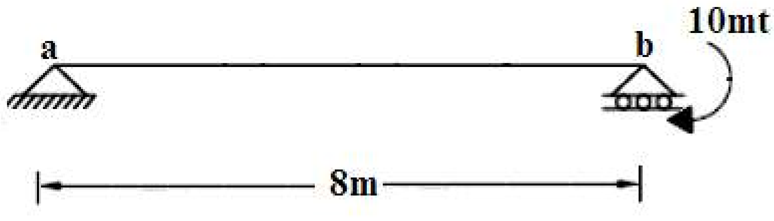


Fig. 3.26.a

Solution

- *Finding the reactions*

a) $\sum F_x = 0 \ggg\ggg\ggg X_a = 0$

b) $\sum M_a = 0 \ggg\ggg\ggg 10 - Y_b x8 = 0 \therefore Y_b = \frac{10}{8} = 1.25t \uparrow$

c) $\sum M_b = 0 \ggg\ggg\ggg 10 + Y_a x8 = 0 \therefore Y_a = \frac{-10}{8} = 1.25t \downarrow$

Check $\sum F_y = 1.25 - 1.25 = 0 \ggg\ggg\ggg$ ok

- *For drawing B.M.D*

M_b from left side = -1.25×8 = -10 m.t

M_b from right side = 0

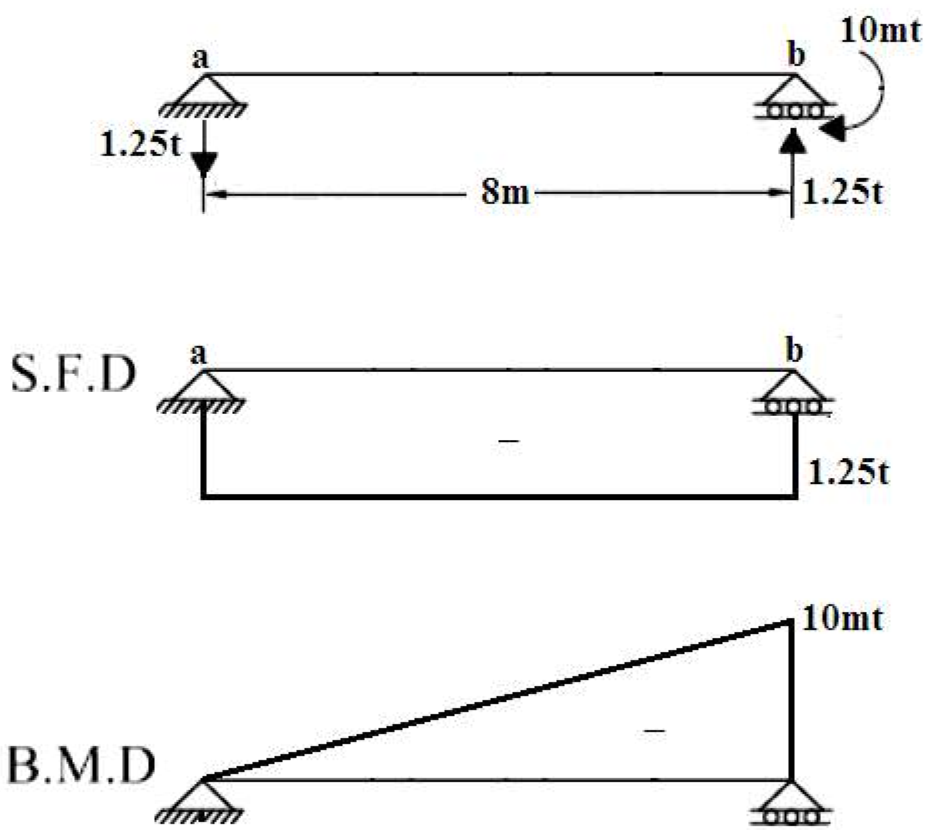


Fig. 3.26.b

Example (19)

Draw the normal force, shear force and bending moment diagram due to the applied load for the given simple beam in Fig. 3.27.

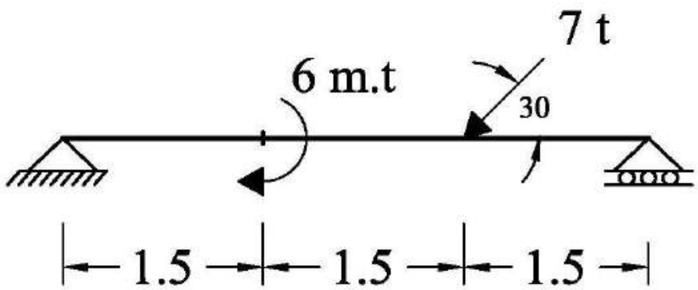


Fig. 3.27.a

Solution

- *Finding the reactions*

a) $\sum F_x = 0 \ggg\ggg X_a = 6.06t \rightarrow$

b) $\sum M_a = 0 \ggg\ggg 6 + 3.5x3 - Y_b x4.5 = 0$

$$\therefore Y_b = \frac{16.5}{4.5} = 3.67t \uparrow$$

c) $\sum M_b = 0 \ggg\ggg 6 - 3.5x1.5 + Y_a x5 = 0$

$$\therefore Y_a = \frac{-0.75}{4.5} = 0.17t \downarrow$$

Check $\sum F_y = 3.5 + 0.17 - 3.67 = 0 \ggg\ggg$ ok

- *For drawing B.M.D*

M_1 from left side = -0.17×1.5 = -0.255 m.t

M_1 from right side=3.67×3-3.5x1.5=5.75

M_2 from right side = 3.67×1.5= 5.57 m.t

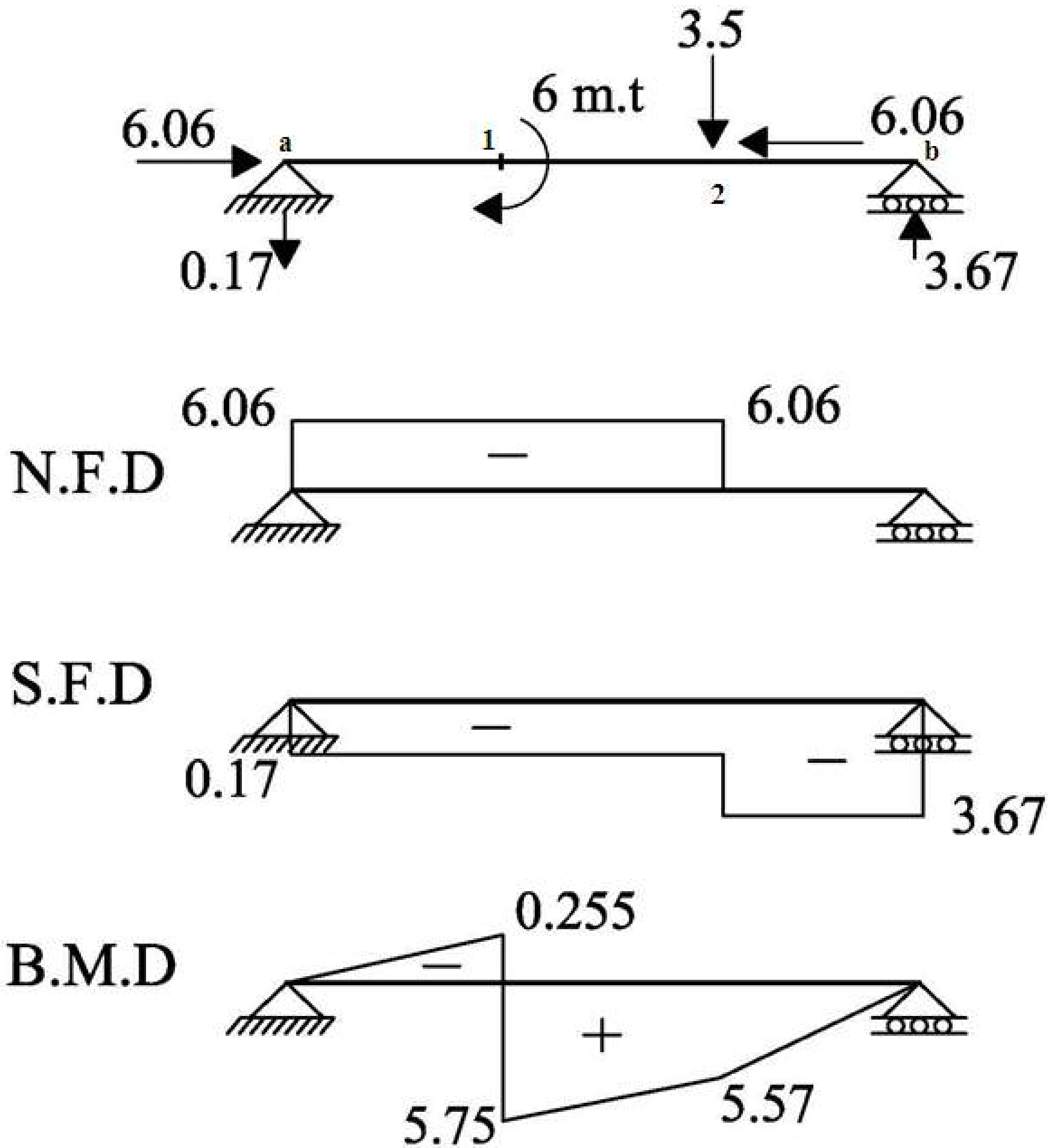


Fig. 3.26.b

Example (20)

Draw the shear force and bending moment diagram due to the applied load for the given cantilever in Fig. 3.27.

Solution

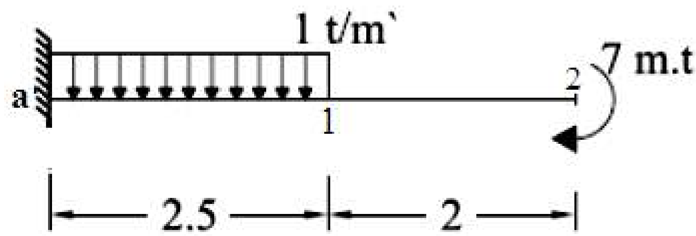


Fig. 3.27.a

- *Finding the reactions*

a) $\sum F_x = 0 \ggg\ggg X_a = 0 \rightarrow$

b) $\sum M_{at\ a} = 0 \ggg\ggg 7 + 2.5x1.25 - M_a = 0$

$\therefore M_a = 10.125m.t$

c) $\sum F_y = 2.5 - Y_a = 0 \gg Y_a = 2.5t \uparrow$

- *For drawing B.M.D*

M_a from left side = -10.125 m.t M_1 from left side=-10.125-2.5x1.25+2.5x2.5=-7 m.t M_2 from right side = -7 m.t

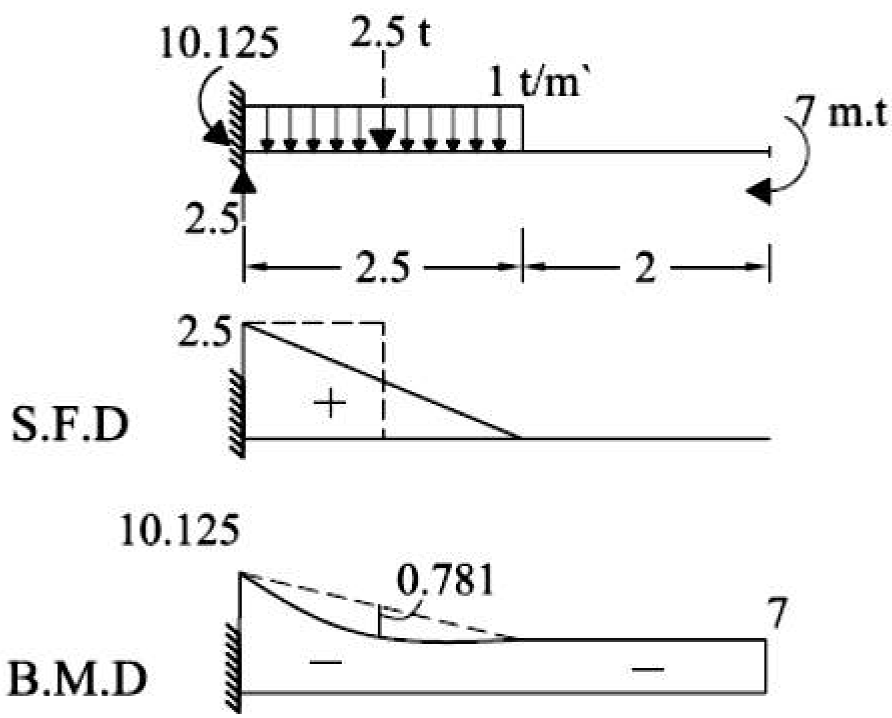


Fig. 3.27.b

Example (21)

Draw the shear force and bending moment diagram due to the applied load for the given cantilever in Fig. 3.28.

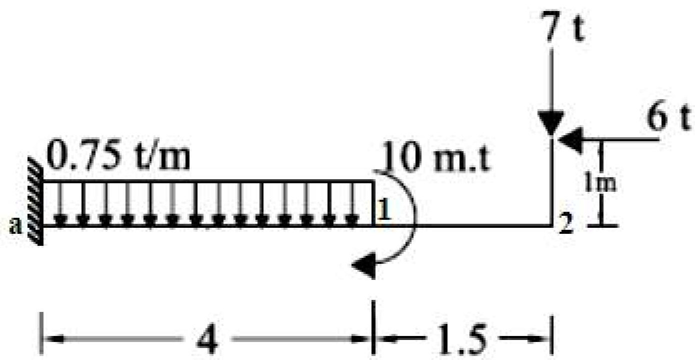


Fig. 3.28.a

Solution

- *Finding the reactions*

The fixed support has three reactions Y_a, X_a and M_a

$\sum F_x = 0 \gg\gg\gg X_a = 6 \rightarrow$

$\sum M_{at\ a} = 0 \gg\gg\gg 3x2 + 10 + 7x5.5 - 6x1 - M_a = 0$

$\therefore M_a = 48.5mt$

a) $\sum F_y = 7 + 3 - Y_a = 0 \gg Y_a = 10\ \uparrow$

- *For drawing B.M.D*

M_1 from left side = -48.5-3×2+10x4 = -14.5 m.t

M_1 from right side=-7×1.5+6=-4.5 M_2 from right side = 6×1= 6 m.t

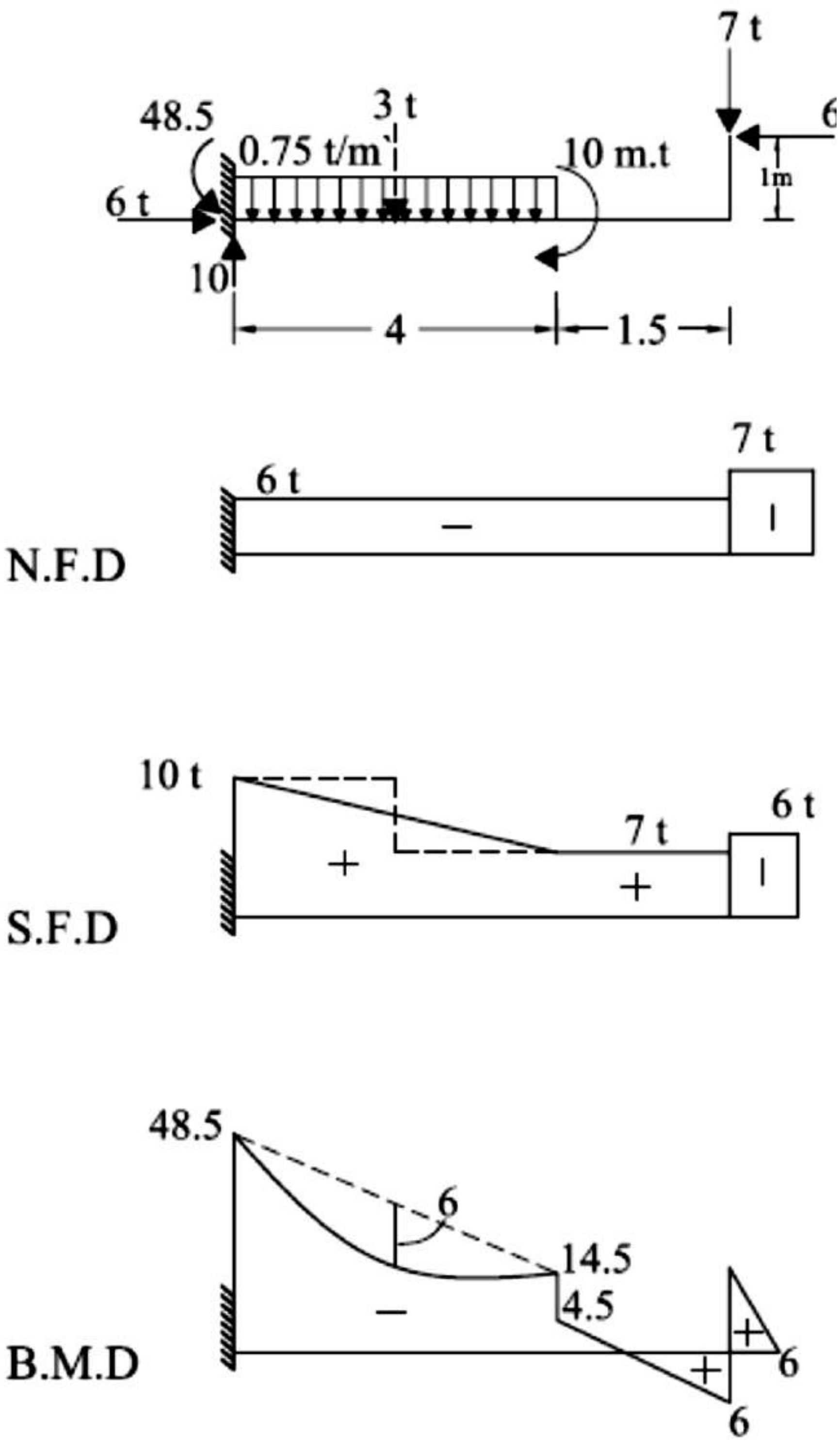

7 t
48.5
3 t
6
0.75 t/m'
10 m.t
1m
6 t
10
4
1.5
N.F.D
6 t
7 t
S.F.D
10 t
7 t
6 t
B.M.D
48.5
6
14.5
4.5
6
6

Fig. 3.28.b

Example (22)

Draw the normal force, shear force and bending moment diagram due to the applied load for the beam with two overhangs in Fig. 3.29.

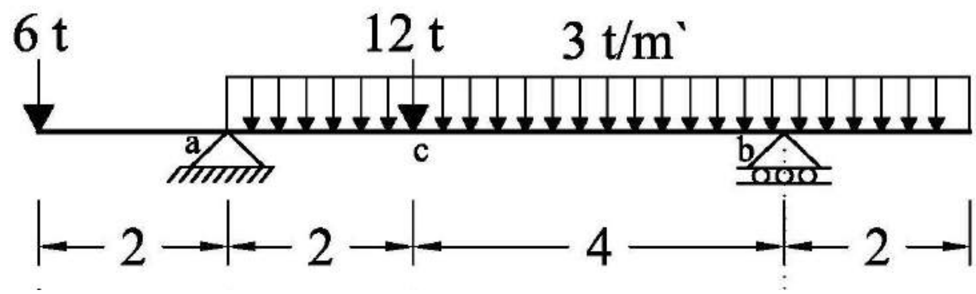


Fig. 3.29.a

Solution

$\sum F_x = 0.0 \qquad X_a = 0.0$

$\sum M_a = 0.0 \qquad -6x2 + 6x1 + 12x2 + 12x4 - 6Y_b + 6x7 = 0 \quad Y_b = 18t \uparrow$

$\sum M_b = 0.0 \qquad 6x1 - 6x5 - 12x2 - 12x4 + 6Y_a - 6x2 = 0 \qquad Y_a = 24t \uparrow$

Check $\sum F_y = 24 + 18 - 12 - 12 - 6 - 6 - 6 = 0 \quad Ok$

For drawing B.M.D

M_a from left side = -6×2 = - 12 m.t

M_c from left side = -6×4 -6×1 +24×2 = 18 m.t

M_b from right side = -6×1 = - 6 m.t

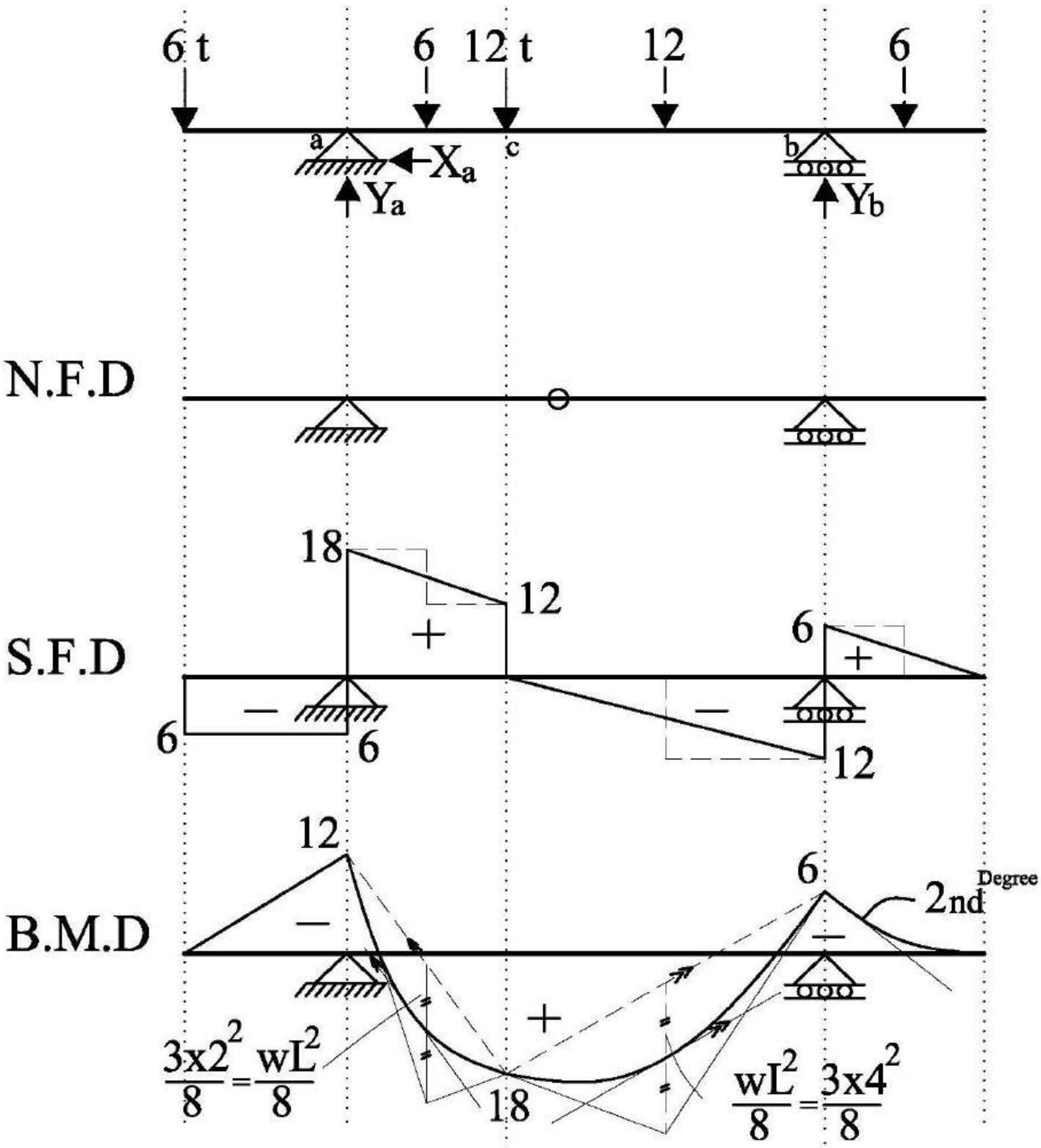


Fig. 3.29.a

<u>Example (23)</u>

Draw the normal force, shear force and bending moment diagram due to the applied load for the beam with one overhang in Fig. 3.30.

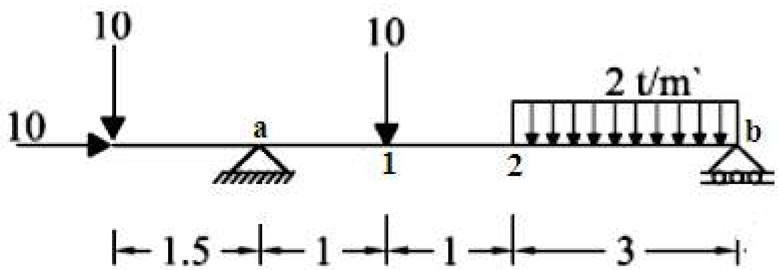


Fig. 3.30.a

<u>*Solution*</u>

$\sum F_x = 0.0 \qquad X_a = 10 \leftarrow$

$\sum M_a = 0.0 \qquad -10x1.5 + 10x1 + 6x3.5 - 5Y_b = 0 \quad Y_b = 3.2t \uparrow$

$\sum M_b = 0.0 \qquad 6x1.5 + 10x4 + 10x6.5 - 5Y_a = 0 \quad Y_a = 22.8t \uparrow$

Check $\sum F_y = 22.8 + 3.2 - 20 - 6 = 0 \quad Ok$

- *For drawing B.M.D*

M_a from left side = -10×1.5 = - 15 m.t

M_1 from left side = -10×2.5 +22.8×1= -2.2 m.t

M_2 from right side = 3.2×3-6×1.5 = 0.6 m.t

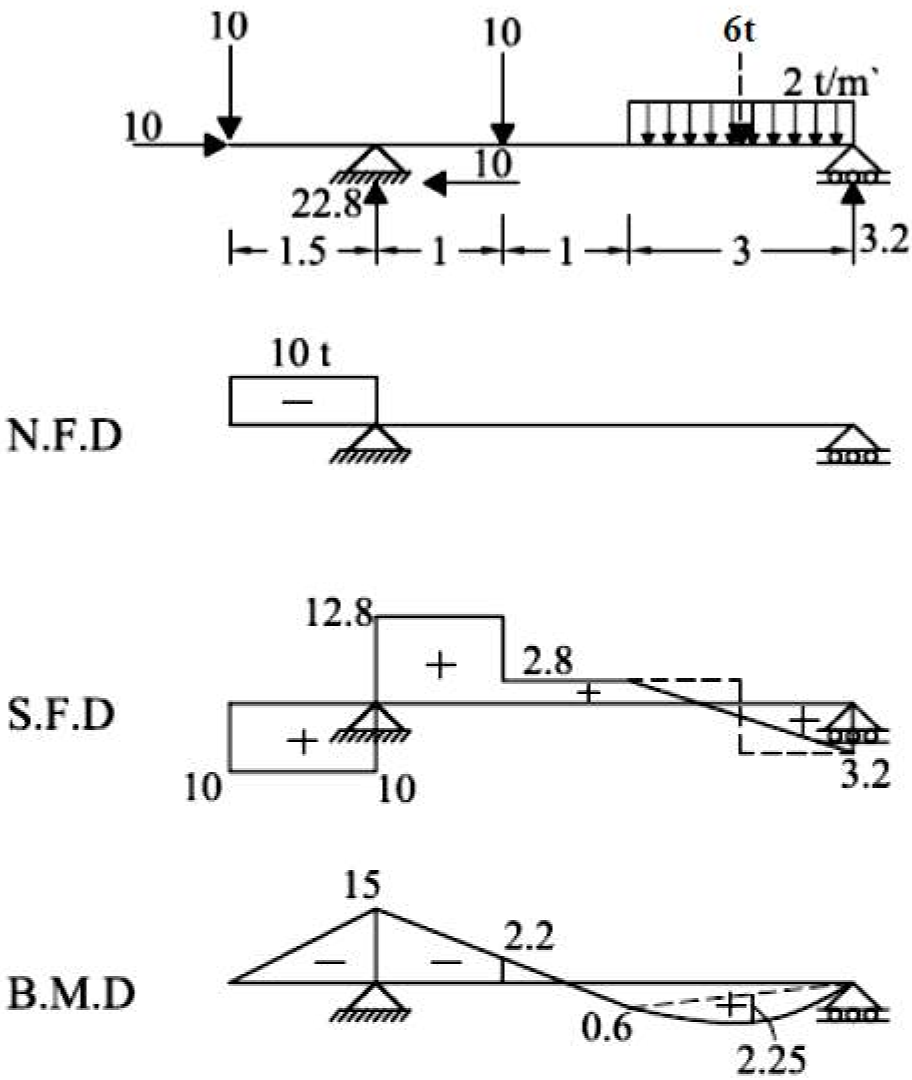


Fig. 3.30.b

Example (24)

Draw the normal force, shear force and bending moment diagram due to the applied load for the simple beam in Fig. 3.31.

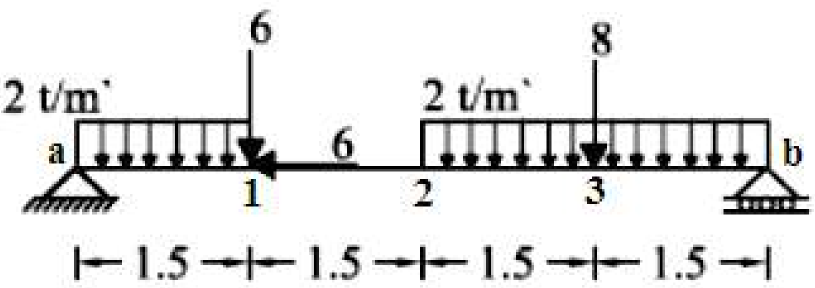


Fig. 3.32.a

Solution

- *Finding the reactions*

$\sum F_x = 0.0 \qquad X_a = 6t \rightarrow$

$\sum M_a = 0.0 \qquad 3x0.75 + 6x1.5 + 6x4.5 + 8x4.5 - 6Y_b = 0$

$$Y_b = 12.375t \uparrow$$

$\sum M_b = 0.0 \qquad 8x1.5 + 6x1.5 + 6x4.5 + 3x5.25 - 6Y_a = 0$

$$Y_a = 10.625t \uparrow$$

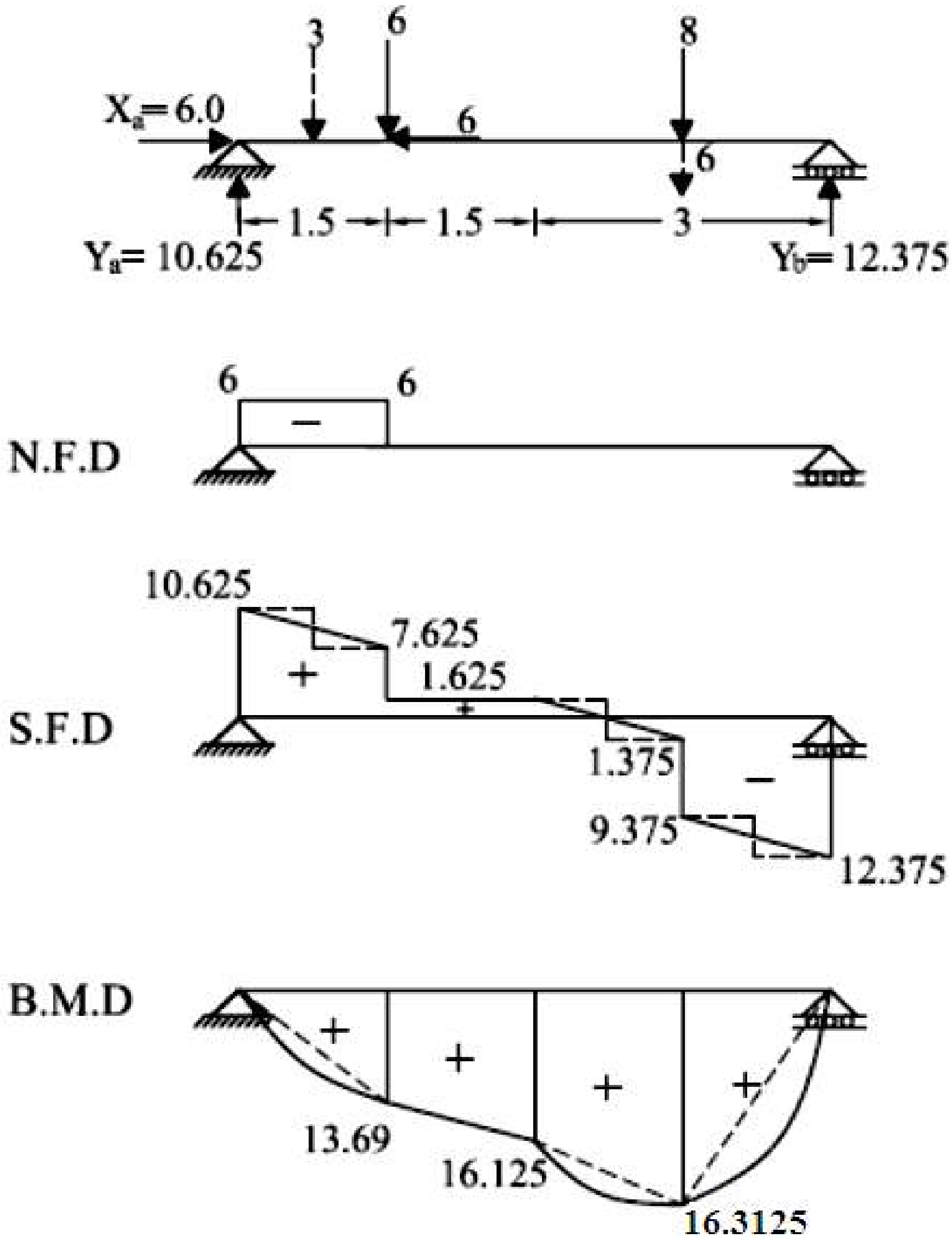
Check $\sum F_y = 12.375 + 10.625 - 3 - 6 - 6 - 8 = 0 \quad Ok$

- *For drawing B.M.D*

M_1 from left side = -3×0.75+10.625×1.5 = 13.688 m.t

$M_2$from left side = -6×1.5 -3×2.25 +10.625×3 = 16.125 m.t

M_3 from right side =12.375×1.5-3×0.75 = 16.3125 m.t



3
6
8
X_a= 6.0
6
6
1.5
1.5
3
Y_a= 10.625
Y_b= 12.375
N.F.D
6
6
−
S.F.D
10.625
7.625
1.625
+
+
1.375
−
9.375
12.375
B.M.D
+
+
+
+
13.69
16.125
16.3125

Fig. 3.31.a

Example (25)

Draw the shear force and bending moment diagram due to the applied load for the given beam with two overhangs in Fig. 3.32.

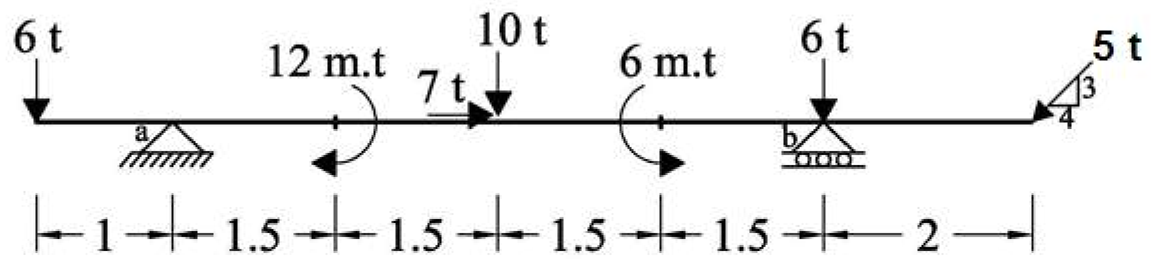


Fig. 3.32.a

➢ *Finding the reactions*

$\sum F_x = 0.0 \qquad\qquad X_a = 3t \leftarrow$

$\sum M_a = 0.0 \qquad 12 + 10x3 - 6 + 6x6 - 6Y_b + 3x8 - 6x1 = 0$

$$Y_b = 15t \uparrow$$

$\sum M_b = 0.0 \qquad 12 - 6x7 - 10x3 - 6 + 3x2 - 6Y_a = 0$

$$Y_a = 10t \uparrow$$

Check $\sum F_y = 15 + 10 - 10 - 6 - 3 - 6 = 0 \quad Ok$

➢ *For drawing B.M.D*

M_a from left side = -6×2 = - 12 m.t

M_c from left side = -6×4 -6×1 +24×2 = 18 m.t

M_b from right side = -6×1 = - 6 m.t

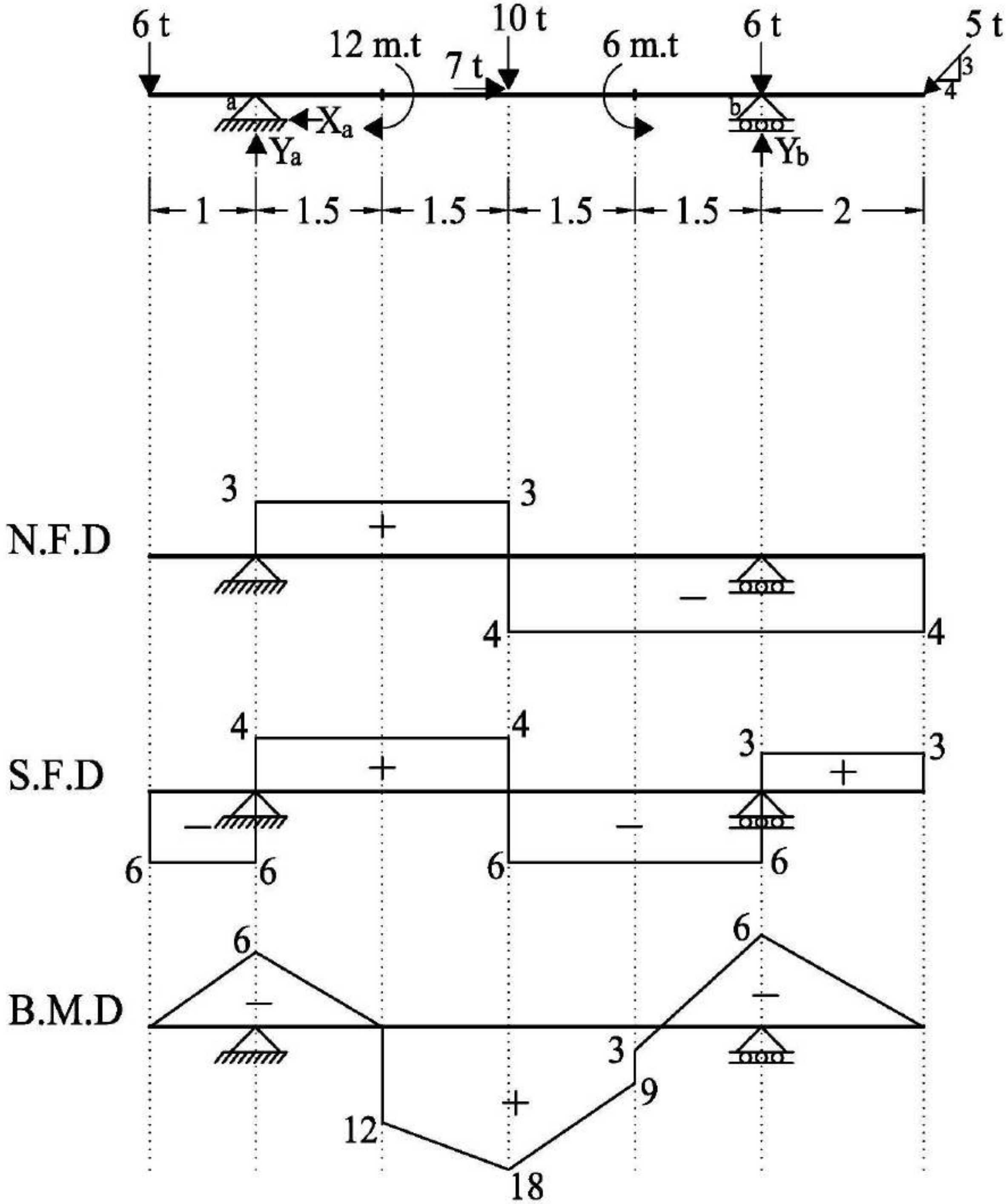

6 t
12 m.t
7 t
10 t
6 m.t
6 t
5 t
3
4
a
b
X_a
Y_a
Y_b
1
1.5
1.5
1.5
1.5
2
N.F.D
3
3
+
−
4
4
S.F.D
4
4
3
3
+
+
−
−
6
6
6
6
B.M.D
6
6
−
−
+
3
9
12
18

Fig. 3.32.b

Example (26)

Draw the shear force and bending moment diagram due to the applied load for the given beam with two overhangs in Fig. 3.33.

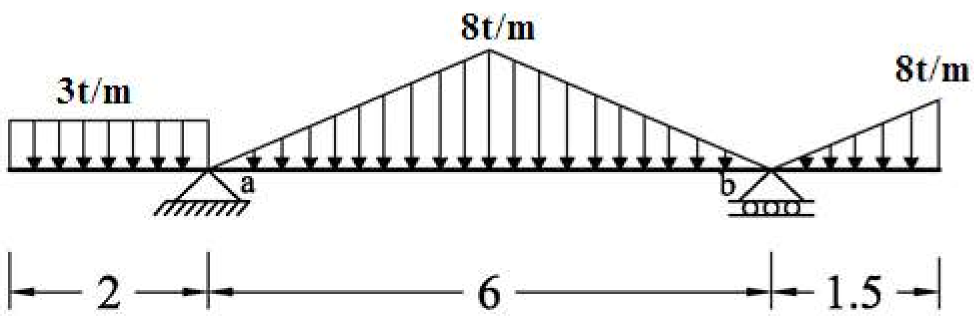


Fig. 3.33.a

Solution

- *Finding the reactions*

$\sum F_x = 0.0 \qquad X_a = 0$

$\sum M_a = 0.0 \quad 24x3 - 6x1 + 6x7 - 6Y_b = 0 \qquad Y_b = 18t \uparrow$

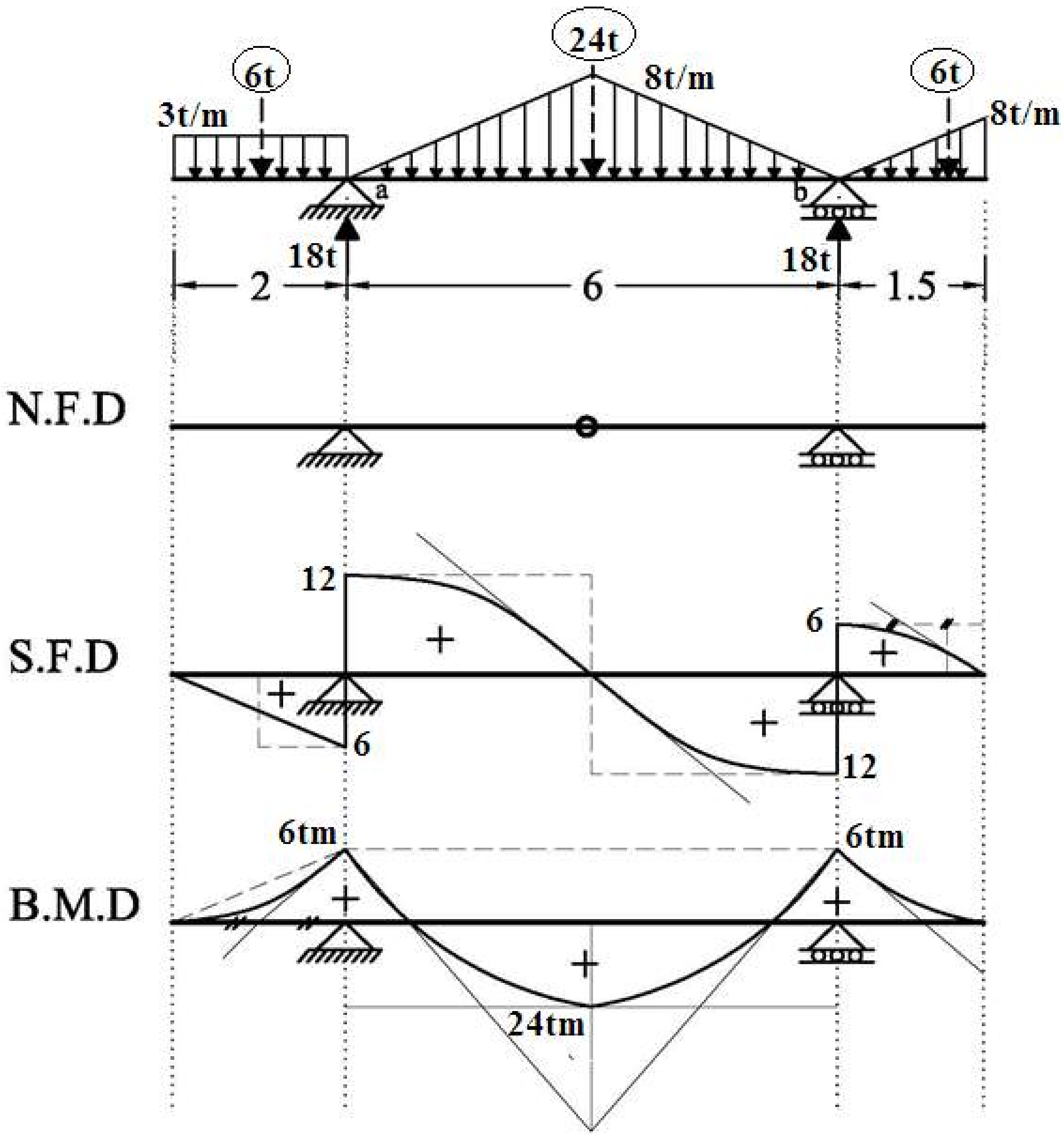
$\sum M_b = 0.0 \quad 24x3 + 6x7 - 6x1 - 6Y_a = 0 \qquad Y_a = 18t \uparrow$

Check $\sum F_y = 24 + 6 + 6 - 18 - 18 = 0 \quad Ok$

- *For drawing B.M.D*

M_a from left side = -6×1 = - 6 m.t

M_b from right side = -6×1 = - 6 m.t



6t
24t
6t
3t/m
8t/m
8t/m
a
b
18t
18t
2
6
1.5
N.F.D
S.F.D
12
6
6
12
B.M.D
6tm
6tm
24tm

Fig. 3.33.b

<u>Example (27)</u>

Draw the normal force, shear force and bending moment diagrams due to the applied load for the given beam with two overhangs in Fig. 3.34.

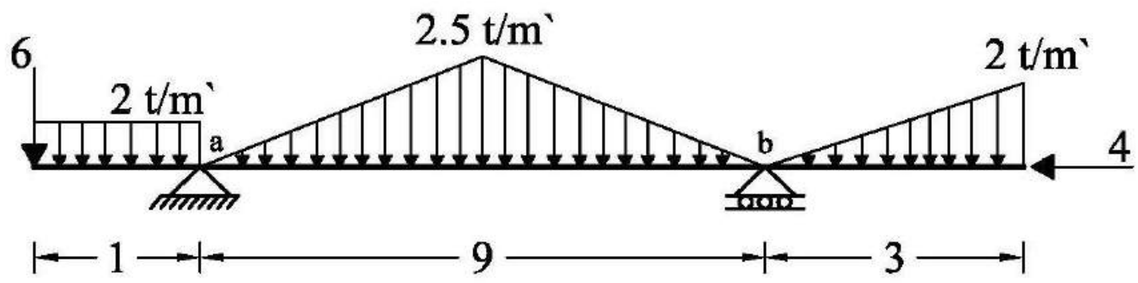


Fig. 3.34.a

<u>Solution</u>

- *Finding the reactions*

$\sum F_x = 0.0 \qquad\qquad X_a = 4t \rightarrow$

$\sum M_a = 0.0 \qquad 11.25x4.5 + 3x11 - 6x1 - 2x0.5 - 9Y_b = 0$

$$Y_b = 8.51t \uparrow$$

$\sum M_b = 0.0 \qquad 11.25x4.5 + 6x10 + 2x9.5 - 3x2 - 9Y_a = 0$

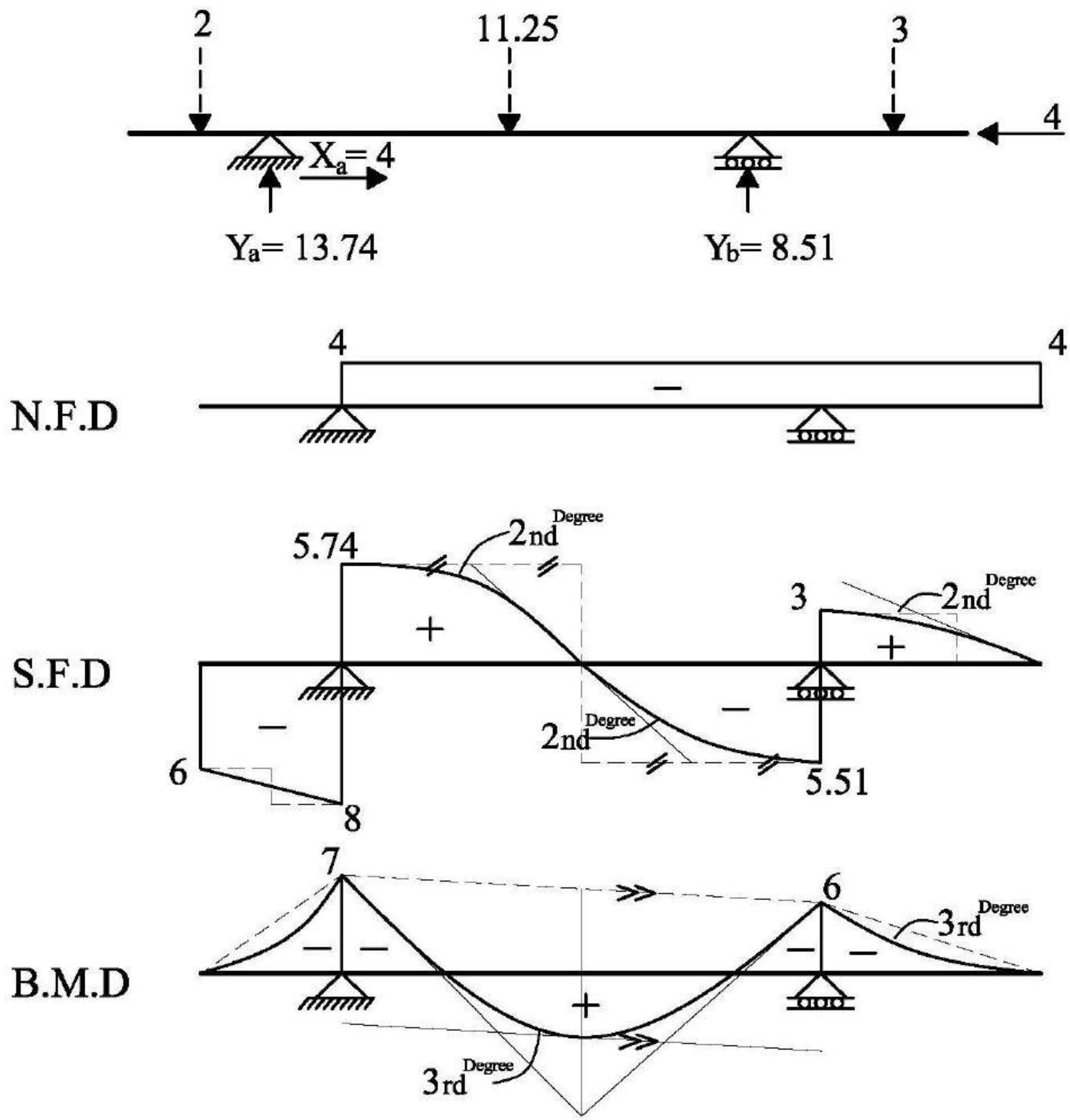
$$Y_a = 13.74t \uparrow$$

Check $\sum F_y = 11.25 + 2 + 6 + 3 - 8.51 - 13.74 = 0 \quad Ok$

- *For drawing B.M.D*

M_a from left side = -6×1-2×0.5 = - 7 m.t

M_b from right side = -3×2 = - 6 m.t



2
11.25
3
4
Xa= 4
Ya= 13.74
Yb= 8.51
N.F.D
4
4
S.F.D
5.74
2nd Degree
3
2nd Degree
2nd Degree
6
8
5.51
B.M.D
7
6
3rd Degree
3rd Degree

Fig. 3.34

Example (27)

Draw the normal force, shear force and bending moment diagrams due to the applied load for the given beam with two overhangs in Fig. 3.35.

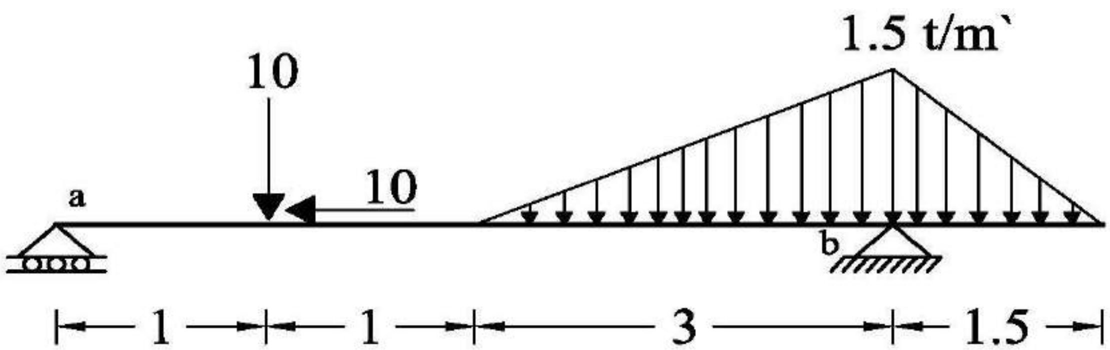


Fig. 3.35.a

Solution

➤ *Finding the reactions*

$\sum F_x = 0.0 \qquad X_a = 10t \rightarrow$

$\sum M_a = 0.0 \qquad 10x1 + 2.25x4 + 1.13x5.5 - 5Y_b = 0$

$Y_b = 5.04t \uparrow$

$\sum M_b = 0.0 \qquad 10x4 + 2.25x1 - 1.13x0.5 - 5Y_a = 0$

$Y_a = 8.34t \uparrow$

Check $\sum F_y = 10 + 2.25 + 1.13 + -8.34 - 5.04 = 0 \quad Ok$

➤ *For drawing B.M.D*

M_1 from left side = 8.34×1= 8.34 m.t

M_2 from left side = 8.34×2-10×1 = 6.68 m.t

M_b from left side = -1.13×0.5 = -0.565 m.t

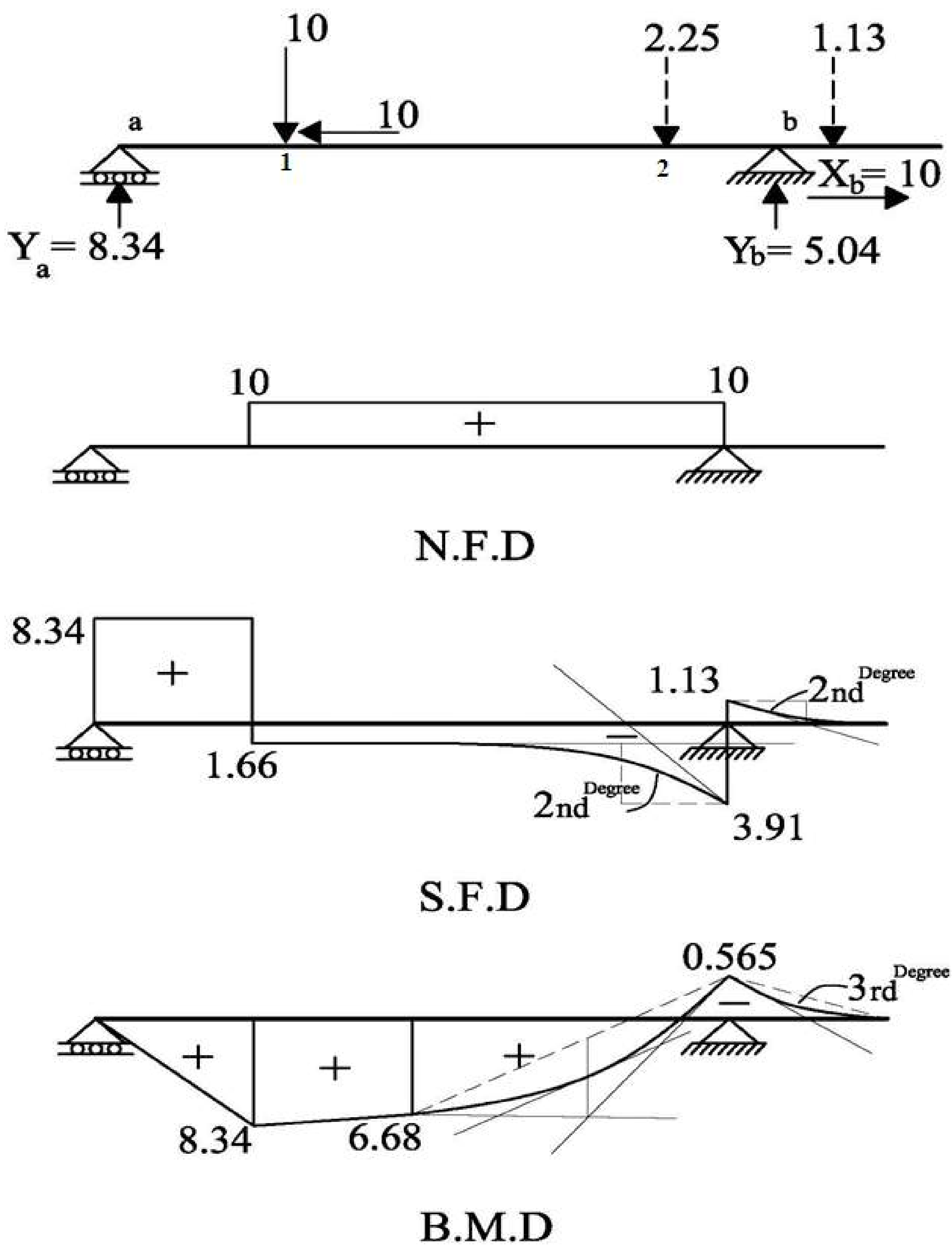

10
10
2.25
1.13
a
b
1
2
$X_b = 10$
$Y_a = 8.34$
$Y_b = 5.04$
10
10
+
N.F.D
8.34
+
1.66
1.13
2nd Degree
2nd Degree
3.91
S.F.D
0.565
3rd Degree
+
+
+
8.34
6.68
B.M.D

Fig.3.35.b

Example (28)

Draw the normal force, shear force and bending moment diagrams due to the applied load for the given beam with two overhangs in Fig. 3.36.

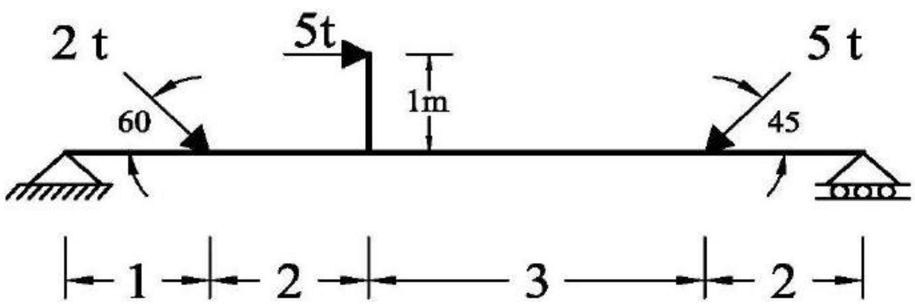


Fig. 3.36.a

Solution

- *Finding the reactions*

$\sum M_a = 0 \quad 1.73\text{x}1 + 5\text{x}1 + 3.54\text{x}6 - 8Y_b = 0 \quad Y_b \ = 3.5t \uparrow$

$\sum M_b = 0 \quad 1.73\text{x}7 - 5\text{x}1 + 3.54\text{x}2 - 8Y_a = 0 \quad Y_a \ = 1.77t \uparrow$

$\sum F_x = 0.0 \qquad X_b = 2.46t \leftarrow$

Check $\sum F_y$ = 1.73+3.54-3.5-1.77= 0.0 … OK

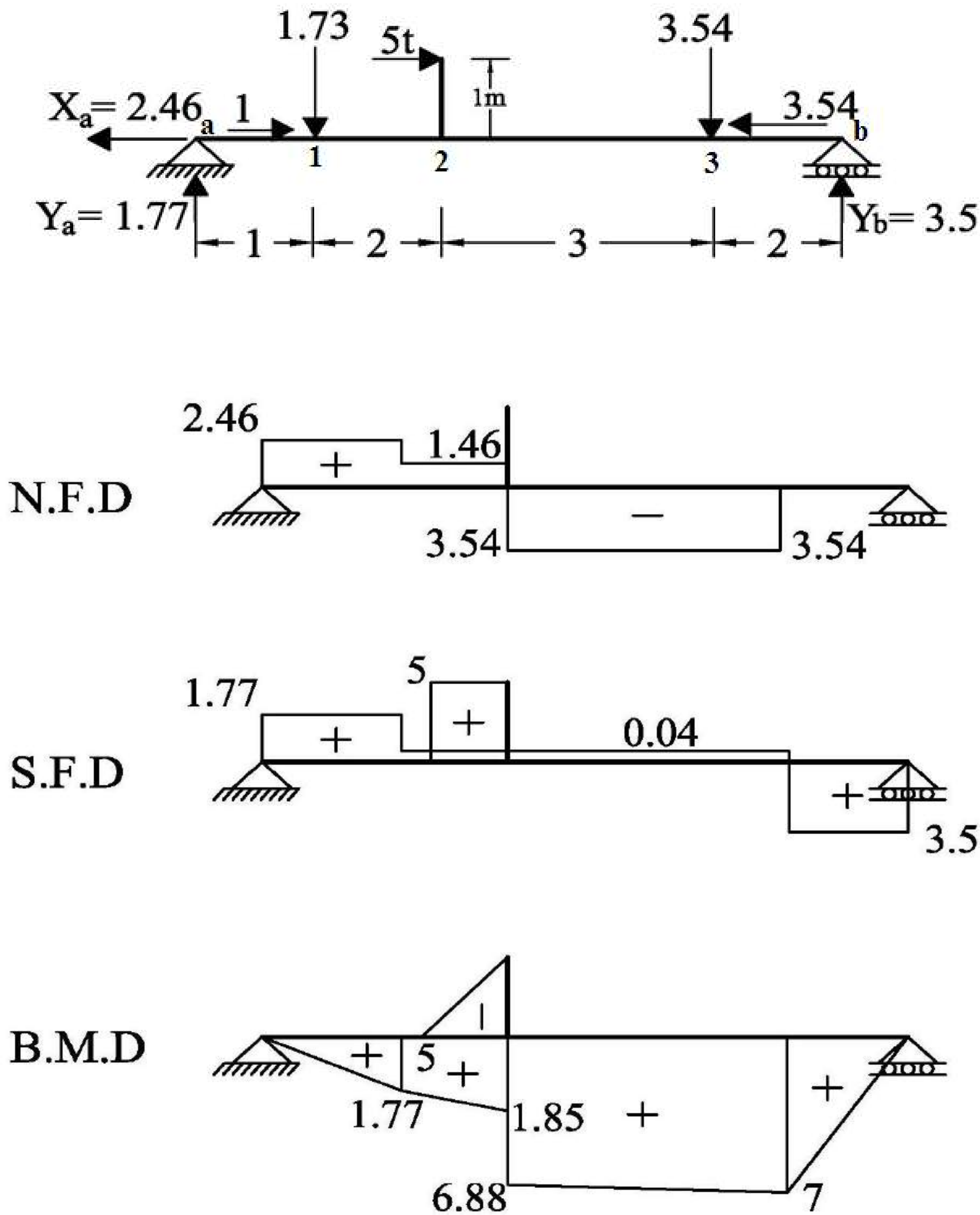
- *For drawing B.M.D*

M_1 from left side = 1.77×1= 1.77 m.t

M_2 from left side = 1.77×3-1.73×2 = 1.85 m.t

M_2 from right side =3.5×5-3.54×3 = 6.88 m.t

M_2 for the cantilever =5×1= -5 m.t



1.73
5t
3.54
1m
Xa= 2.46
a
1
3.54
b
1
2
3
Ya= 1.77
Yb= 3.5
1
2
3
2
N.F.D
2.46
1.46
+
−
3.54
3.54
S.F.D
1.77
5
+
+
0.04
+
3.5
B.M.D
+
5
+
1.77
1.85
+
+
6.88
7

Fig. 3.36.b

Example (29)

Draw the normal force, shear force and bending moment diagram due to the applied load for the given compound beam in Fig. 3.37.

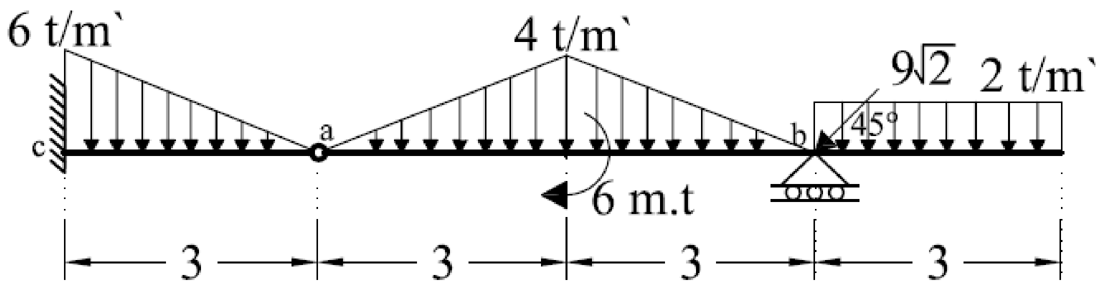


Fig. 3. 37.a

Solution

- *For part a-b*

$F_x = 0.0 \qquad X_a = 9t \rightarrow$

$\sum M_a = 0.0 \gg 6x2 + 6 + 6x4 + 9x6 + 6x7.5 - 6Y_b = 0.0 \gg Y_b = 23.5t \uparrow$

$\sum M_b = 0.0 \gg 6x1.5 + 6 - 6x2 - 6x4 + 6Y_a = 0.0 \qquad \gg \quad Y_a = 3.5t \uparrow$

$Check \quad \sum F_y = 6 + 6 + 6 + 9 - 3.5 - 23.5 = 0 \quad Ok$

- *For part c-a*

$\sum F_y = 0 \quad Y_c = 12.5t \uparrow$

$\sum M_c = 0 \quad M_c = 19.5t\ m.t$

$\sum F_x = 0 \quad X_a = 9t \rightarrow$

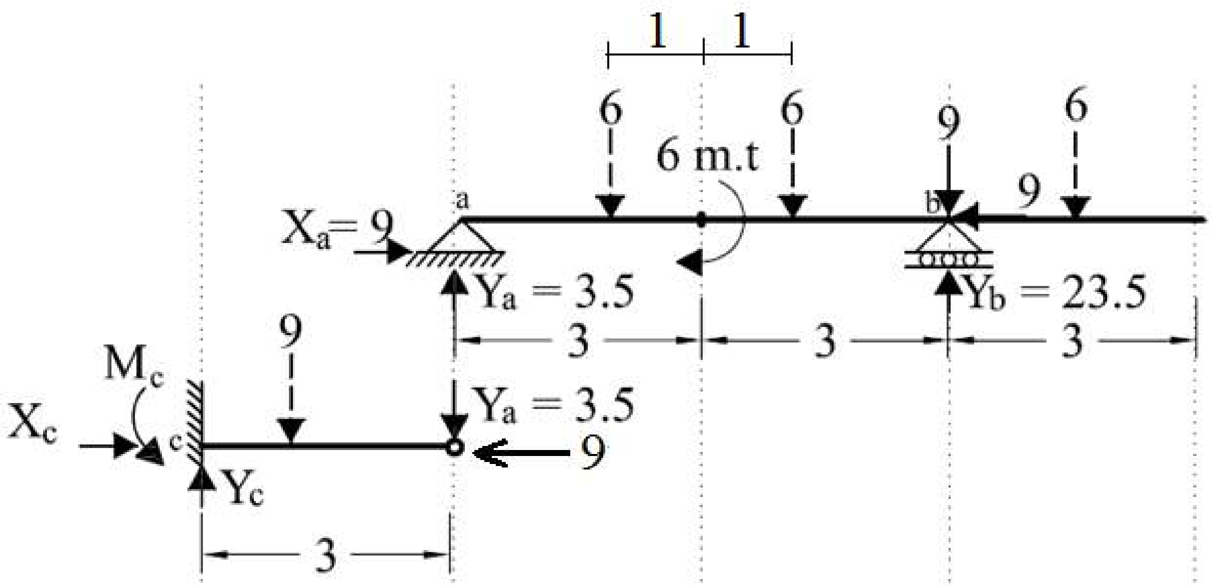
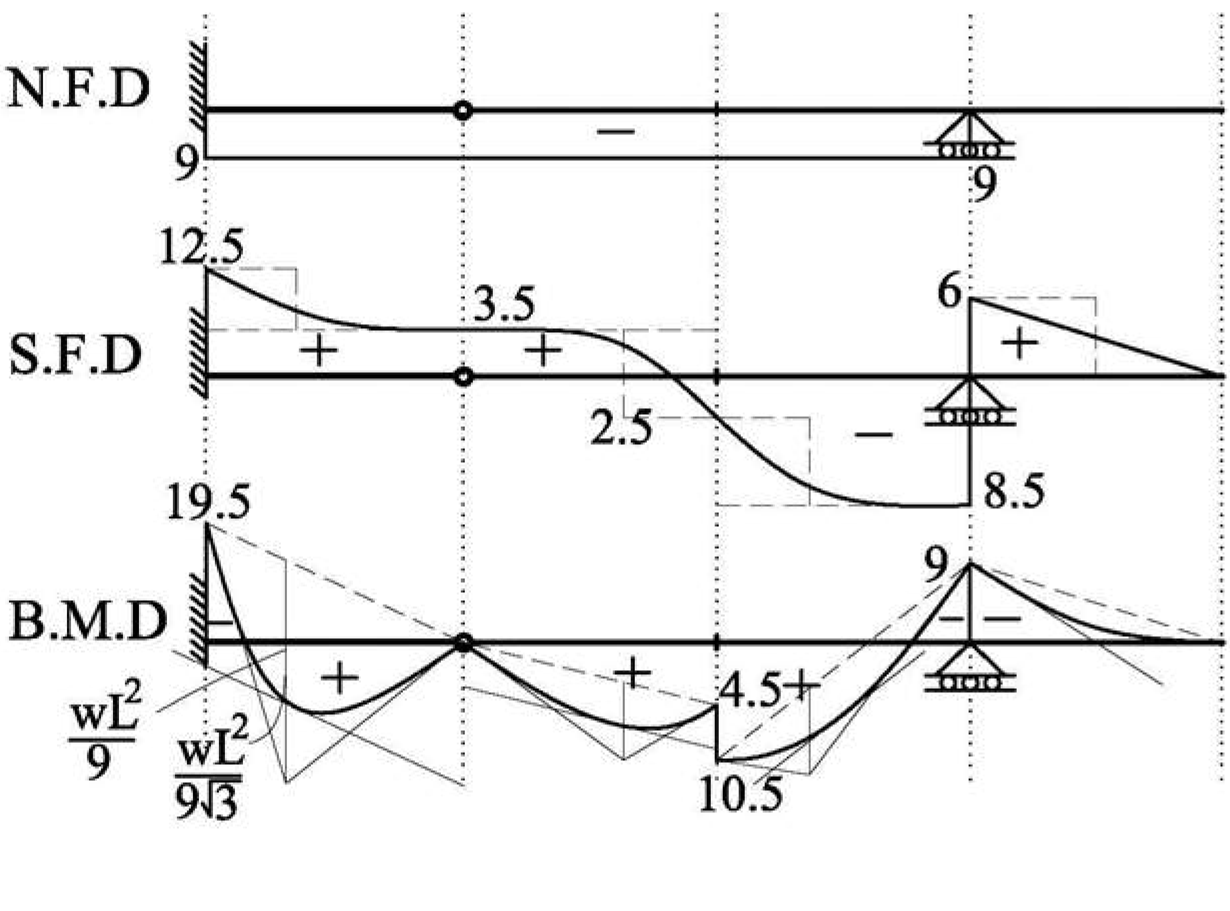


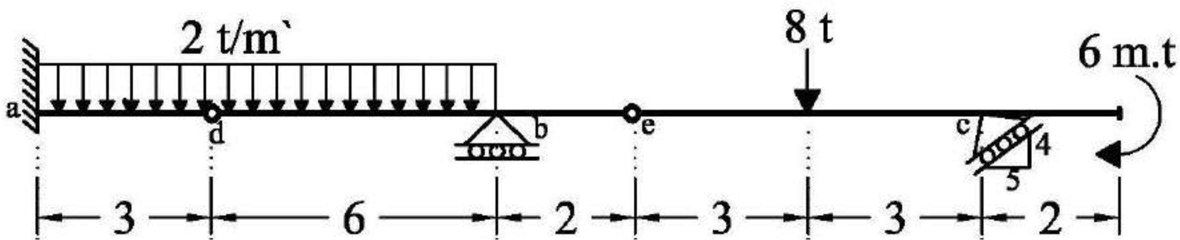
Fig. 3. 37.b



Example (30)

For the given compound beam in Fig. 3.38, draw the normal force, shear force and bending moment diagram due to the applied load.

Fig. 3. 38



<u>*Solution*</u>

- <u>*For part ec*</u>

$\sum M_e = 0.0 \quad \gg 8x3 + 6 - 6Y_c = 0.0 \qquad \gg Y_c = 5t \uparrow$

$\sum M_c = 0.0 \quad \gg \; 6 - 8x3 + 6Y_e = 0.0 \qquad \gg \; Y_e = 3t \uparrow$

$Check \quad \sum F_y = 8 - 3 - 5 = 0 \quad Ok$

$X_c = R\ \sin\Theta$ and $Y_c = R\ \cos\Theta$

$$\frac{X_c}{Y_c} = \frac{\sin\Theta}{\cos\Theta} = \tan\Theta \quad \gg X_c = 4t \leftarrow$$

$\sum F_x = 0.0 \qquad X_e = 4t \rightarrow$

- *For part d-b-e*

$\sum F_x = 0.0 \qquad X_d = 4t \rightarrow$

$\sum M_d = 0.0 \quad \gg 12x3 + 3x8 - 6Y_c = 0.0 \qquad \gg Y_b = 10t \uparrow$

$\sum M_b = 0.0 \quad \gg \; 2x3 - 12x3 + 6Y_d = 0.0 \qquad \gg \; Y_d = 5t \uparrow$

$Check \quad \sum F_y = 3 + 12 - 10 - 5 = 0 \quad Ok$

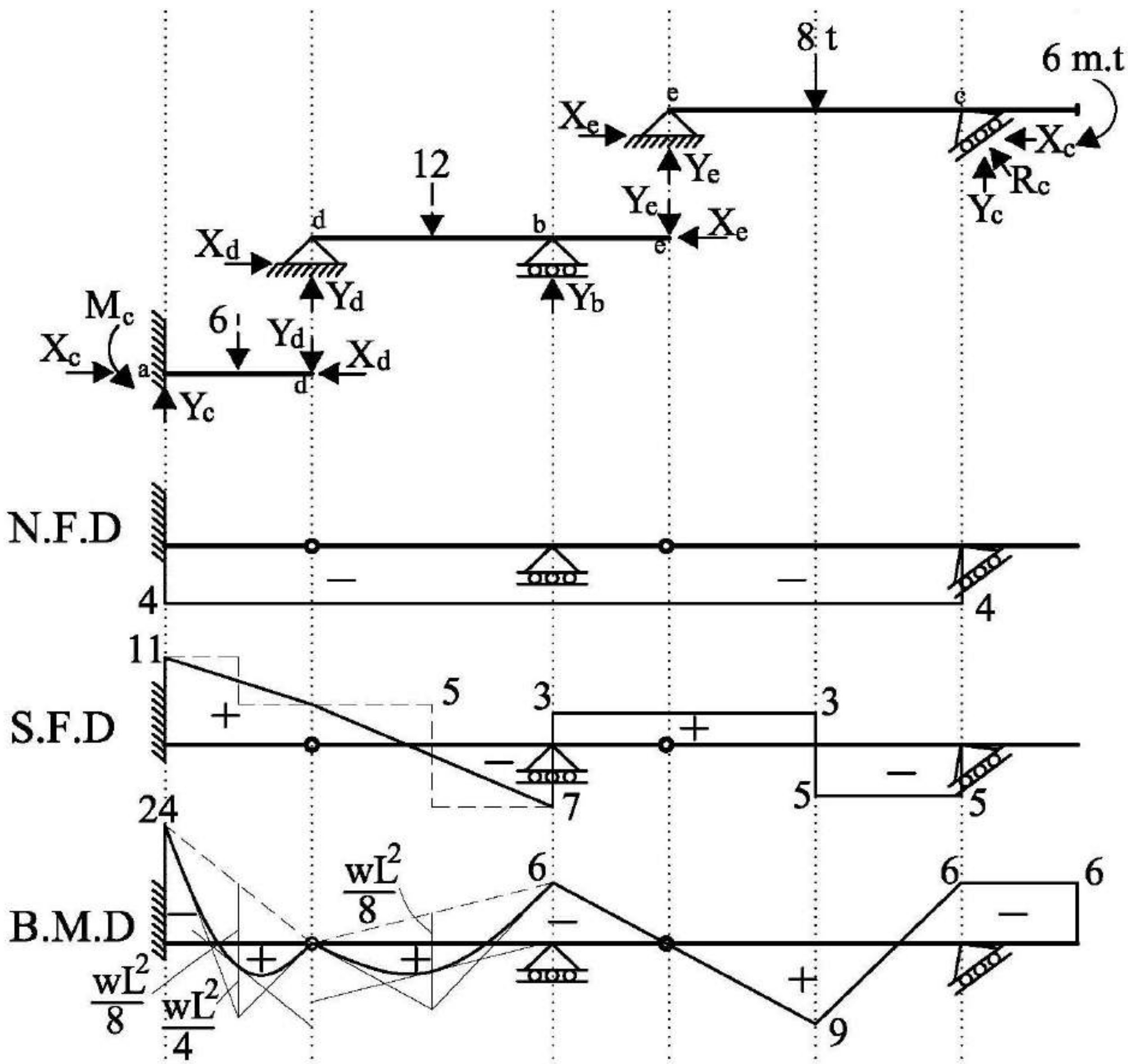
- *For part ad*

$\sum F_y = 0.0$ $\quad\quad$ $Y_c = 11$ t ↑

$\sum M_c = -\sum M_c$ fixed $\quad$ $M_c = 24$m.t

$\sum F_x = 0.0$ $\quad\quad$ Xa = 4 t →

Fig. 3. 38.b



Example (31)

Draw the normal force, shear force and bending moment diagram due to the applied load for the given compound beam in Fig. 3.39.

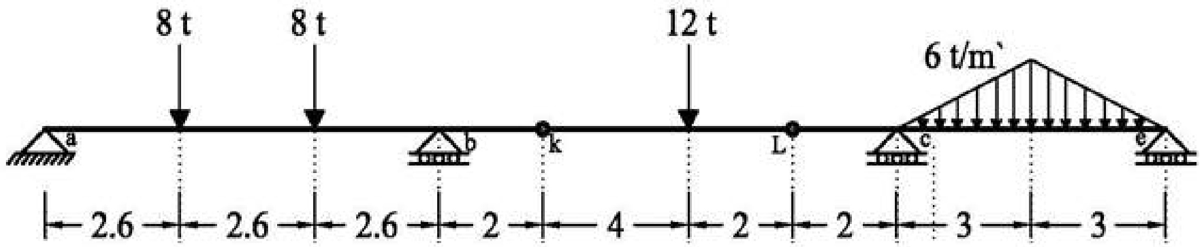


Fig. 3.39.a

- ***<u>Finding the reactions</u>***
- *For part k-L*

$\sum M_k = 0.0 \quad \gg 12x4 - 6Y_L = 0.0 \qquad \gg Y_L = 8t \uparrow$

$\sum M_L = 0.0 \quad \gg 12x2 - 6Y_k = 0.0 \qquad \gg Y_k = 4t \uparrow$

$Check \quad \sum F_y = 12 - 8 - 4 = 0 \quad Ok$

- *For part a-b-k*

$\sum M_a = 0.0 \quad \gg 8x2.6 + 8x5.2 + 4x9.8 - 7.8Y_b = 0.0 \qquad \gg Y_b = 13t \uparrow$

$\sum M_L = 0.0 \quad \gg 8x2.6 + 8x5.2 - 4x2 - 7.8Y_a = 0.0 \qquad \gg Y_a = 7t \uparrow$

$Check \quad \sum F_y = 8 + 8 + 4 - 13 - 7 = 0 \quad Ok$

- *For part L-c-e*

$\sum M_c = 0.0 \quad \gg 18x3 - 8x2 - 6Y_e = 0.0 \qquad \gg Y_e = 6.33t \uparrow$

$\sum M_L = 0.0 \quad \gg 18x3 + 8x8 - 6Y_c = 0.0 \qquad \gg Y_c = 19.67t \uparrow$

$Check \quad \sum F_y = 18 + 8 - 6.33 - 19.67 = 0 \quad Ok$

- ***<u>For drawing B.M.D</u>***
- *For part a-b-k*

  M_1 from left side $= 7x2.6 = 18.2$ m.t

  M_2 from left side $= 7x5.2 - 8x2.6 = 15.4$ m.t

M_b from right side $= 4x2 = -8$ m. t

- *For part kL*

M_3 from left side $= 4x4 = 16$ m. t

- *For part Lce*

M_c from left side $= -8x2 = 16$ m. t

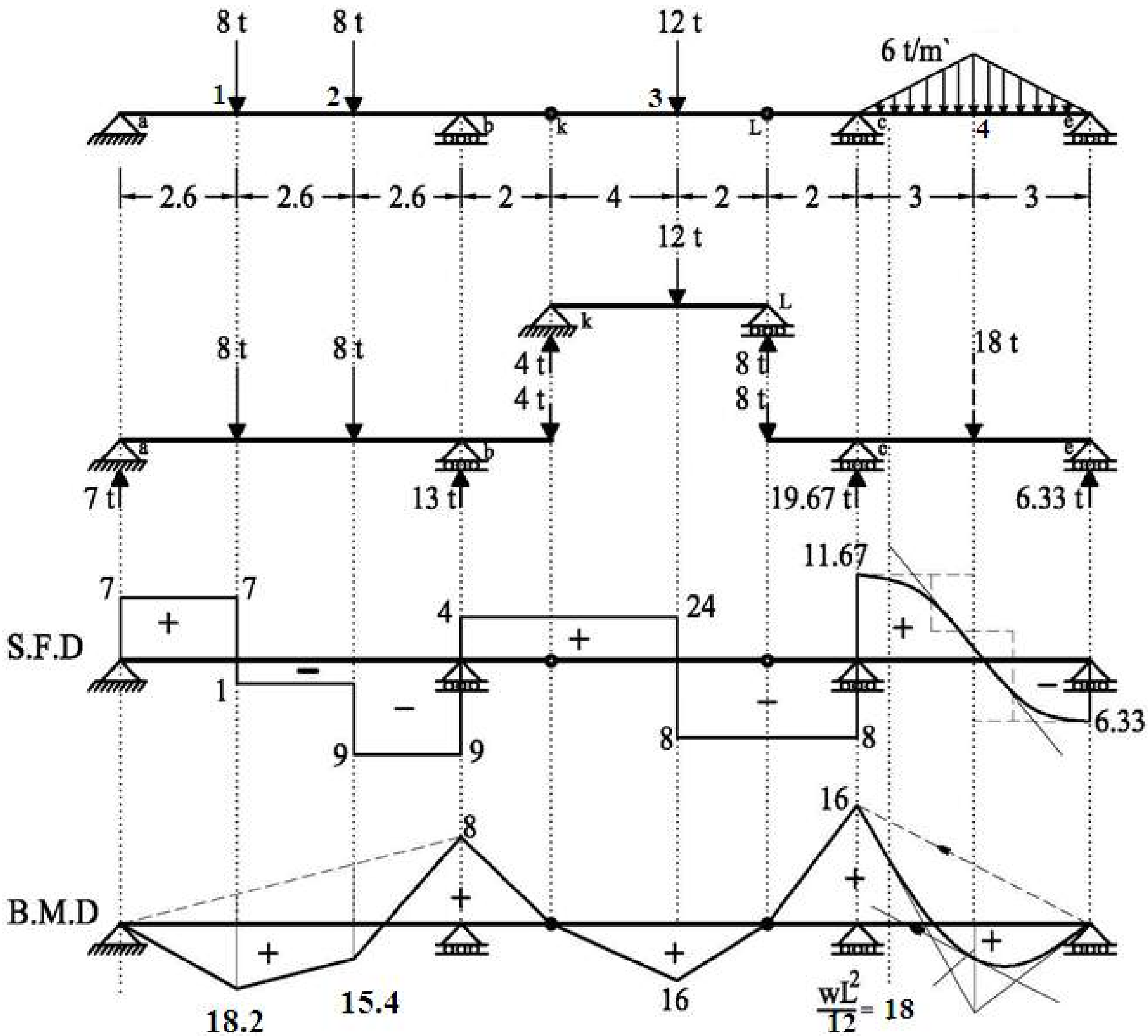


Fig. 3.39.b

Example (32)

For the given inclined beam in Fig. 3.8, draw the normal force, shear force and bending moment diagram due to the applied load.

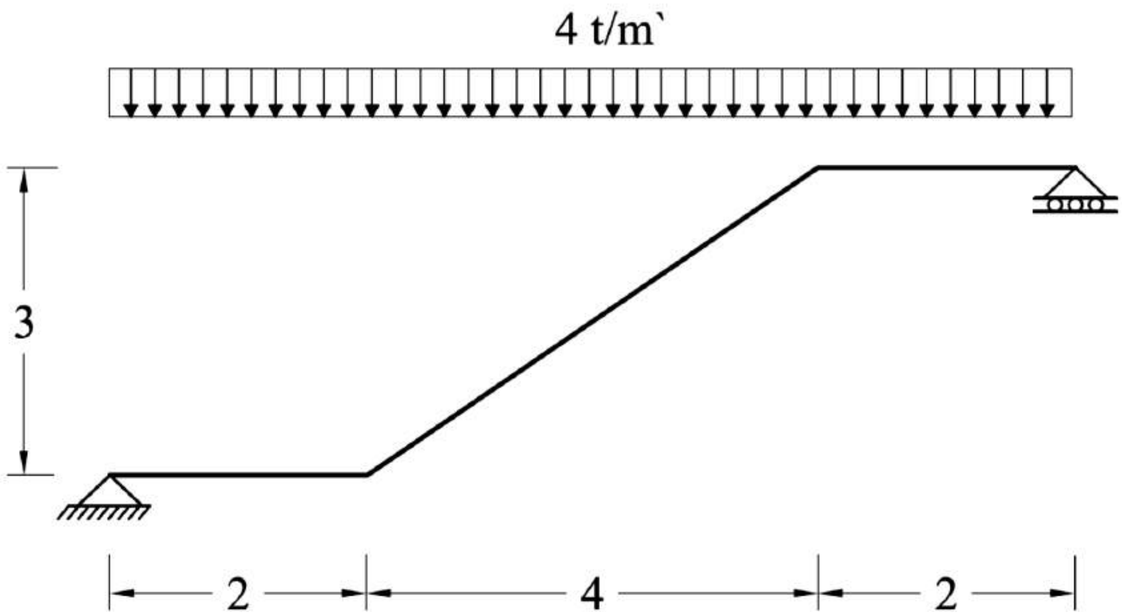


Fig. 3.40.a

- *Finding the reactions*

$\sum M_a = 0.0 \quad \gg 8x1 + 16x4 + 8x7 - 8Y_b = 0.0 \qquad \gg Y_b = 16t \uparrow$

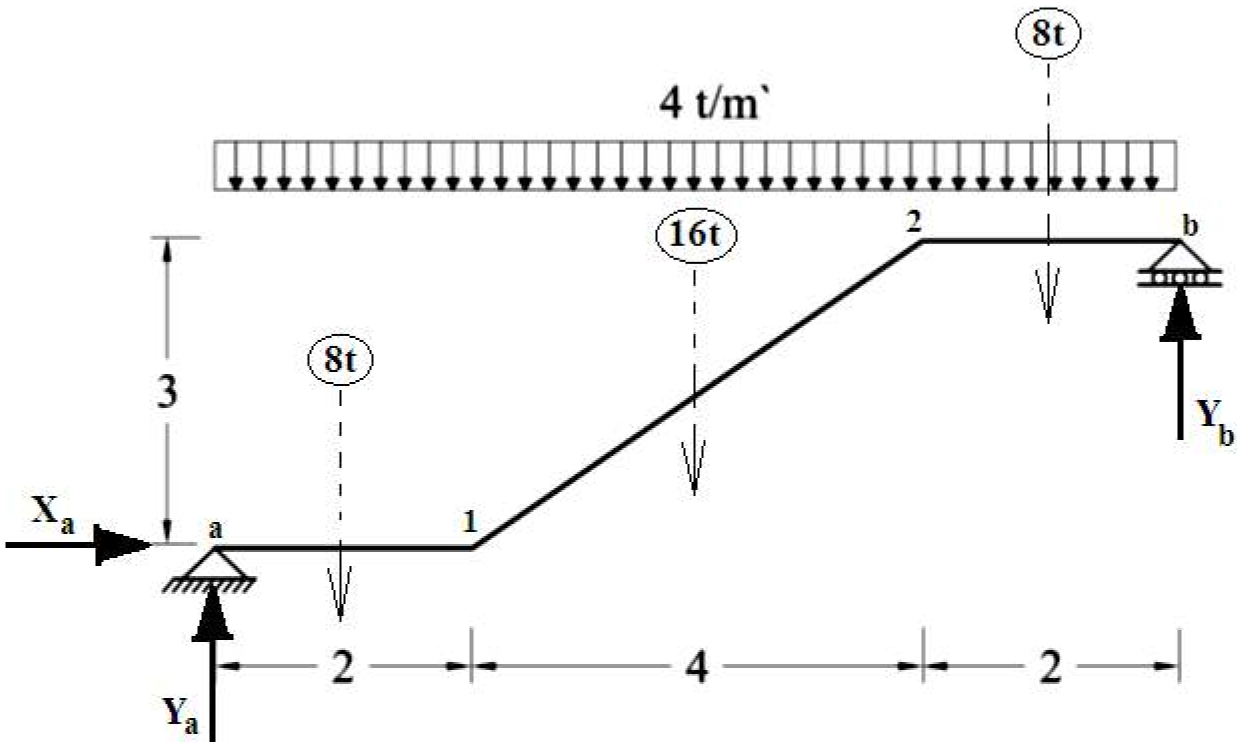
$\sum M_b = 0.0 \quad \gg \; 8x1 + 16x4 + 8x7 - 8Y_a = 0.0 \qquad \gg \; Y_a = 16t \uparrow$

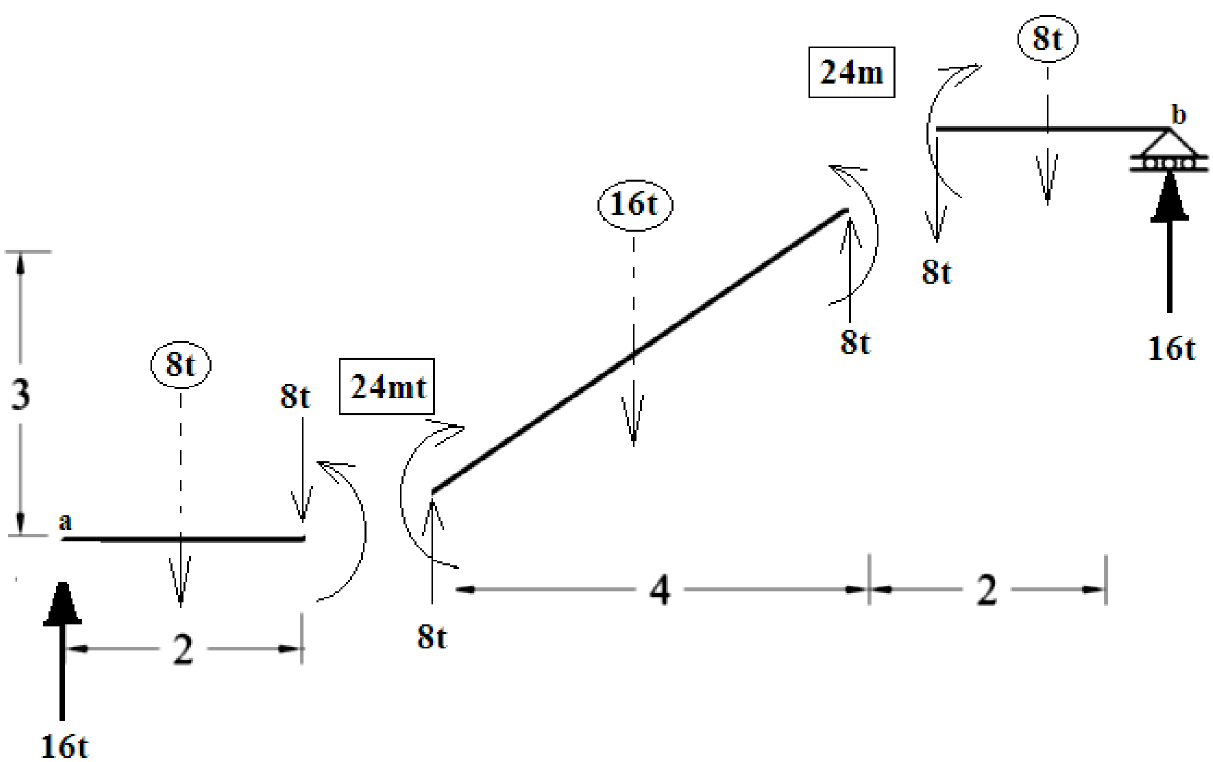
$Check \quad \sum F_y = 16 + 8 + 8 - 16 - 16 = 0 \quad Ok$

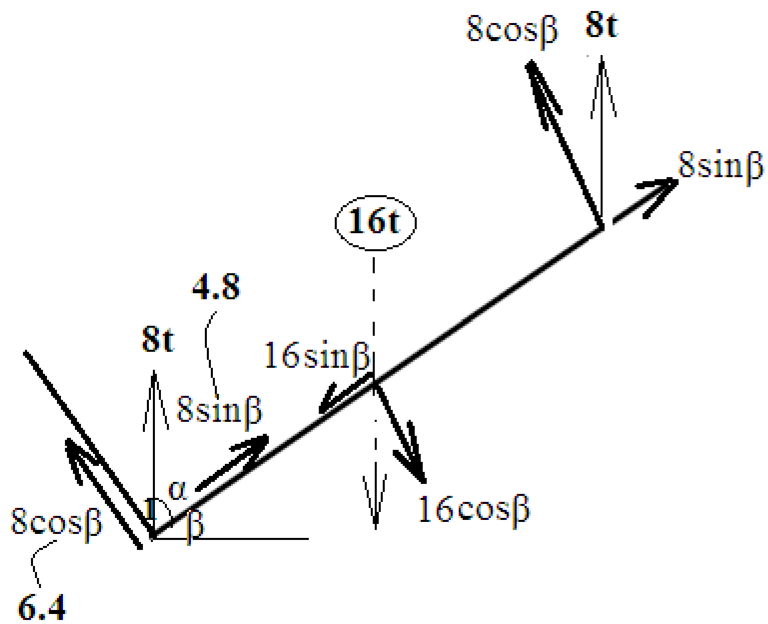
- *For drawing B.M.D*

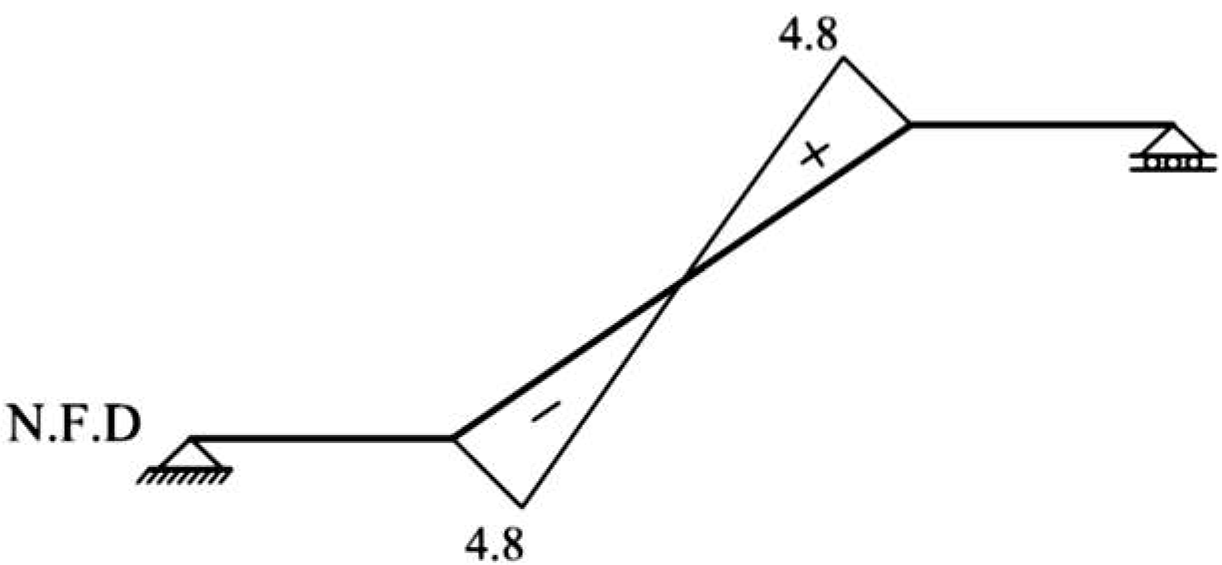
M_1 from left side $= \; 16\text{x}2 - 8\text{x}1 \; = \; 24 \text{ m.t}$

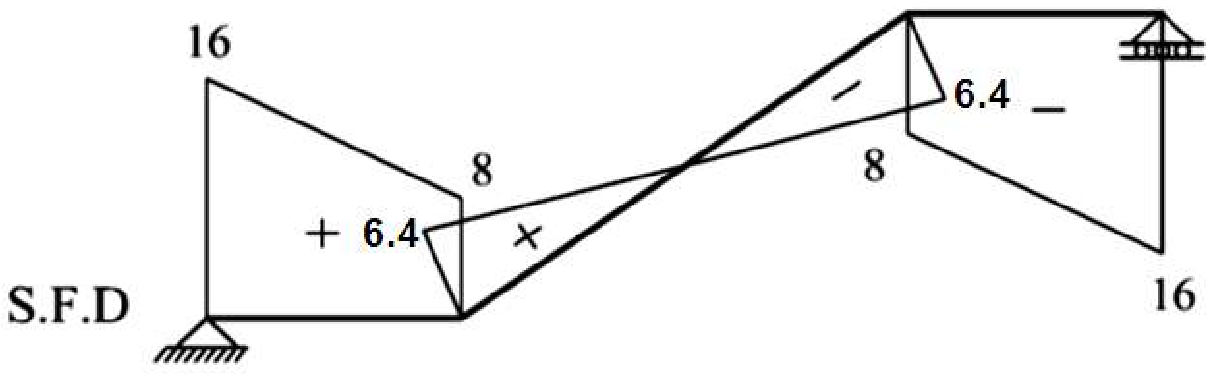
M_2 from right side $= \; 16\text{x}2 - 8\text{x}1 \; = \; 24 \text{ m.t}$


8t
4 t/m`
16t
2
b
8t
3
Y_b
X_a
a
1
2
4
2
Y_a


8t
24m
b
16t
8t
8t
16t
3
8t
8t
24mt
a
4
2
2
8t
16t


8cosβ
8t
8sinβ
16t
4.8
8t
16sinβ
8sinβ
α
β
8cosβ
16cosβ
6.4


4.8
+
N.F.D
−
4.8



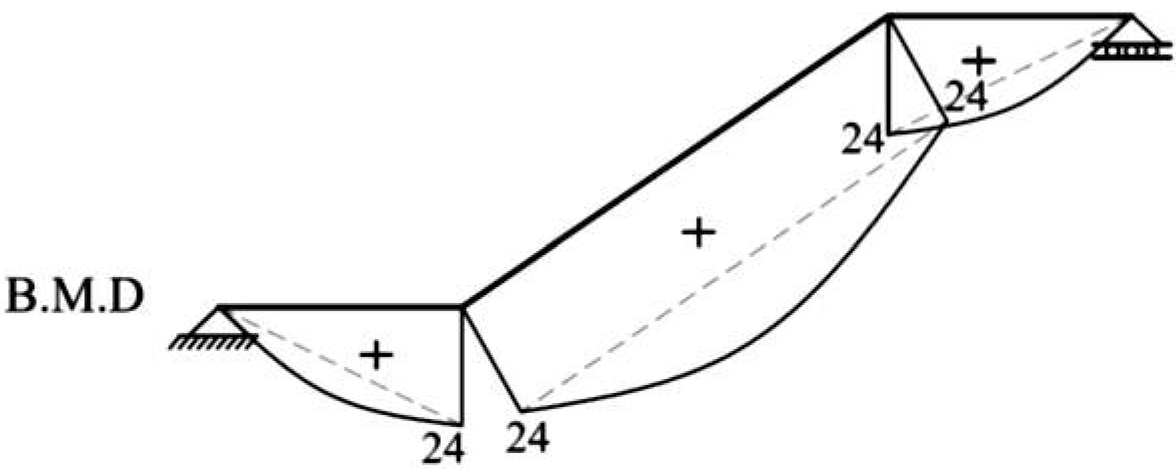


Fig. 3.40.b

Example (33)

For the given trussed beam in Fig. 3.41, draw the normal force, shear force and bending moment diagram due to the applied load.

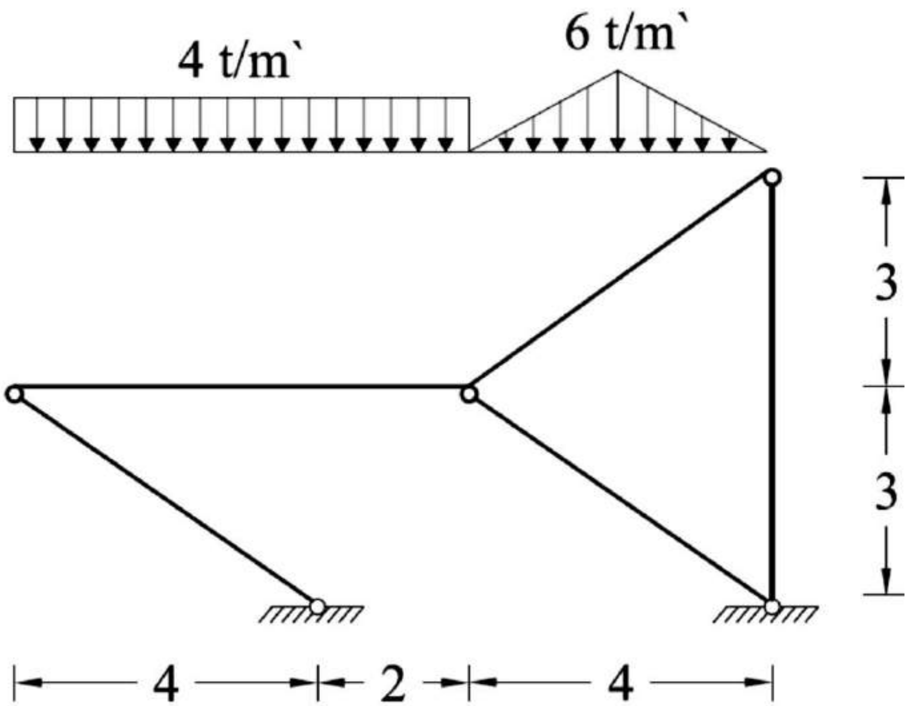


Fig. 3.41.a

Solution

- *Finding the reactions*

$\sum M_a = 0.0 \quad \gg -24x1 + 12x4 - 6Y_b = 0.0 \qquad \gg Y_b = 4t \uparrow$

$\sum M_b = 0.0 \quad \gg 24x7 + 12x2 - 6Y_a = 0.0 \qquad \gg Y_a = 32t \uparrow$

$Check \quad \sum F_y = 24 + 12 - 4 - 32 = 0 \quad Ok$

$$\mathrm{Y_a} = \frac{3}{5}\mathrm{R_a} \qquad \text{and } \mathrm{X_a} = \frac{4}{5}\mathrm{R_a} \qquad \ggg \mathrm{X_a} = \frac{4}{3}\mathrm{Y_a} \quad = 42.667\ t \leftarrow$$

$$\mathrm{R_a} = \frac{5}{3}\mathrm{Y_a} \quad \gg\gg\gg 53.333\ \mathrm{t\ (comp)}$$

$\sum F_x = 0.0 \qquad X_b = 42.667\ t \longrightarrow$

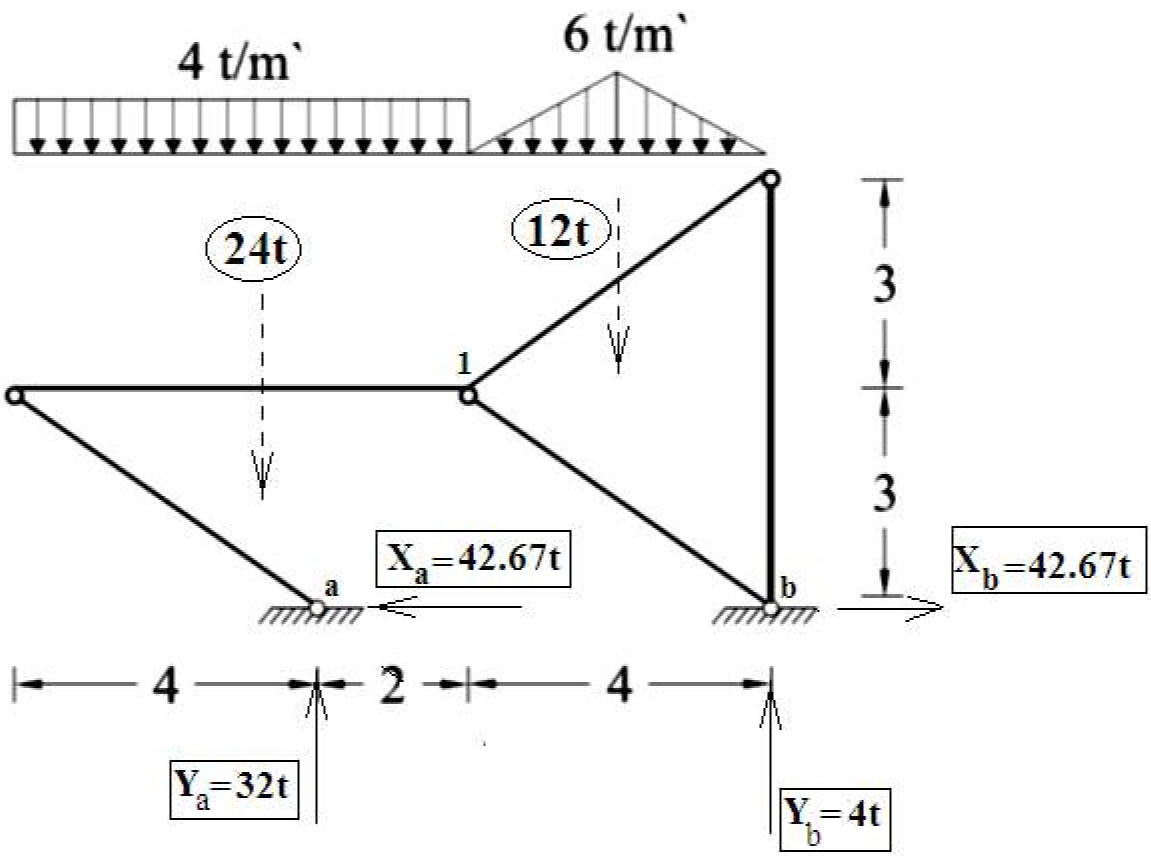


Fig. 3.41.b

To draw the internal forces, consider the equilibrium of joint#a and joint#b as shown in Fig.3.31.c

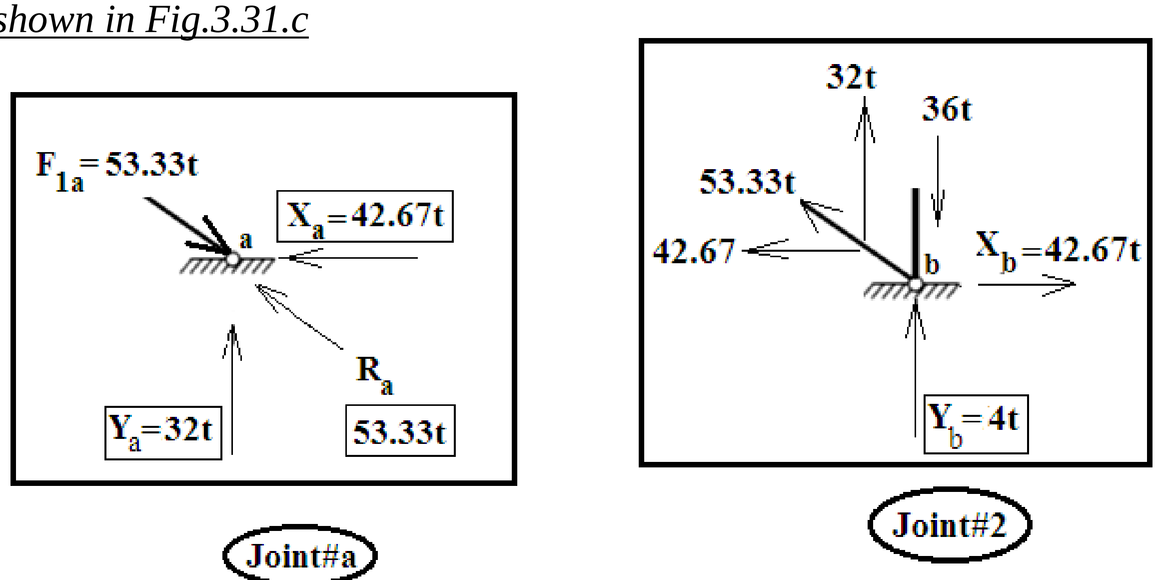
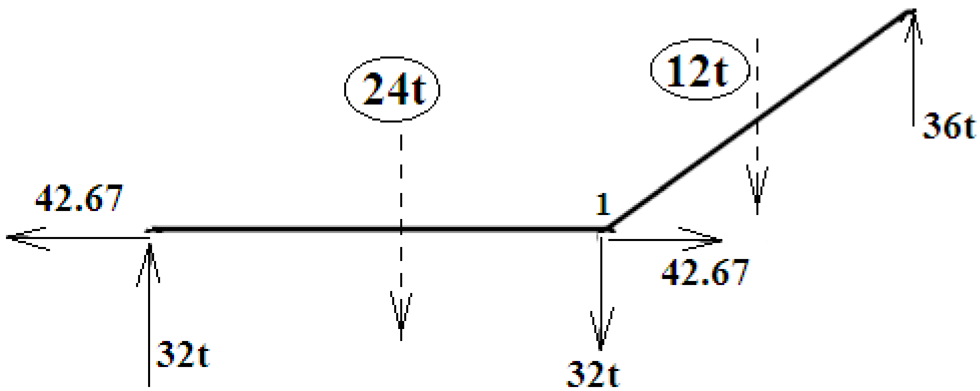
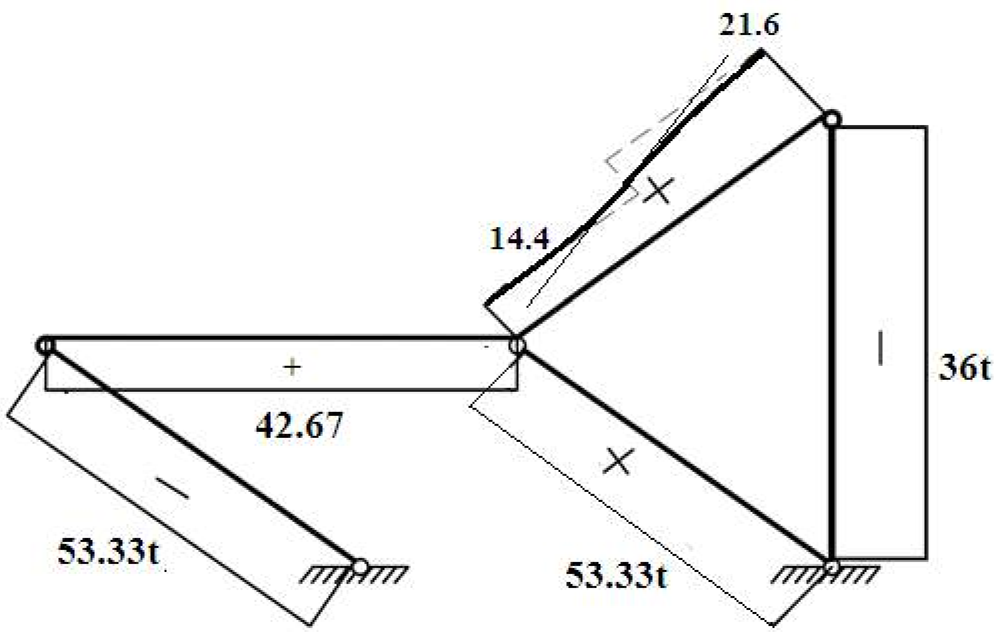
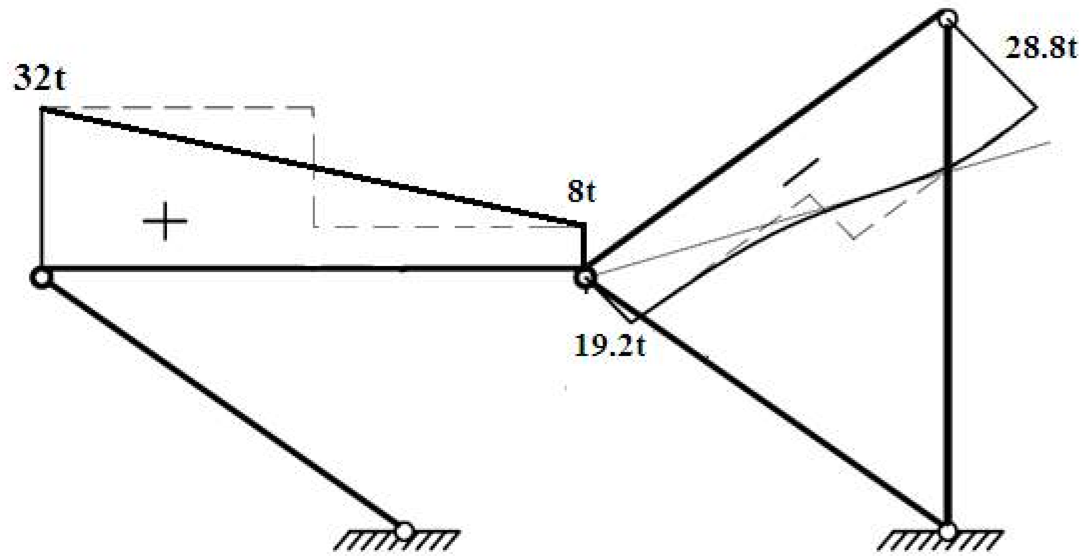


Fig. 3.41.c

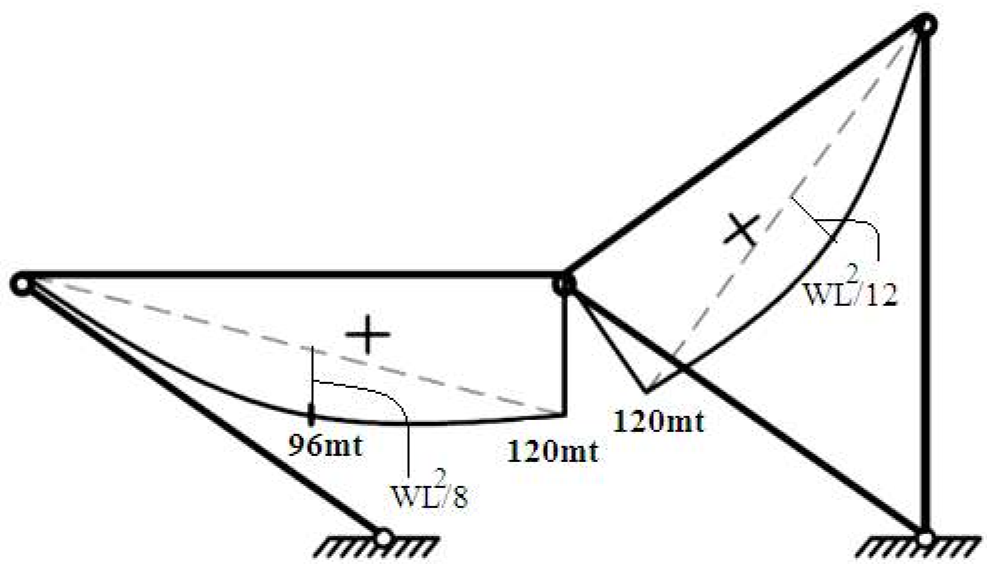




N.F.D



S.F.D



B.M.D

Fig. 3.41.d

Example (34)

For the given continuous beam in Fig. 3.42, draw the normal force, shear force and bending moment diagram due to the applied load if the bending moments at support a, b and are indicated.

M_a = -20.31 m.t $\qquad$ M_b = -20.125 m.t $\qquad$ M_a = -19.90 m.t

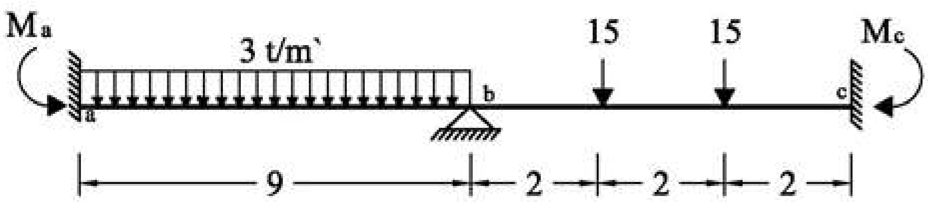


Fig. 3.42.a

Solution

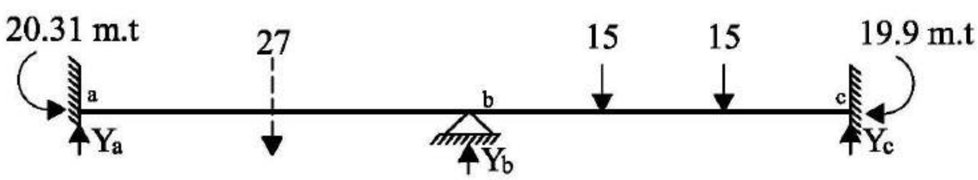


Fig. 3.42.b

$\sum M_b$ from left side $= -20.125 \quad \gg -20.31 - 27x4.5 + 9Y_a =$
$-20.125 \qquad\qquad\qquad\qquad Y_a = 13.52t \uparrow$

$\sum M_b$ from right side $= -20.125 \quad \gg -19.90 - 15x2 - 15x4 + 6Y_c =$
$-20.125 \qquad\qquad\qquad\qquad Y_c = 14.963t \uparrow$

$\sum M_a$ from left side $= -20.31 \quad \gg -27x4.5 - 15x11 - 15x13 - 19.9 +$
$14.96x15 + 9Y_b = -20.31 \qquad\qquad Y_b = 28.52t \uparrow$

$Check \quad \sum F_y = 27 + 15 + 15 - 13.52 - 14.963 - 28.52 = 0 \quad Ok$

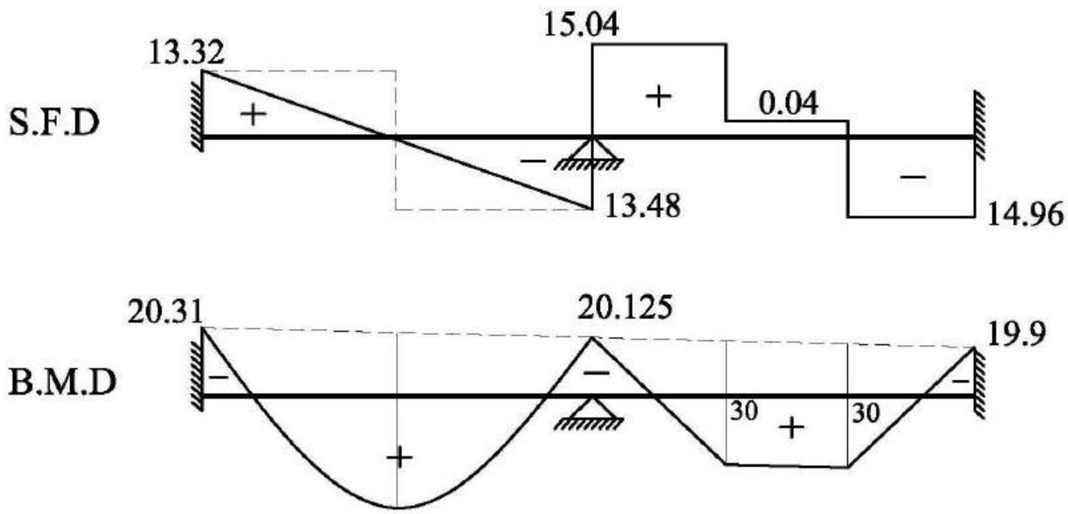


Fig. 3.42.c

Example (35)

For the given a bending moment diagram in Fig. 3.43, put the static system of a beam to give this bending diagram.

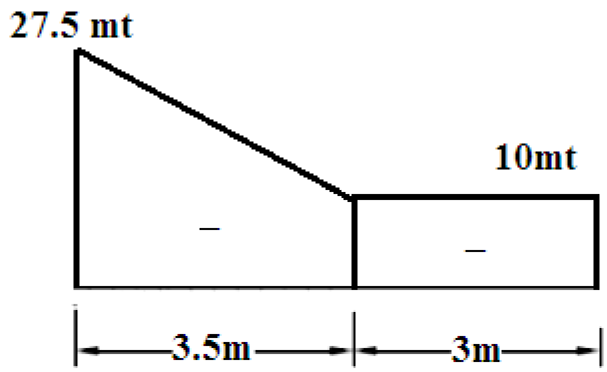


Fig. 3.43.a

Solution

1- The structural system of the beam that has bending moment diagram indicated in Fig. 3.43.a must be cantilever beam because this moment is negative moment.
2- The location of fixed support is at point#a.
3- The bending moment in portion 1-2 is constant so that shear force in this part is zero and there is no force in this part and the applied load is bending moment and it equals 10mt ($Q = \frac{dM}{dx}$ $and\ p = \frac{dQ}{dx}$).
4- The bending moment diagram in portion a-1is a straight line so that the shear force diagram is constant and the applied load is concentrated load at point#1. The value of the concentrated load P1 can be calculated as below:-
5- $M_a = 10 + Px3.5 \ggg P = \frac{(27.5-10)}{3.5} = 5t \downarrow$

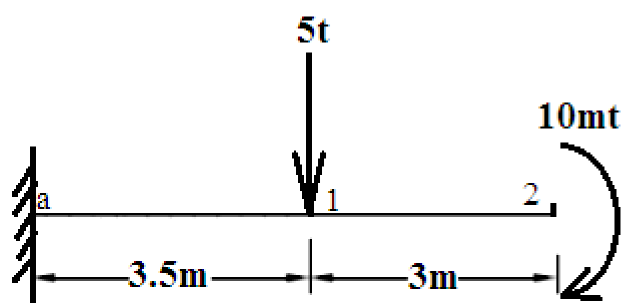


Fig. 3.43.b

Example (36)

The given diagram in Fig. 3.44 is a bending moment. Put the applied load on the beam to give this bending diagram.

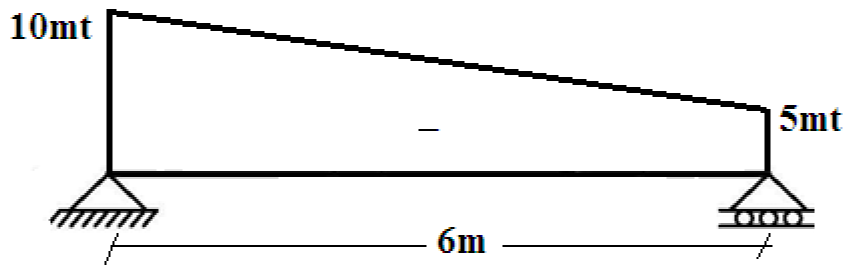


Fig. 3.44.a

Solution

In the natural case, the bending moment is zero at the end hinge and roller supports. In the beam shown in Fig. 3.44.a, the beam has bending moments at the two supports. This means that there are two moments at the two supports; M_a=-10mt and M_b=-5mt. Additionally there are any different in the bending moment in span a-b or the slop of the bending moment is zero. This means that there are not applied load in this span.

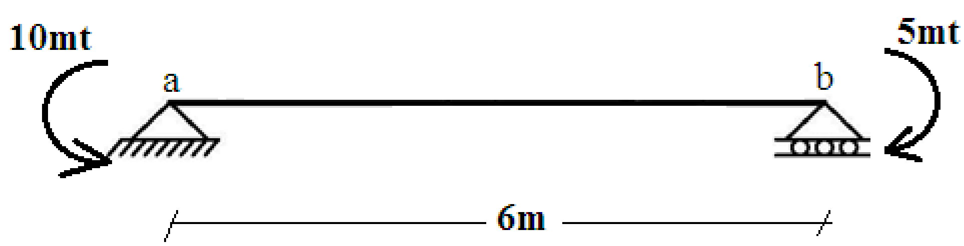


Fig. 3.44.b

__Example (38)__

The given diagram in Fig. 3.45 is a bending moment. Conclude the applied load on the beam to give this bending diagram.

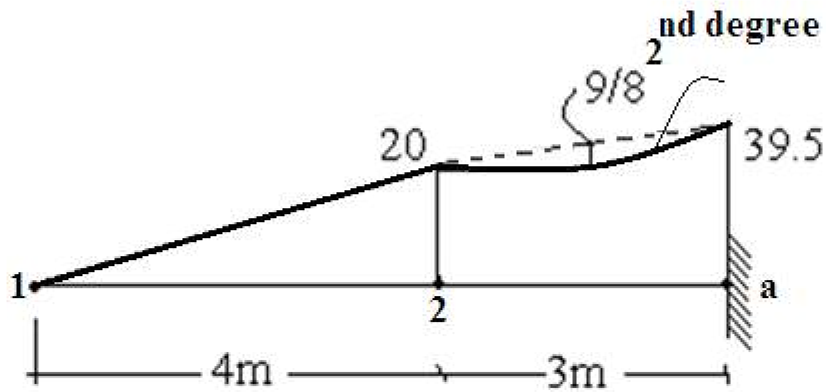


Fig. 3.45.a

Solution

1- The bending moment in portion 1-2 is a straight line started with zero at a and with value at point#2 so that shear force in this part is constant and the applied is concentrated load (P) at point#1 ($Q = \frac{dM}{dx}$ and $p = \frac{dQ}{dx}$).

2- The value of the concentrated load can be calculated from the moment at point#2

$$M_2 = Px4 \ggg P = 5t \downarrow$$

3- The bending moment diagram in portion a-2 is a parabola from the second degree so that the shear force diagram is a straight line and the applied load is uniform load in this portion.

4- The intensity of the uniform load can be calculated as below:-

$$\frac{WL^2}{8} = \frac{9}{8} \therefore w = 1\ t/m$$

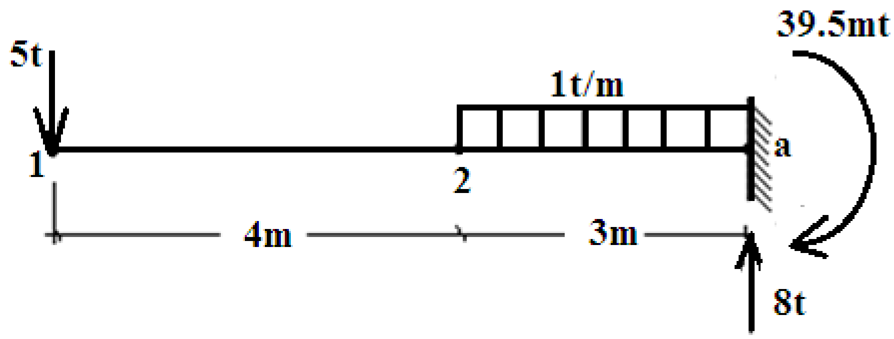


Fig. 3.45.b

Example (38)

For the given compound beam in Fig. 3.46, conclude the applied load (P) to give negative moment at support b equals the maximum positive moment in span a-b.

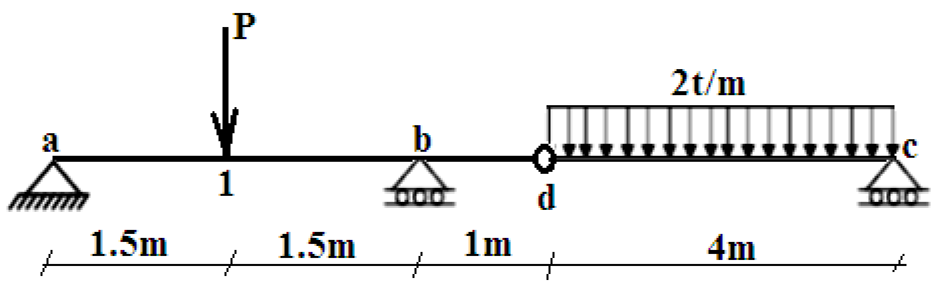


Fig. 3.46.a

Solution

- *For part d-c*

$F_x = 0.0 \quad X_a = 0.0$

$\sum M_d = 0.0 \quad \gg 8x2 - 4Y_c = 0.0 \qquad \gg Y_c = 4t \uparrow$

$\sum M_c = 0.0 \quad \gg \; 8x2 - 4Y_d = 0.0 \qquad \gg \; Y_d = 4t \uparrow$

$Check \quad \sum F_y = 8 - 4 - 4 = 0 \quad Ok$

- *For part a-d*

  $M_d \; from \; right \; side = -4x1 = -4mt$

$$M_d = M_1 \; (Negative \; moment = Positive \; moment)$$

$$M_1 = Y_a x1.5 \gg Y_a = \frac{4}{1.5} = 2.67t \uparrow$$

$$Y_d = 6.67 \text{ t} \uparrow$$

$M_d = 2.67x3 - 1.5P$

$-4 = 2.66x3 - 1.5P \ggg P = 8t \uparrow$

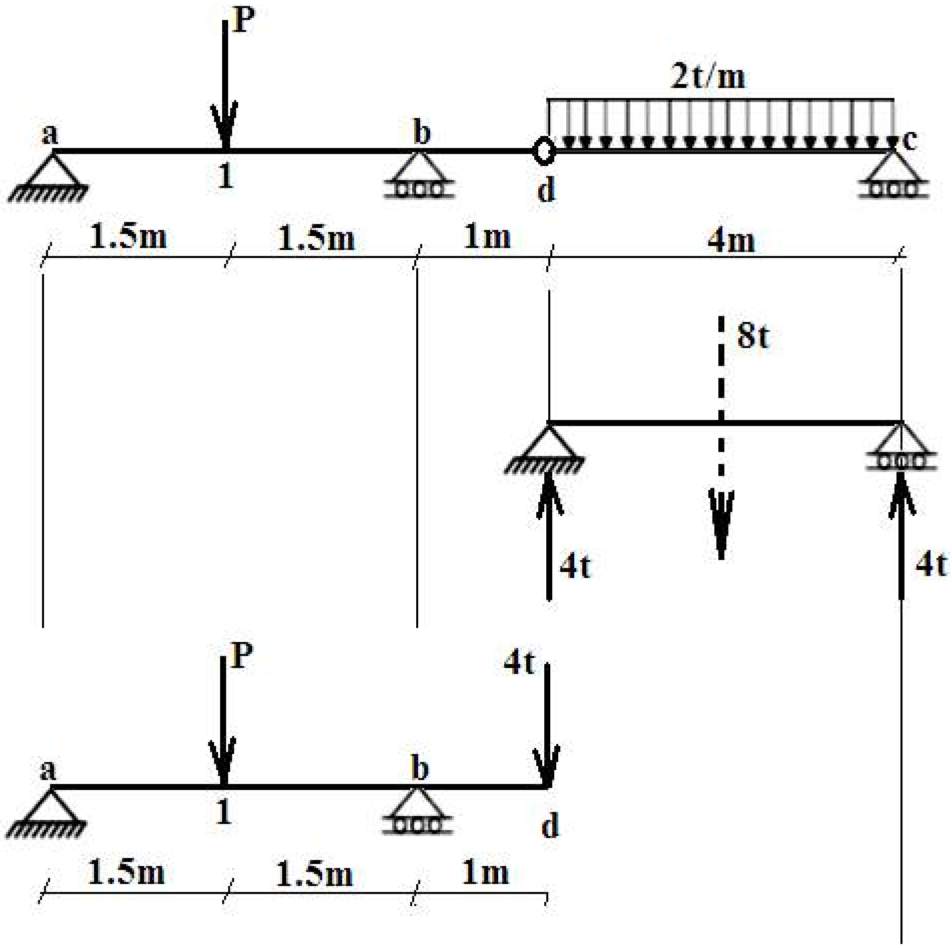


Fig. 3.46.b

Chapter (4)

INTERNAL FORCES IN THE FRAME STRUCTURES

4.1. Introduction

In the frame structure system, the beam girders are connected to the columns to transfer the bending moment and the other components or members (floors and roof panels) are not connected to the columns (and called secondary members). Framing systems are the basic structure used in the majority of new residential construction to get larger spans to caver large area without inter columns as shown in Fig. 4.1. It may comprise of wood, steel or concrete members.

Fig. 4.1: Steel frame building

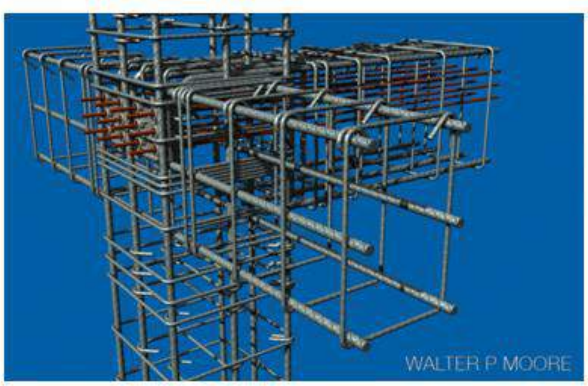
The difference between the ordinary building and frame structures is the connection between the column and beam. There are two types of connection between them as indicated below:-

1. Flexible Joint or Pin Joint

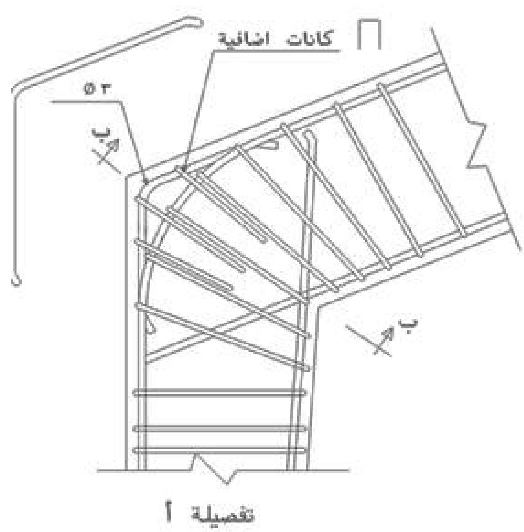
   It is a hinge which can transfer forces only. In this case, the column and beam end moments are both equal to zero ($M_{col} = M_{beam} = 0$). This connection is constructed by separating the reinforcements of the columns and beams in the case of reinforced concrete building. This connection is illustrated in Fig. 4.2 and it is modeled in Fig. 4.3.

2. Rigid Joint

   In this joint, bending moment can be transmitted through the connection. Also in a rigid connection, the end moments and rotations are equal ($M_{beam} = M_{col} \neq 0$). This connection is constructed by inserting the reinforcements of the columns in the beams. This connection is illustrated in Fig. 4.2 and it is modeled in Fig. 4.3.

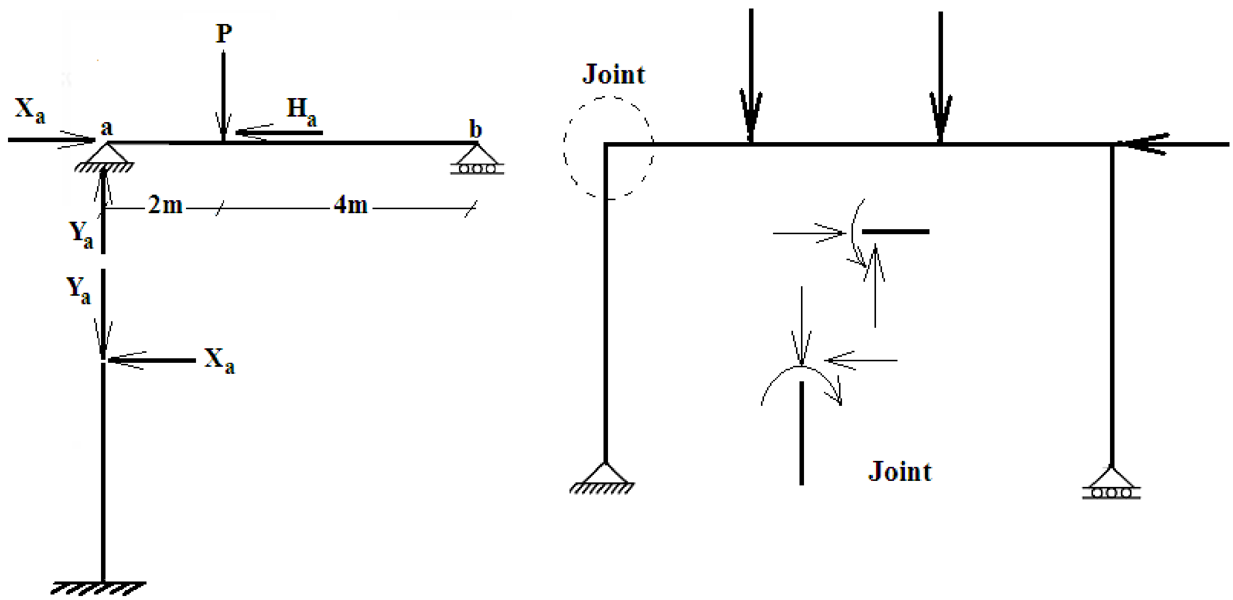


a) Connection in beam



b) Connection in frame

Fig. 4.2: Connection between reinforced concrete beam and column



a) Ordinary building b) Framed structures

Fig. 4.3: Modeling of the connection between the beam and column

4.2. Statically Determinate System of the Frame

A statically determinate system is a system in which all the reactions of supports can be determined by means of equations of equilibrium ($\sum F_x, \sum F_y$ and $\sum M$) and the internal forces also can be found by method of sections. A statically indeterminate system means that the reactions and internal forces cannot be analyzed by the application of the equations of static alone. The below equation is used to find the degree of indeterminacy in frame structures.

$$I = (3M + R) - 3J - n$$

Where:-

I is the degree of indeterminacy

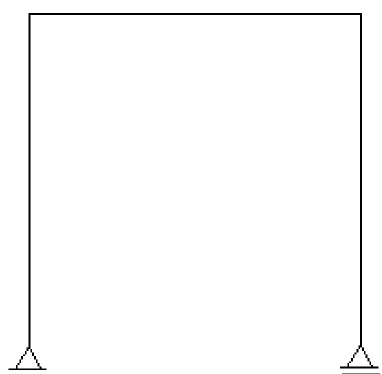
M is the number of members

R is the number of the reactions

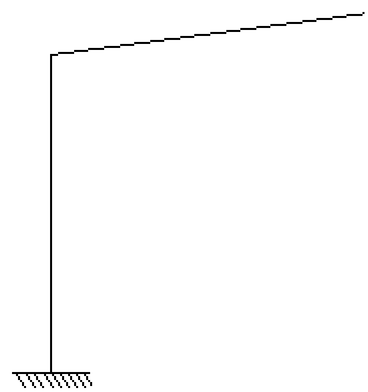
J is the number of the joints

n is the number of the intermediate hinge

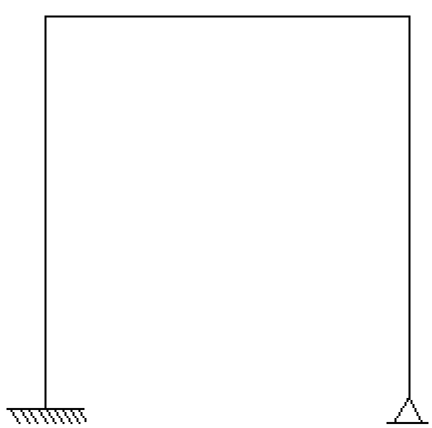
Examples: Calculate the degree of indeterminacy in the below frames:-



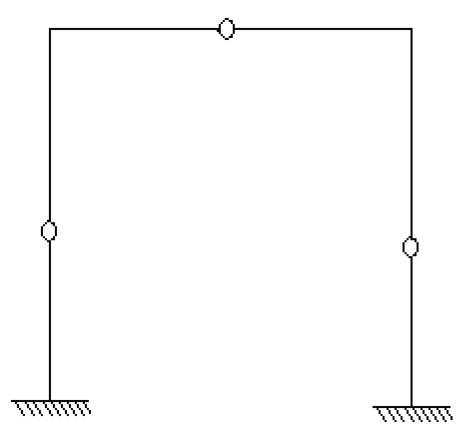
a) Frame#1



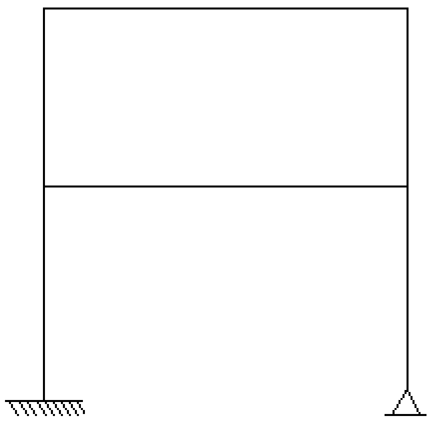
b) Frame#2



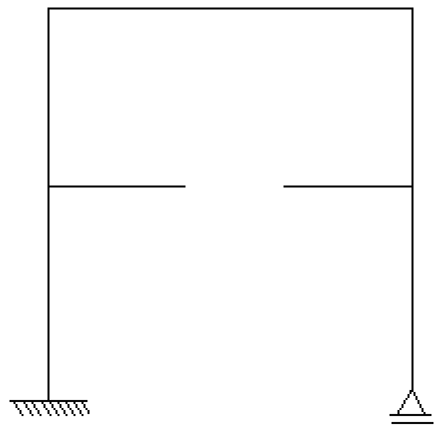
c) Frame#3



d) Frame#4



e) Frame#5



f) Frame#6

$For\ (a) \gg I = (3x3 + 3) - 3x4 - 0 = 12 - 12 = 0$

$For\ (b) \gg I = (3x2 + 3) - 3x3 = 9 - 9 = 0$

$For\ (c) \gg I = (3x3 + 4) - 3x4 - 0 = 13 - 12 = 1$

$For\ (d) \gg I = (3x6 + 6) - 3x7 - 3 = 24 - 21 - 3 = 0$

$For\ (e) \gg I = (3x6 + 4) - 3x6 - 0 = 22 - 18 = 4$

$For\ (f) \gg I = (3x4 + 4) - 3x4 - 0 = 13 - 12 = 1$

4.3. Solved Examples

Example (1)

For the given frame in Fig. 4.4, draw the normal force, shear force and bending moment diagrams.

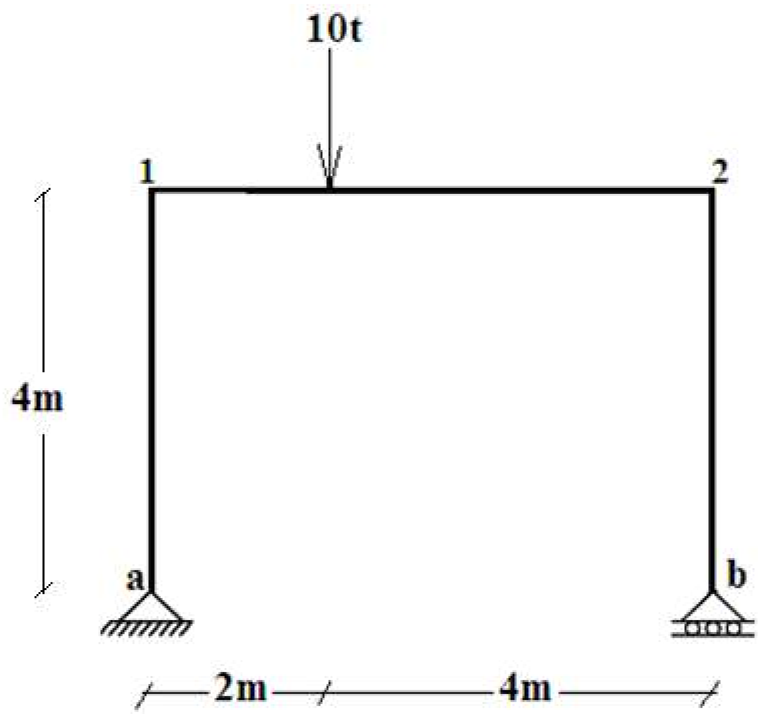


Fig. 4.4.a

Solution

- *Finding the reactions*

a) $\sum \mathrm{F_x} = 0 \ggg\ggg \mathrm{X_a} = 0$

b) $\sum M_a = 0 \ggg\ggg 10x2 - Y_b x6 = 0$

$$\therefore Y_b = \frac{20}{6} = 3.33t \uparrow$$

c) $\sum M_b = 0 \ggg\ggg 10x4 - Y_a x6 = 0$

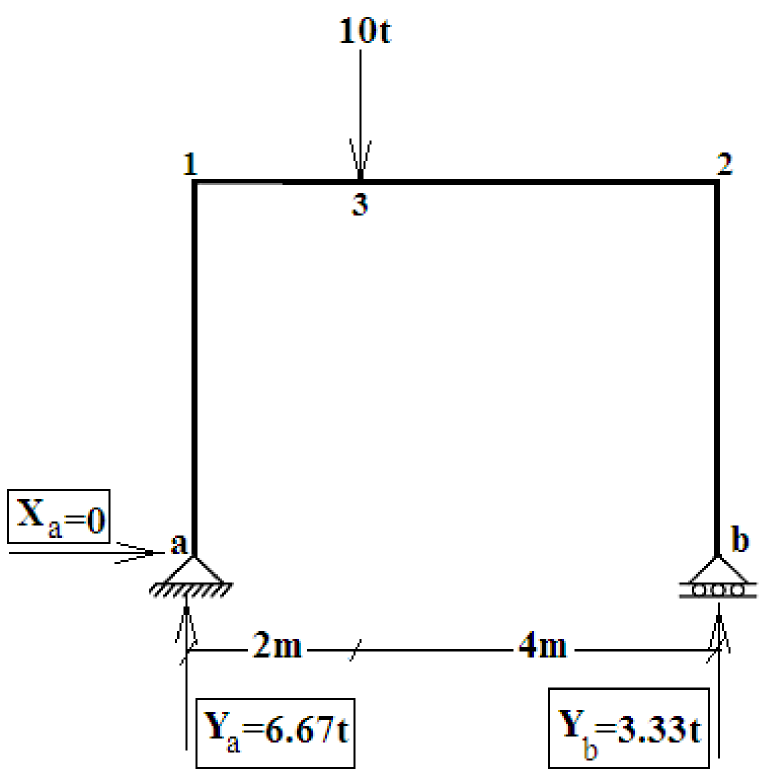
$$\therefore Y_a = \frac{40}{6} = 6.67\ t \uparrow$$

d) $\underline{Check} \sum \mathrm{F_y} = 6.67 + 3.33 - 10 = 0 \ggg\ggg \text{ok}$

- *For drawing B.M.D*

M_1 from left side = 0 M_2 from right side = 0

M_3 from left side = 6.67x2= 13.33 m.t



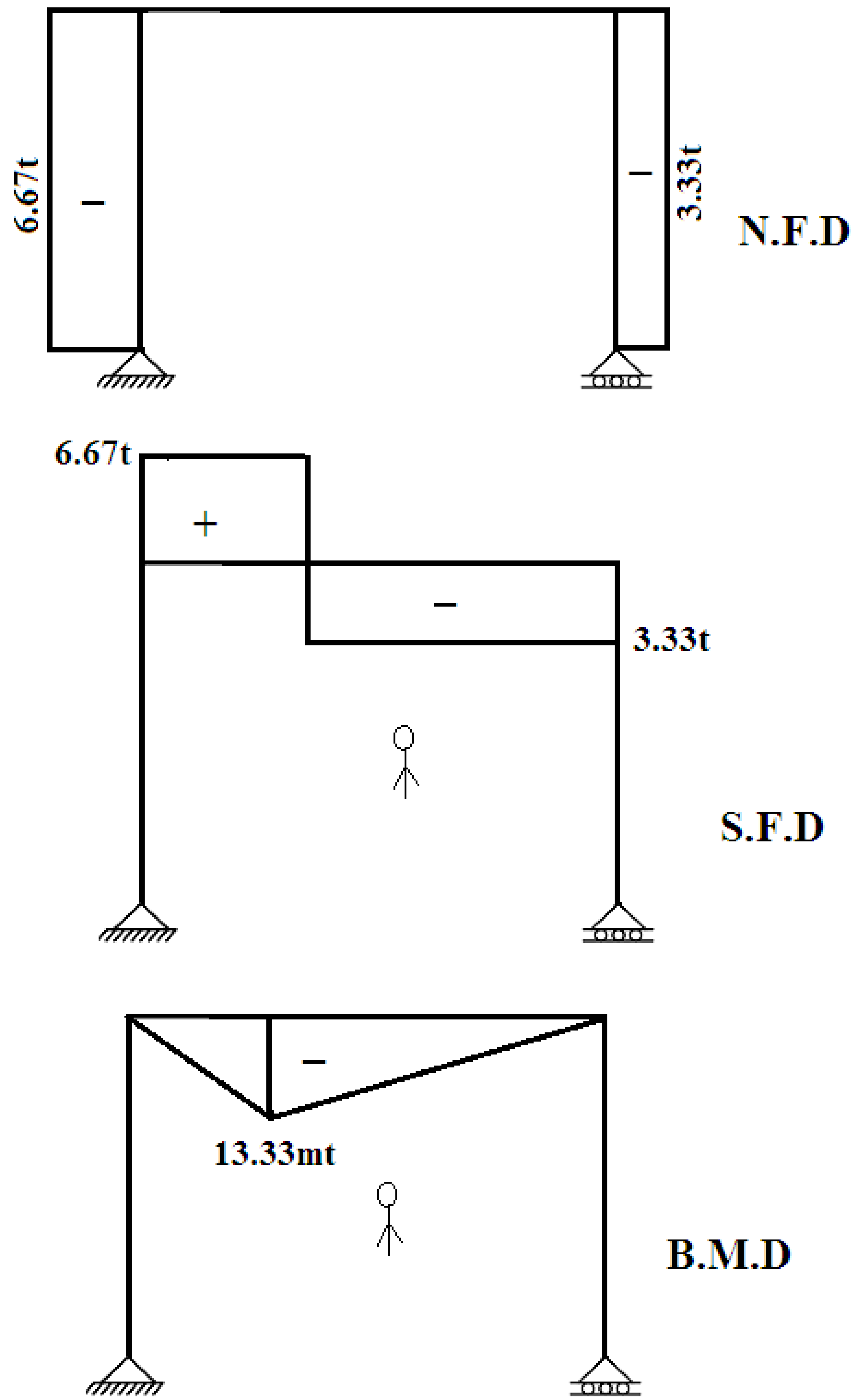


Fig. 4.4.b

Example (2)

For the given frame in Fig. 4.5, draw the normal force, shear force and bending moment diagrams.

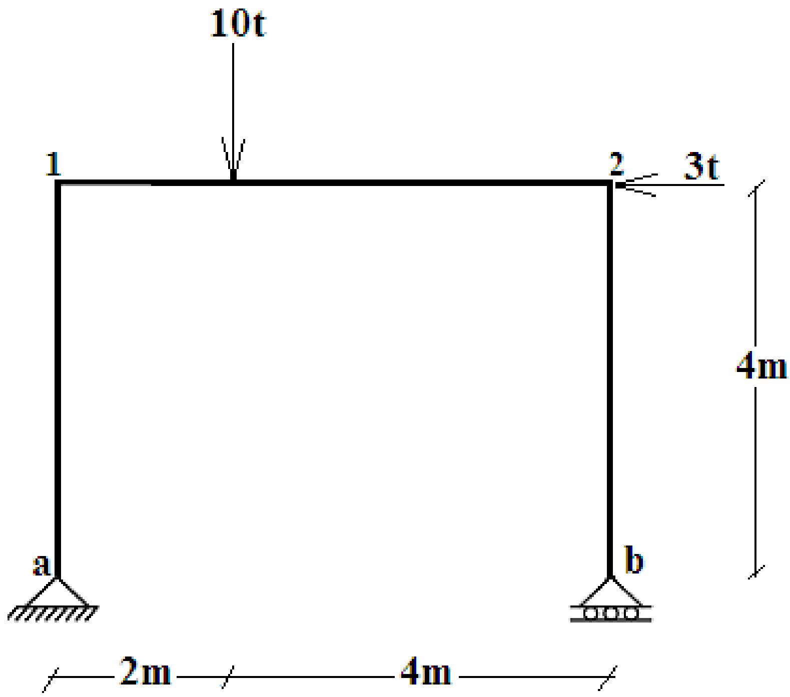


Fig. 4.5.a

Solution

- *Finding the reactions*

a) $\sum F_x = 0 \ggg\ggg X_a = 3t \rightarrow$

b) $\sum M_a = 0 \ggg\ggg 10x2 - 3x4 - Y_b x6 = 0$

$$\therefore Y_b = \frac{8}{6} = 1.33t \uparrow$$

c) $\sum M_b = 0 \ggg\ggg 10x4 + 3x4 - Y_a x6 = 0$

$$\therefore Y_a = \frac{52}{6} = 8.67\ t \uparrow$$

d) $\underline{Check \sum} F_y = 8.67 + 1.33 - 10 = 0 \ggg\ggg$ ok

- *For drawing B.M.D*

M_1 from left side = -3x4=-12mt

M_2 from right side = 0

M_3 from left side = -3x4+8.67x2= 5.34 m.t

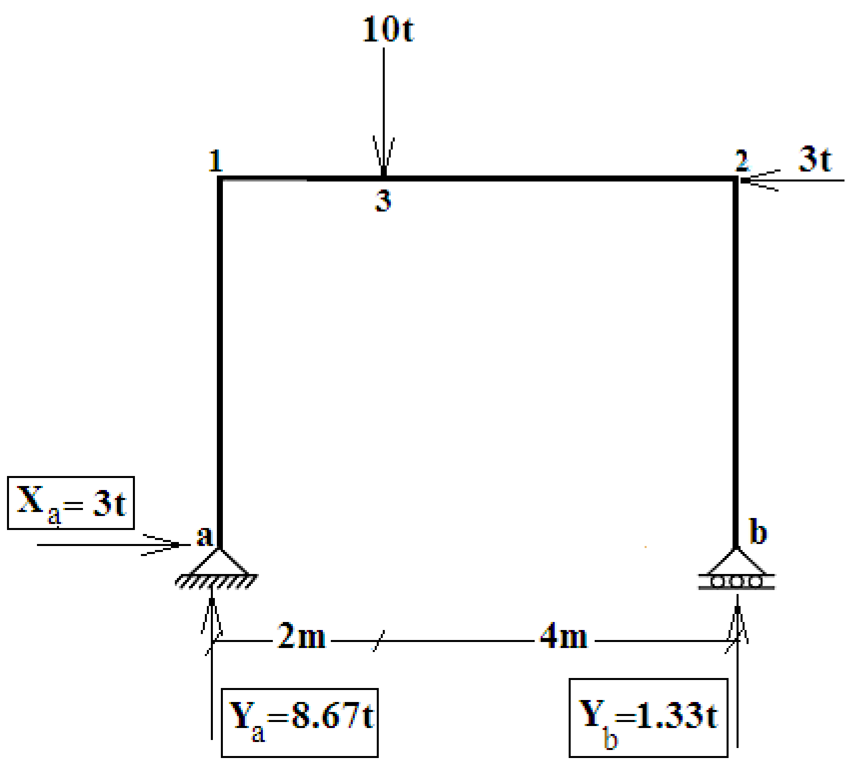
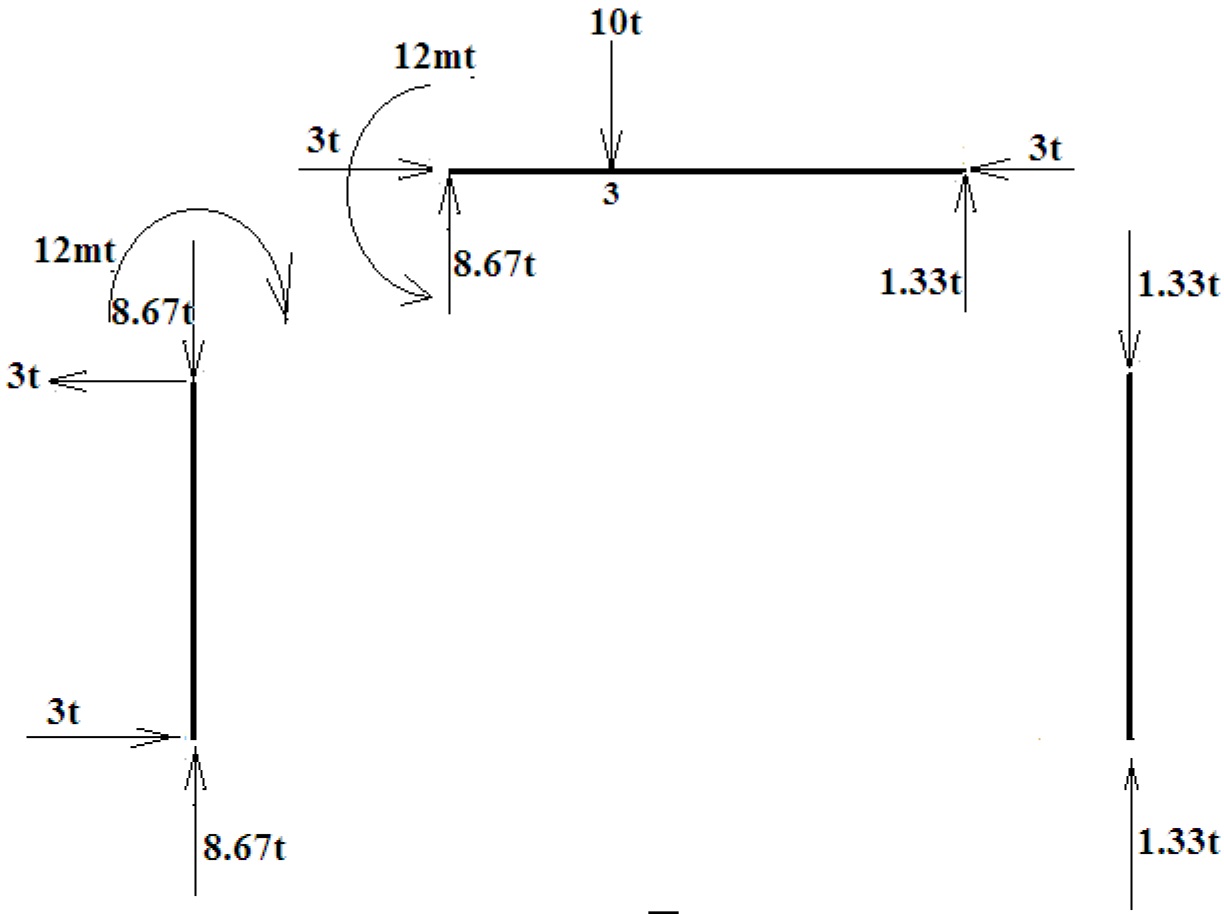
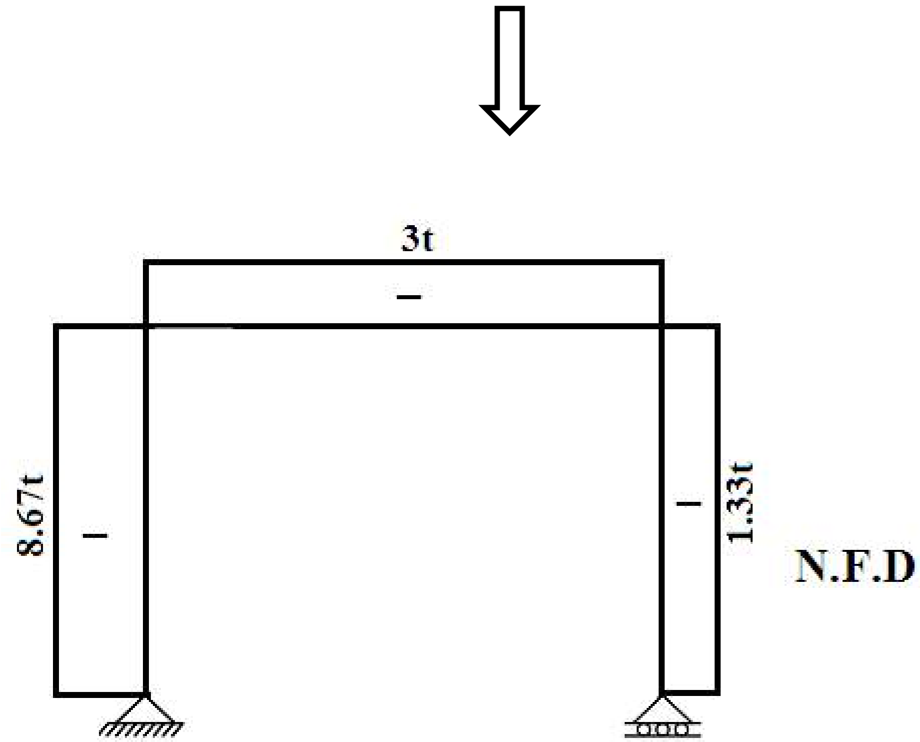


Fig. 4.5.b

⇩


10t
12mt
3t
3
3t
8.67t
1.33t
12mt
8.67t
3t
1.33t
3t
8.67t
1.33t


3t
–
8.67t
–
–
1.33t
N.F.D

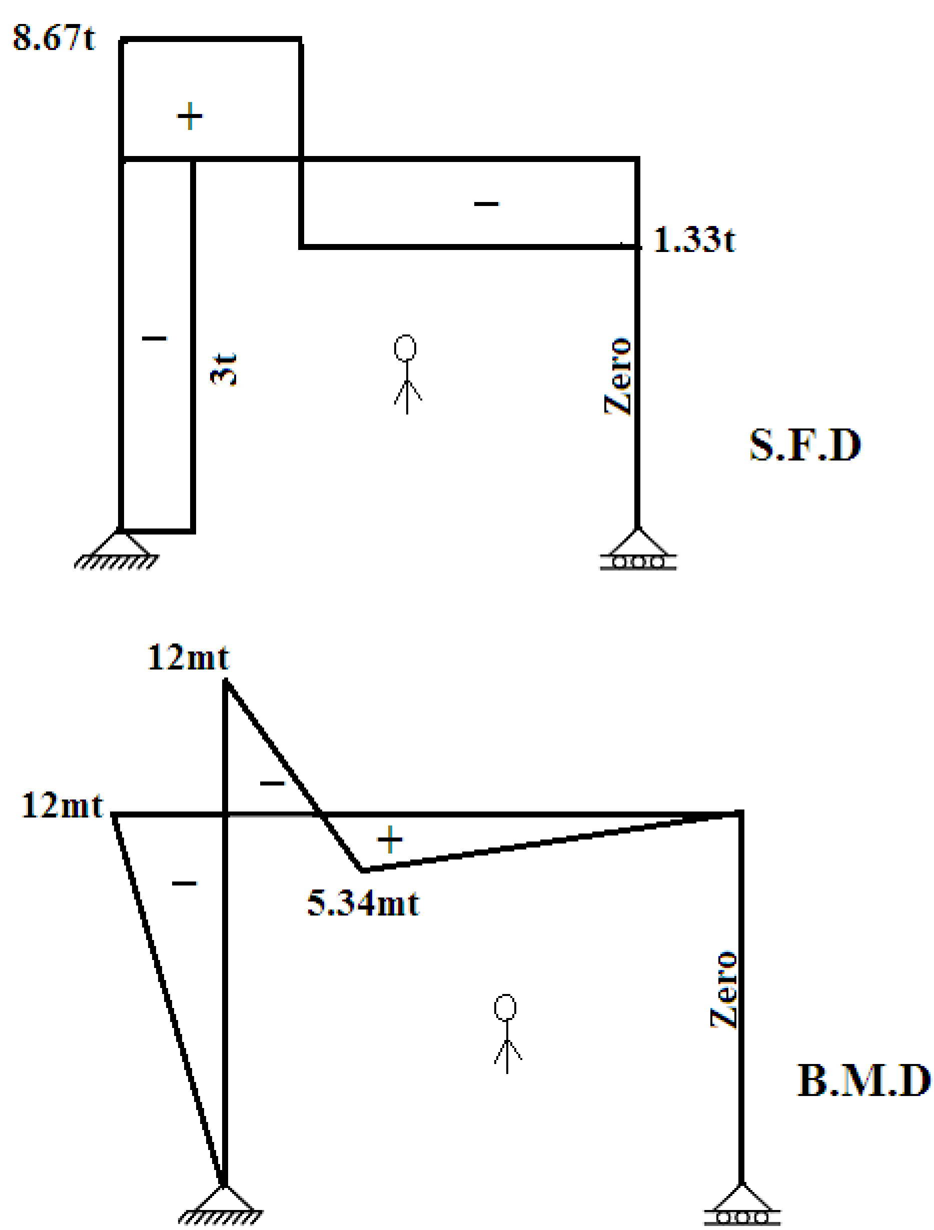


Fig. 4.5.b

Example (3)

Draw the normal, shear force and bending moment diagrams for the given frame in Fig. 4.6.

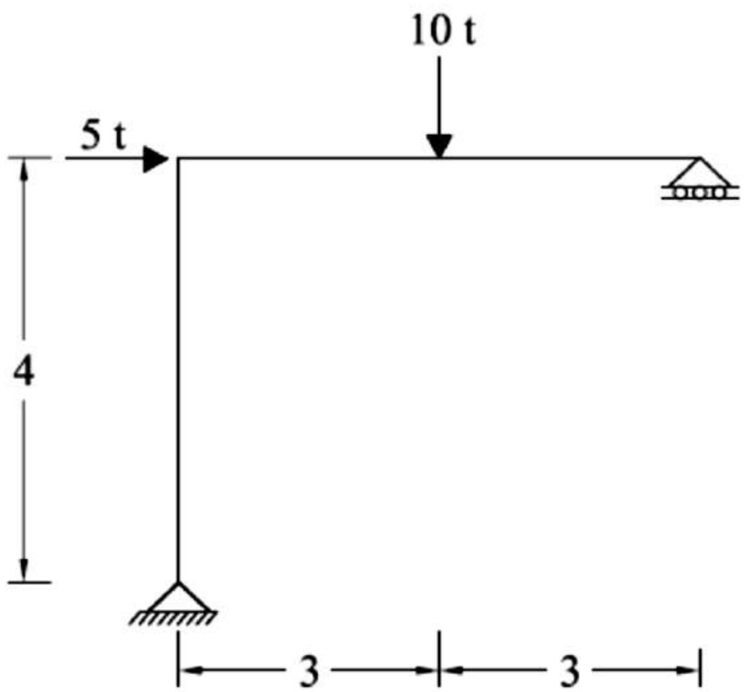


Fig. 4.6.a

Solution

- *Finding the reactions*

a) $\sum F_x = 0 \ggg\ggg X_a = 5t \leftarrow$

b) $\sum M_a = 0 \ggg\ggg 5x4 + 10x3 - Y_b x6 = 0$

$$\therefore Y_b = \frac{50}{6} = 8.33t \uparrow$$

c) $\sum M_b = 0 \ggg\ggg 10x3 - 5x4 - Y_a x6 = 0$

$$\therefore Y_a = \frac{10}{6} = 1.67\ t \uparrow$$

d) $\underline{Check \sum} \mathrm{F_y} = 8.33 + 1.67 - 10 = 0 \gg\gg\gg\gg$ ok

- *For drawing B.M.D*

M_1 from left side = 5x4=20 mt

M_b from right side = 0

M_2 from left side = 5x4+1.67x3= 25 m.t

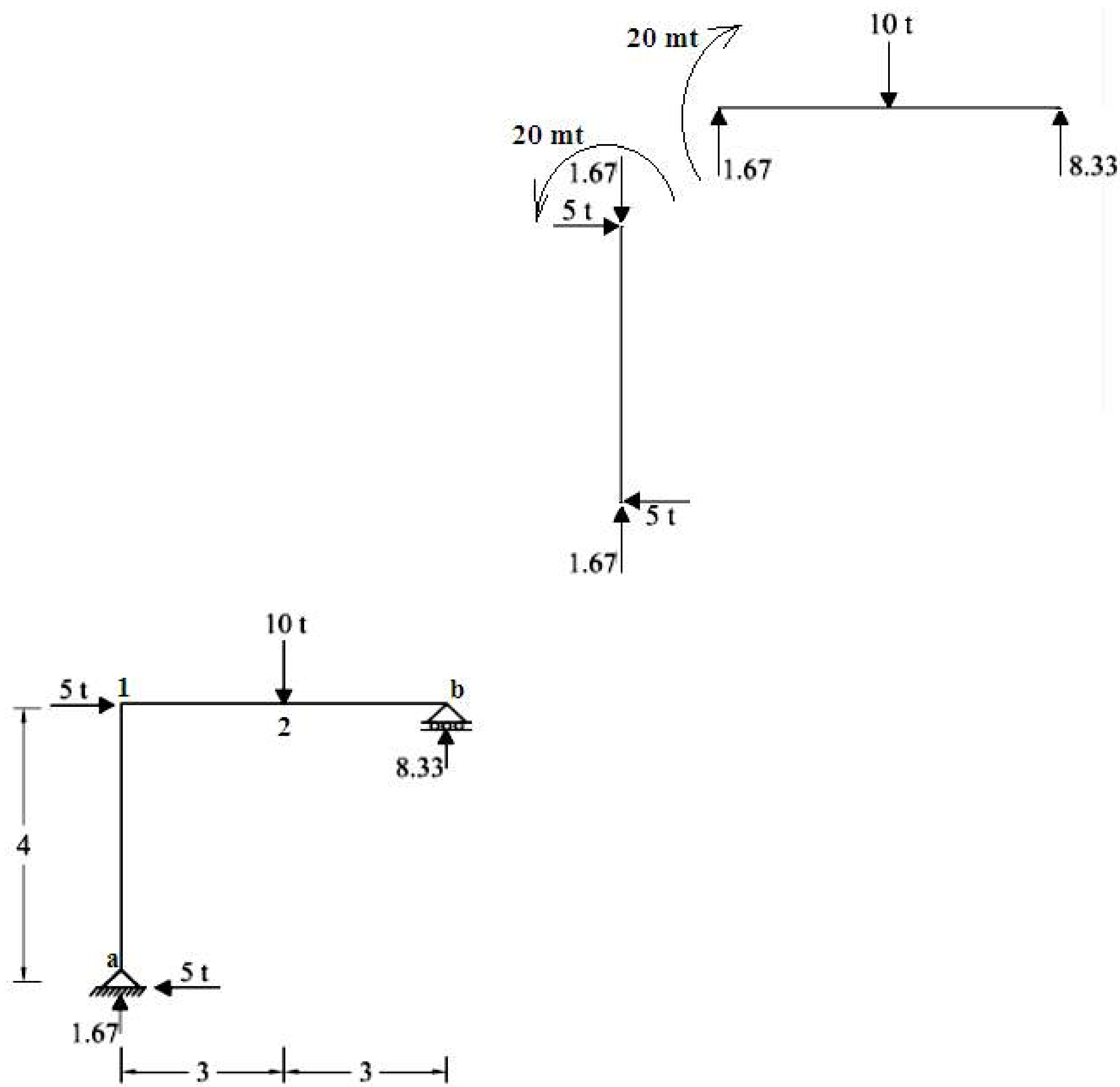
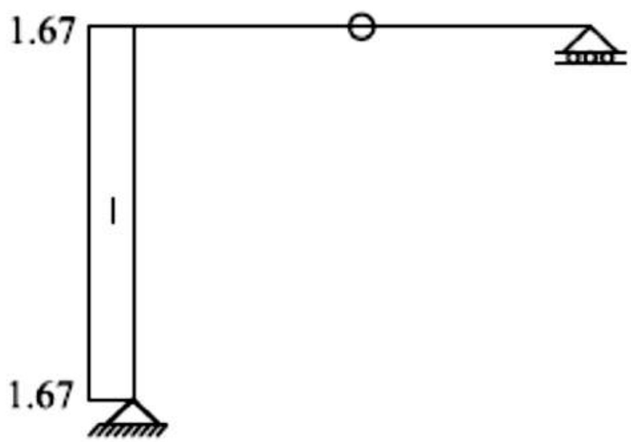
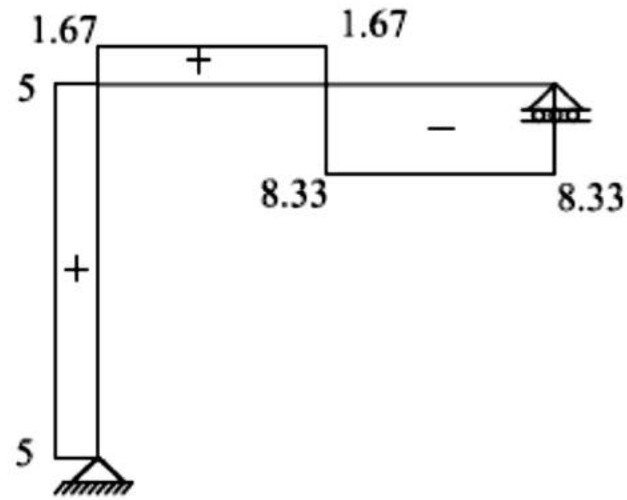


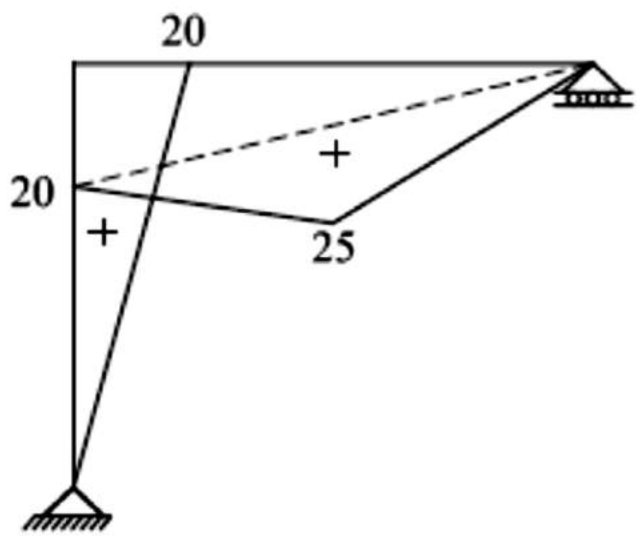
Fig. 4.6.b



N.F.D



S.F.D



B.M.D

Fig. 4.6.c

Example (4)

Draw the normal, shear force and bending moment diagrams for the given frame in Fig. 4.7.

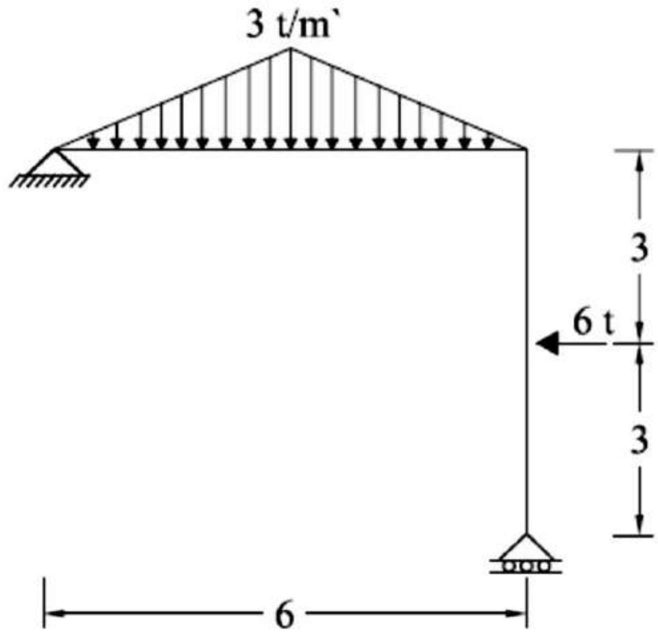


Fig. 4.7.a

Solution

- *Finding the reactions*

a) $\sum F_x = 0 \ggg\ggg\ggg X_a = 6t \rightarrow$

b) $\sum M_a = 0 \ggg\ggg\ggg 9x3 + 6x3 - Y_b x6 = 0$

$$\therefore Y_b = \frac{45}{6} = 7.5t \uparrow$$

$c)\ \sum M_b = 0 \ggg\ggg\ggg 9x3 - 6x3 - Y_a x6 = 0$

$$\therefore Y_a = \frac{9}{6} = 1.5\ t \uparrow$$

d) <u>Check</u> $\sum F_y = 7.5 + 1.5 - 9 = 0 \ggg\ggg\ggg$ ok

- *For drawing B.M.D*

M_1 from left side = 0

M_2 from right side = -6x3= -18 m.t

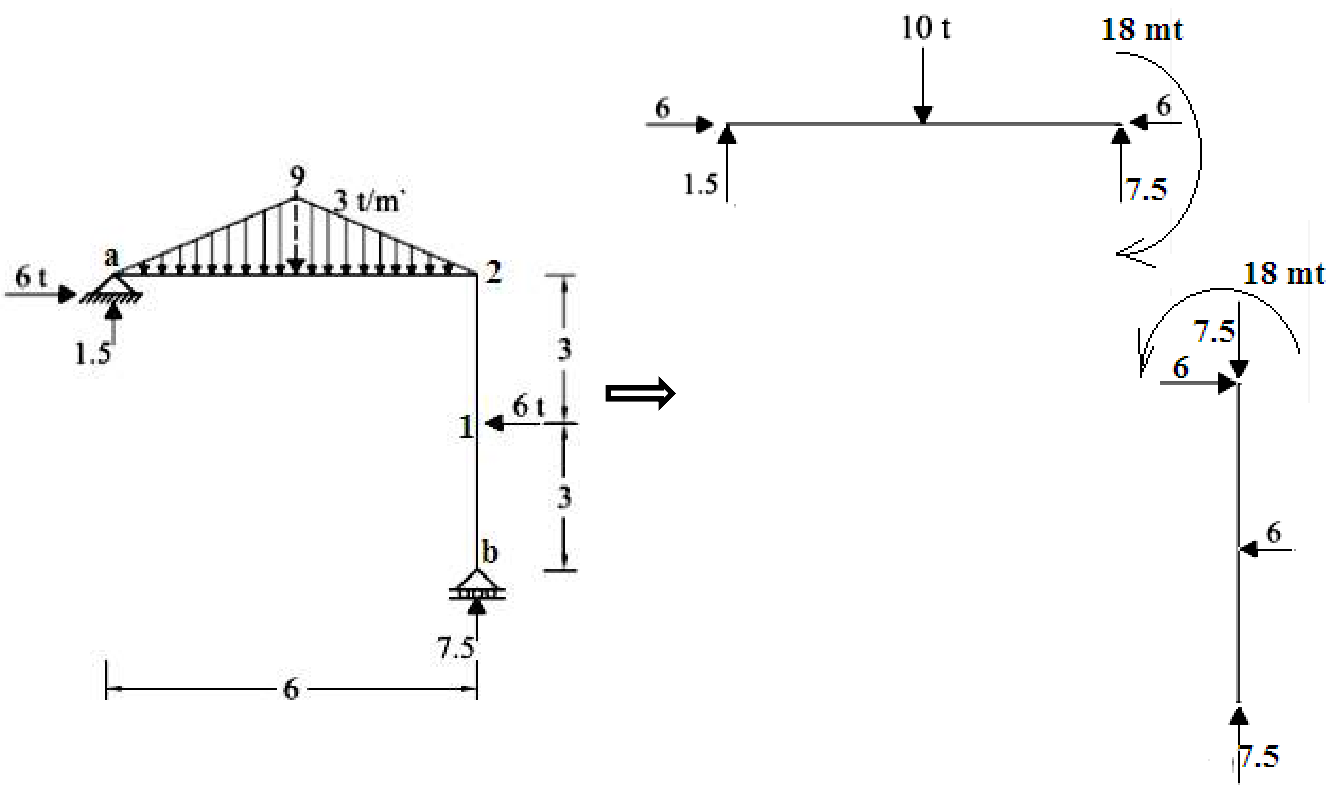
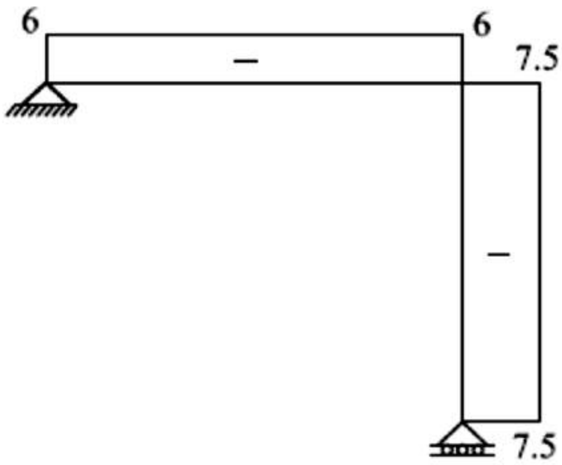
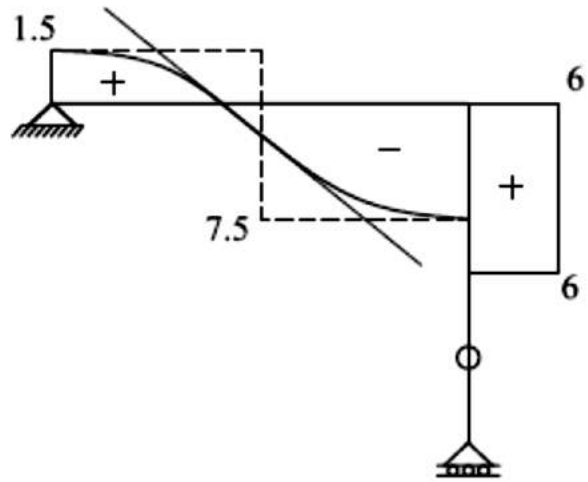


Fig. 4.7.b



N.F.D



S.F.D

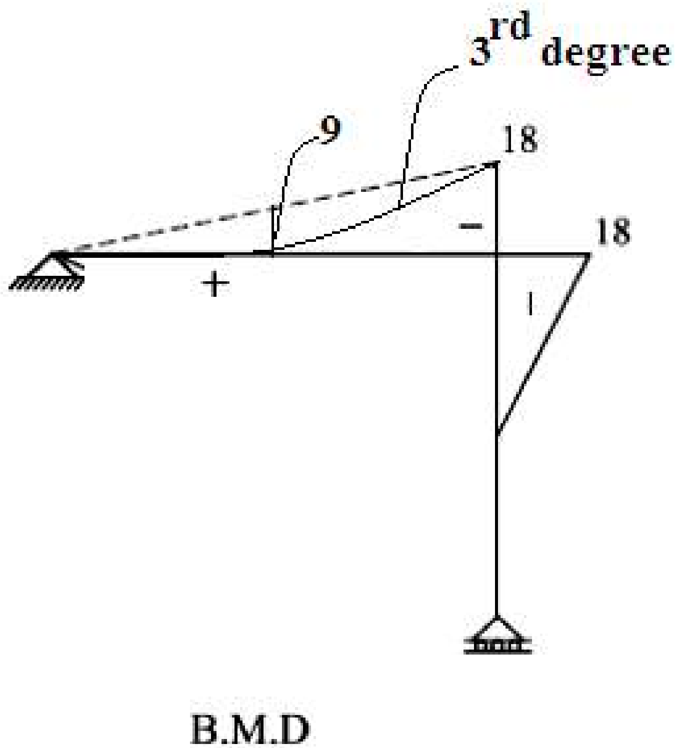


Fig. 4.7.c

Example (5)

For the given frame in Fig. 4.8, draw the normal, shear force and bending moment diagrams.

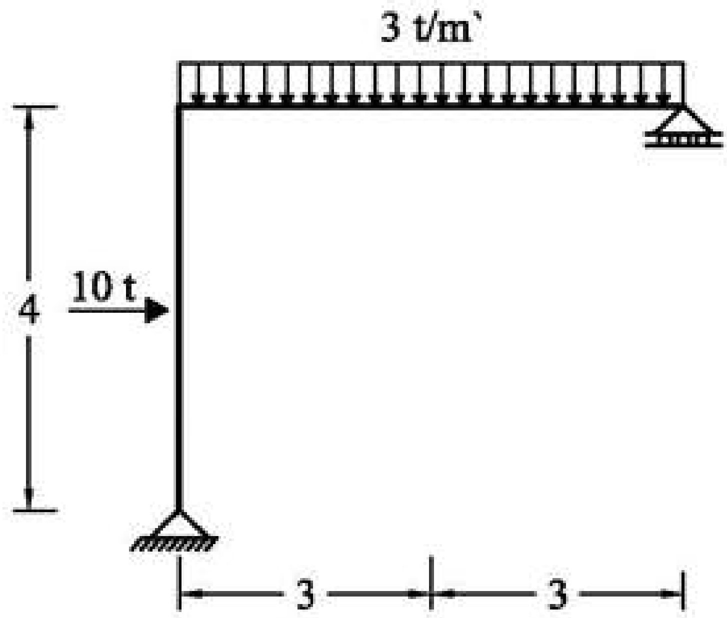


Fig. 4.8.a

Solution

- *Finding the reactions*

a) $\sum F_x = 0 \ggg\ggg X_a = 10t \leftarrow$

b) $\sum M_a = 0 \ggg\ggg 18x3 + 10x2 - Y_b x6 = 0$

$$\therefore Y_b = \frac{74}{6} = 12.33t \uparrow$$

c) $\sum M_b = 0 \ggg\ggg 18x3 - 10x2 - Y_a x6 = 0$

$$\therefore Y_a = \frac{34}{6} = 5.67t \uparrow$$

d) Check $\sum F_y = 12.33 + 5.67 - 18 = 0 \ggg\ggg$ ok

- *For drawing B.M.D*

M_1 from left side = 10×2= 20 m.t

M_2 from left side = 10×4-10x2= 20 m.t

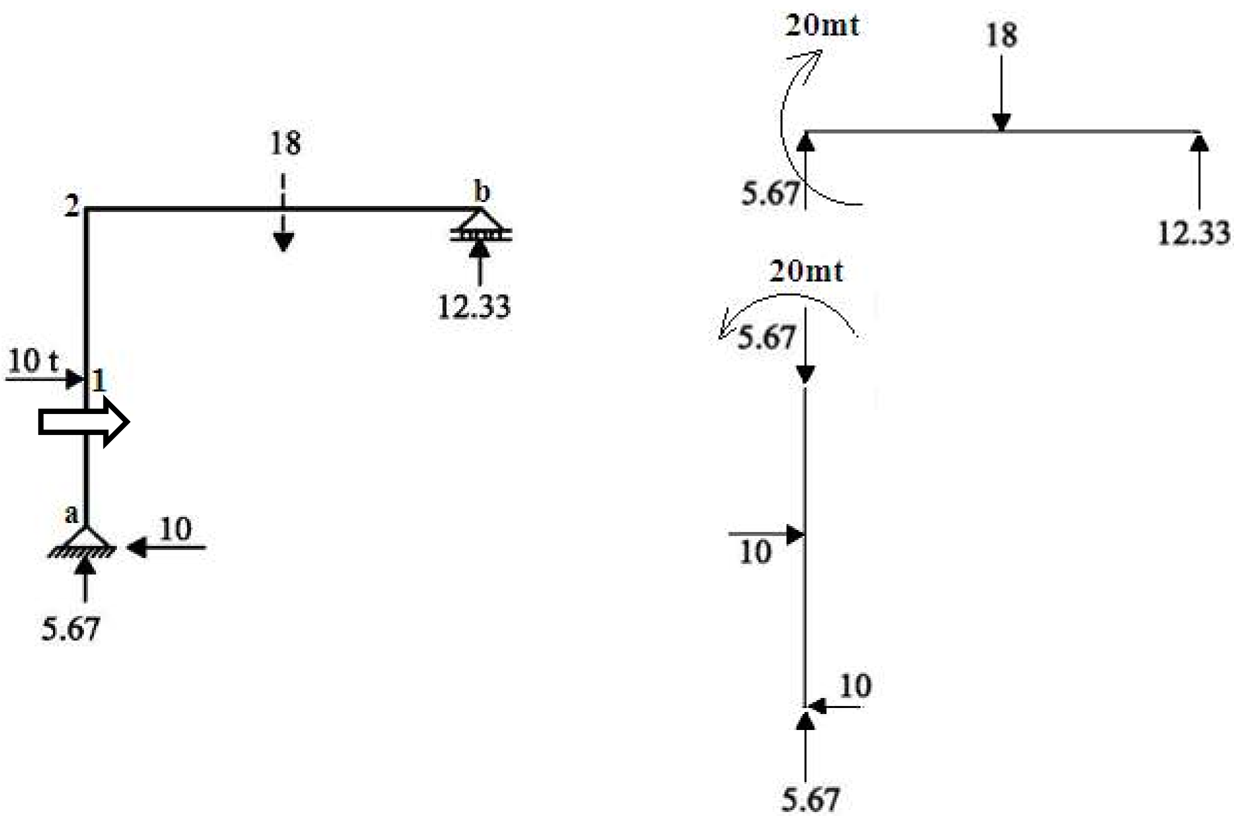
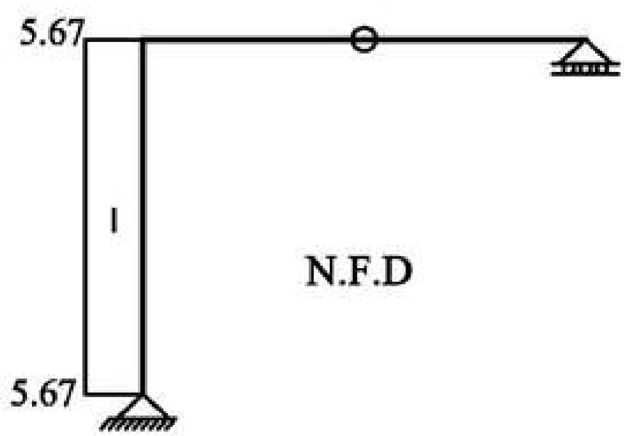


Fig. 4.8.b



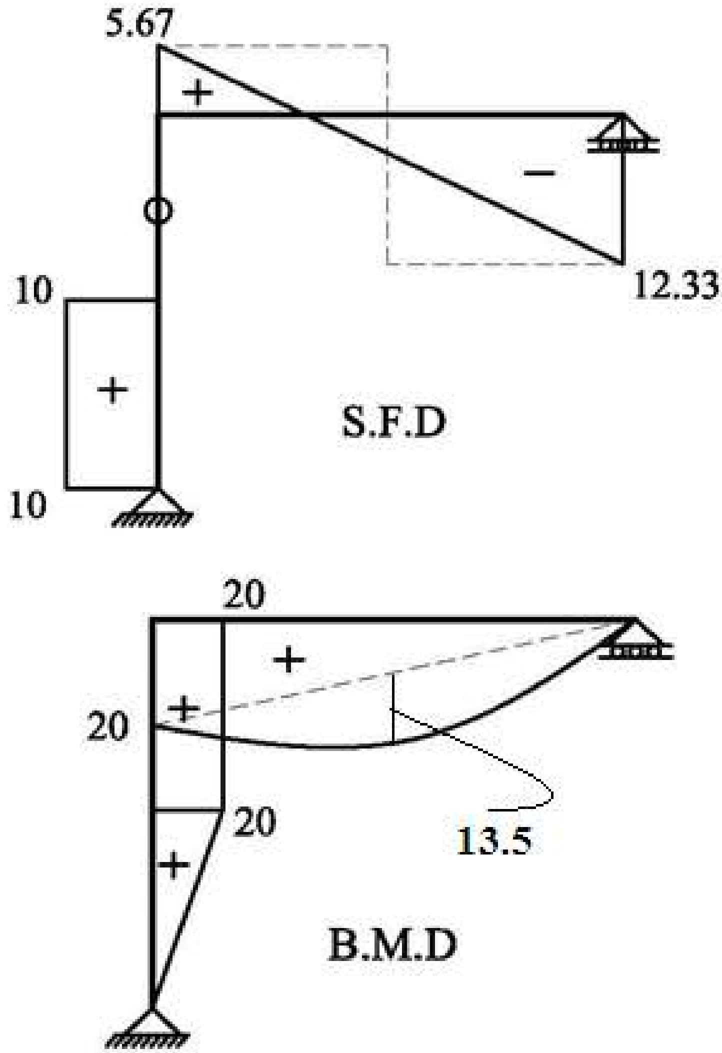


Fig. 4.8.c

<u>Example (5)</u>

For the given frame in Fig. 4.9, draw the normal, shear force and bending moment diagrams.

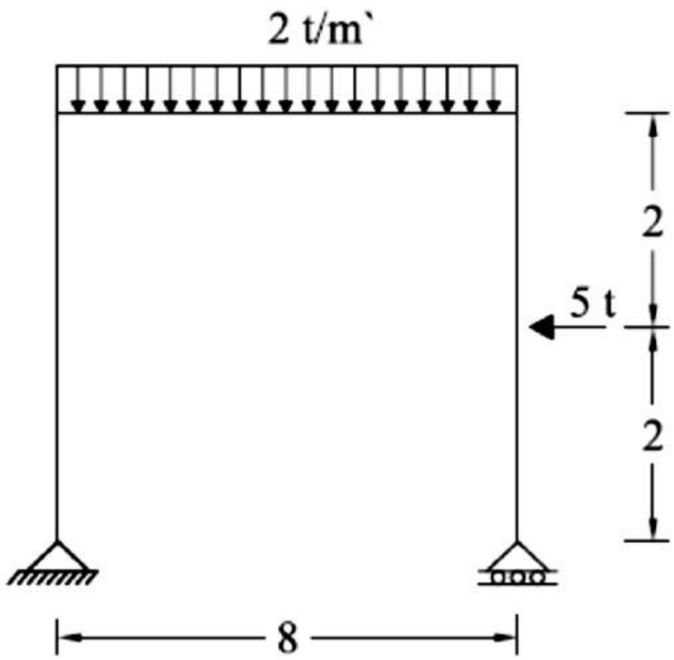


Fig. 4.9.a

<u>*Solution*</u>

➢ *Finding the reactions*

a) $\sum F_x = 0 \ggg\ggg X_a = 5t \rightarrow$

b) $\sum M_a = 0 \ggg\ggg 16x4 - 5x2 - Y_b x8 = 0$

$$\therefore Y_b = \frac{54}{8} = 6.75t \uparrow$$

c) $\sum M_b = 0 \ggg\ggg 16x4 + 5x2 - Y_a x8 = 0$

$$\therefore Y_a = \frac{74}{8} = 9.25t \uparrow$$

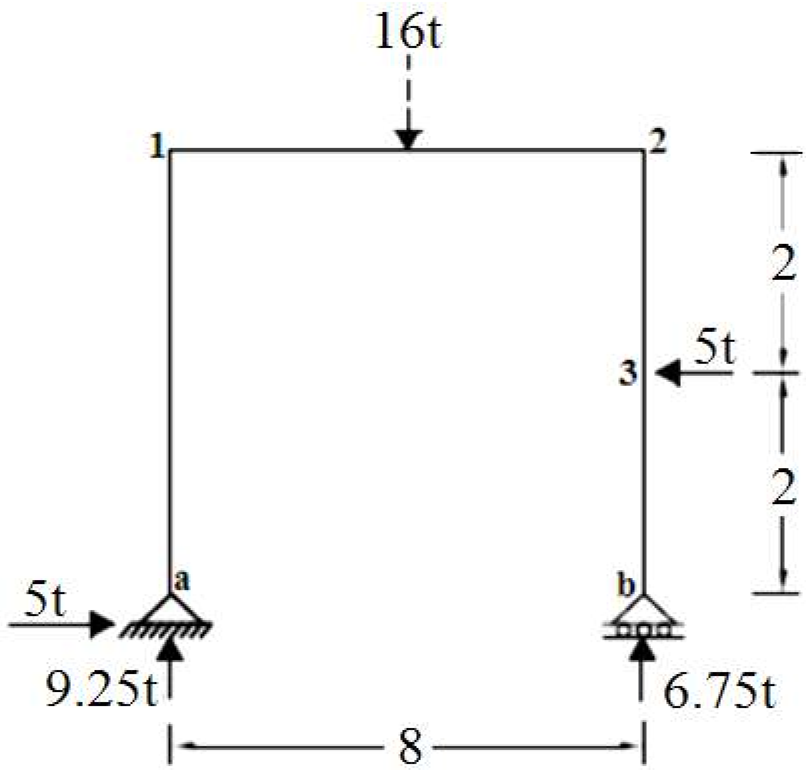
d) <u>*Check*</u> $\sum F_y = 6.75 + 9.25 - 16 = 0 \ggg\ggg$ ok

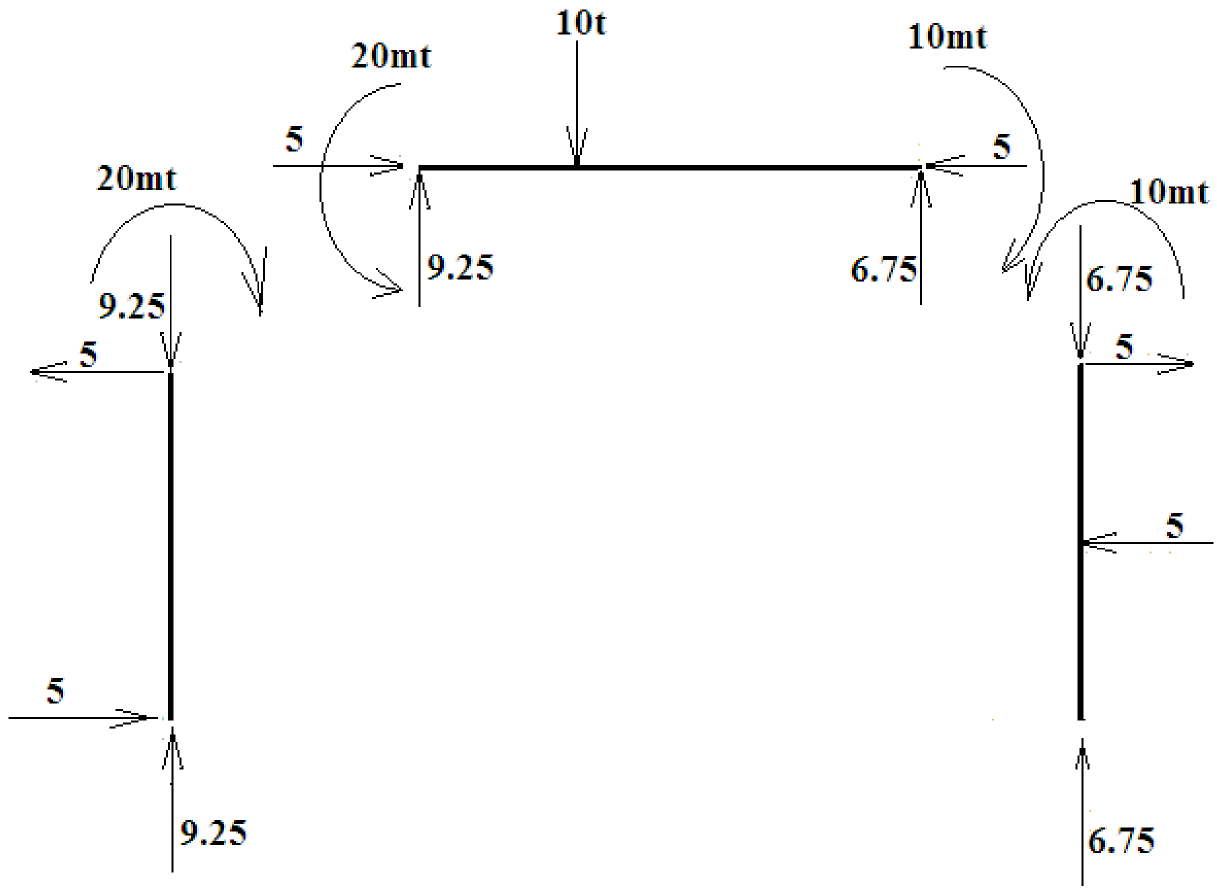
➢ *For drawing B.M.D*

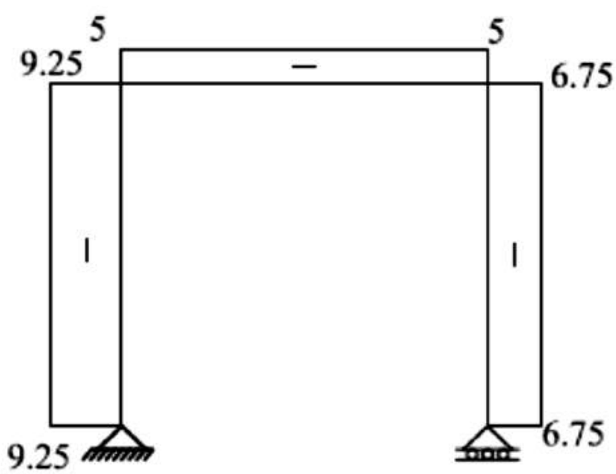
M_1 from left side = -5×4= -20 m.t

M_2 from left side = -5x2= -10 m.t

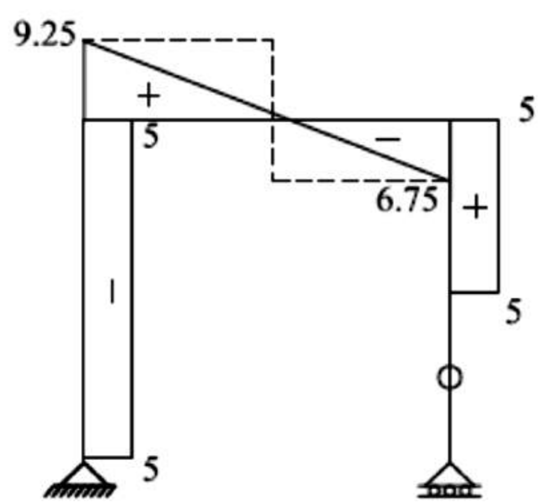
M_3 from right side = 0




10t
20mt
10mt
5
5
9.25
6.75
20mt
10mt
9.25
6.75
5
5
5
5
9.25
6.75


5
5
9.25
6.75
−
−
−
9.25
6.75

N.F.D


9.25
+
5
5
−
6.75
+
−
5
5

S.F.D

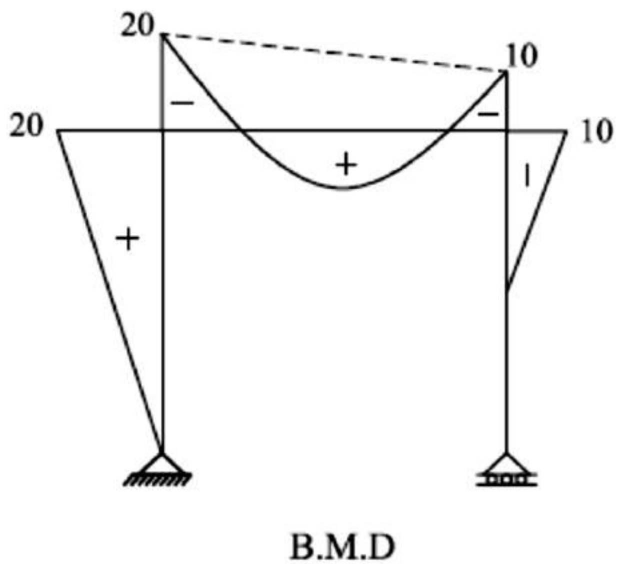


Fig. 4.9.b

Example (6)

For the given frame in Fig. 4.10, draw the normal, shear force and bending moment diagrams.

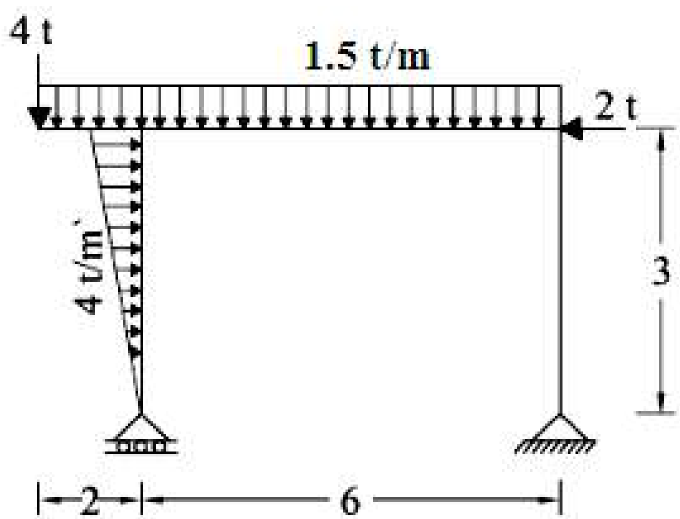


Fig. 4.10.a

Solution

- *Finding the reactions*

a) $\sum F_x = 0 \ggg\ggg X_a = 4t \leftarrow$

b) $\sum M_a = 0 \ggg\ggg 9x3 + 6x2 - 4x2 - 3x1 - 2x3 - Y_b x6 = 0$

$$\therefore Y_b = \frac{22}{6} = 3.67t \uparrow$$

c) $\sum M_b = 0 \ggg\ggg 9x3 - 6x2 + 4x8 + 3x9 - Y_a x6 = 0$

$$\therefore Y_a = \frac{74}{6} = 12.33t \uparrow$$

d) <u>*Check*</u> $\sum F_y = 12.33 + 3.67 - 9 - 3 - 4 = 0 \ggg\ggg$ ok

- *For drawing B.M.D*

M_1 from left side = -6×1= -6 m.t

M_2 from left side = -4x2-3x1= -11m.t

M_3 from equilibrium = -6-11= -17m.t

M_4 from right side = -4x3=-12mt

M_5 from right side = -12mt

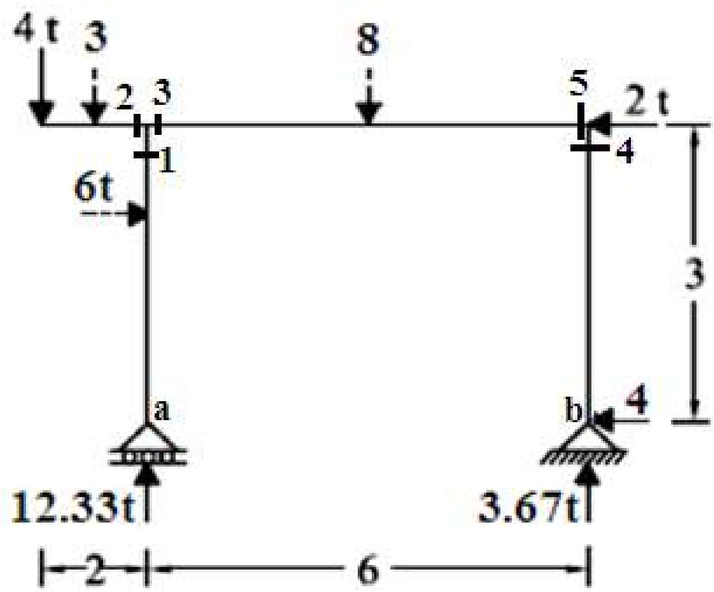
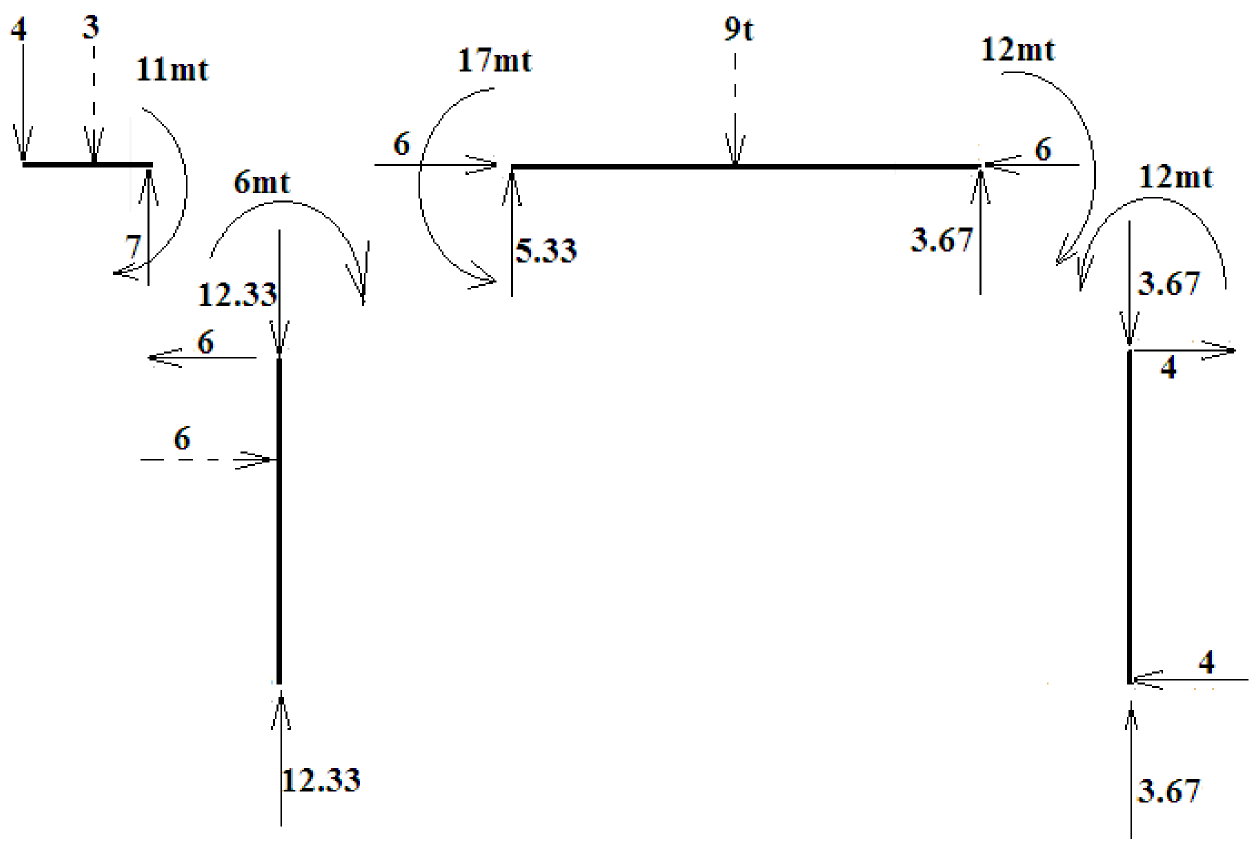

4 t
3
2
3
8
5
2 t
1
4
6t
3
a
b
4
12.33t
3.67t
2
6

Fig. 4.10.b


4
3
11mt
7
9t
17mt
12mt
6
6
5.33
3.67
6mt
12.33
6
6
12mt
3.67
4
4
12.33
3.67

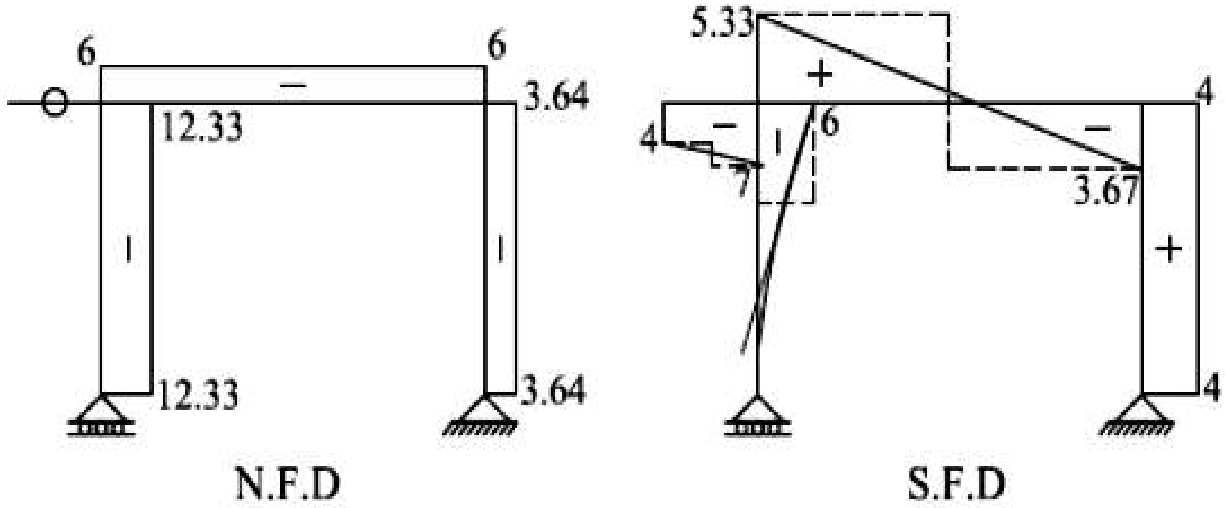


Fig. 4.10.b

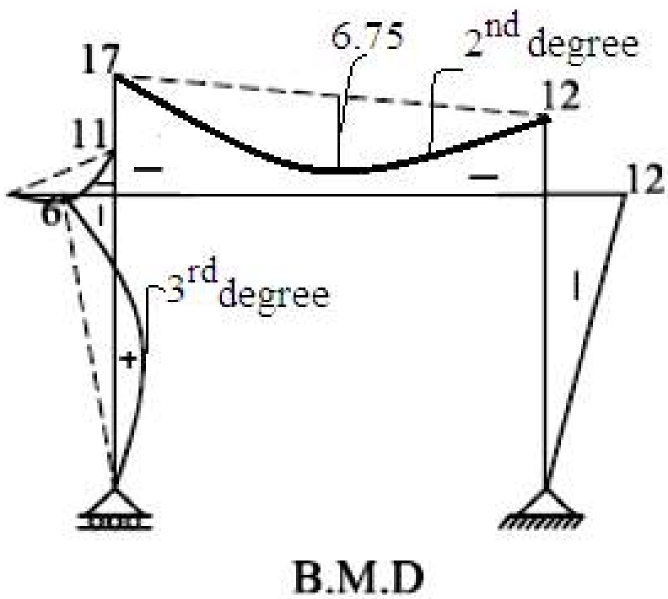


Fig. 4.10.c

Example (7)

For the given frame in Fig. 4.11, draw the normal, shear force and bending moment diagrams.

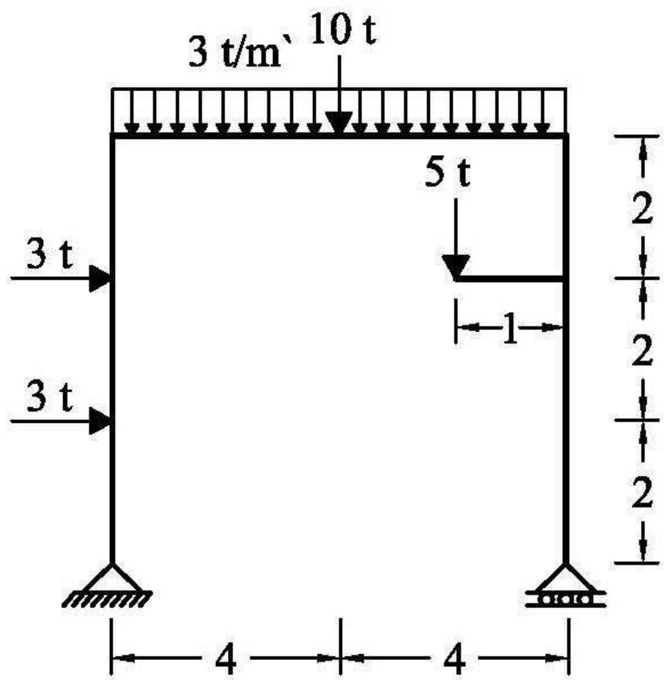


Fig. 4.11.a

Solution

➢ *Finding the reactions*

a) $\sum F_x = 0 \ggg\ggg\ggg X_a = 6t \leftarrow$

b) $\sum M_a = 0 \gg 12x2 + 12x6 + 10x4 + 3x2 + 3x4 + 5x6 - Y_b x8 = 0$

$$\therefore Y_b = \frac{184}{8} = 23t \uparrow$$

c) $\sum M_b = 0 \gg 12x6 + 12x2 + 10x4 - 3x2 - 3x4 + 5x2 - Y_a x8 = 0$

$$\therefore Y_a = \frac{128}{8} = 16t \uparrow$$

d) *Check* $\sum F_y = 12 + 12 + 10 + 5 - 16 - 23 = 0 \ggg\ggg\ggg$ ok

- *For drawing B.M.D*

M_1 from left side = 6×2=12m.t

M_2 from left side = 6x4-3x2=18m.t

M_3 from left side =6x6-3x4-3x2=18m.t

M_4 from left side = -12x2-3x2-3x4+6x6+16x4=58 m.t

M_5 from left side = -12x2-10x4-12x6-3x2-3x4+6x6+16x8=10 m.t

M_6 from right side = 0

M_7 from left side = -5x2=-10 m.t

M_8 from equilibrium = 10 m.t

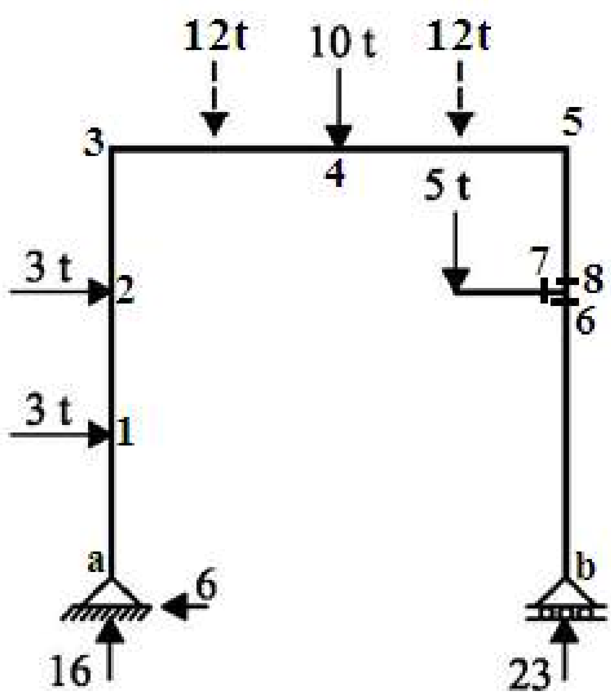
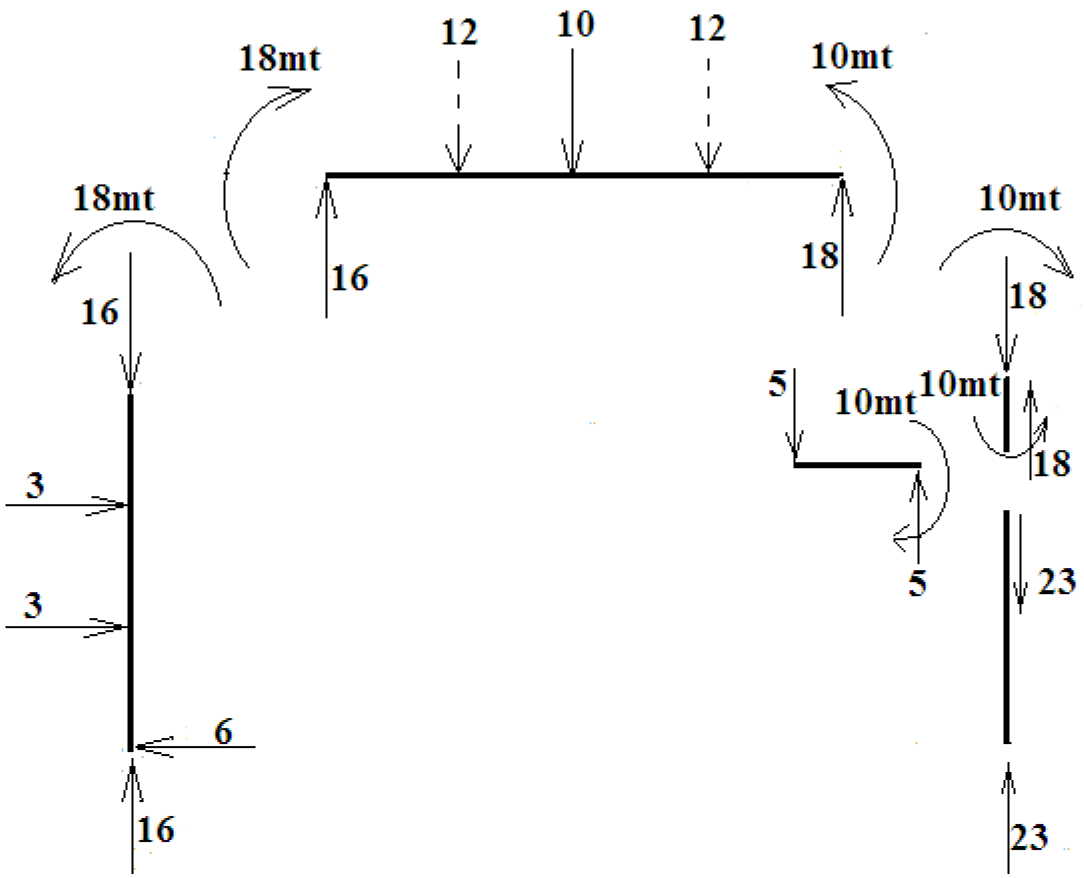
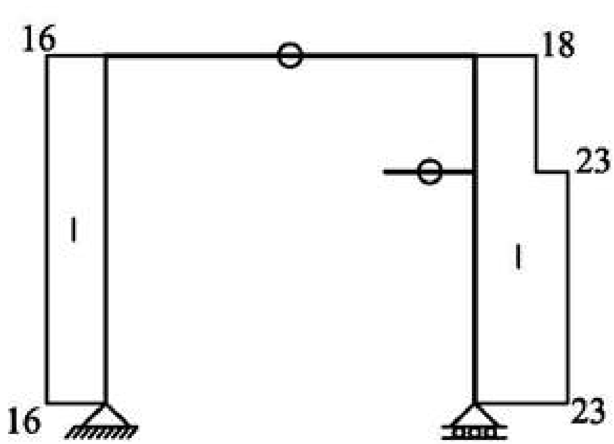
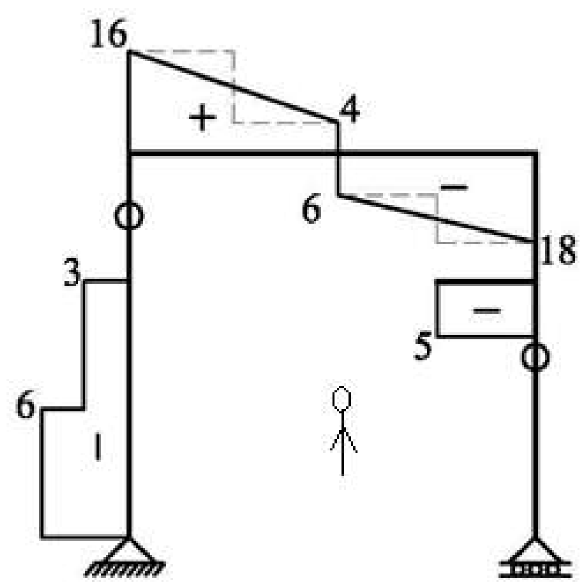


Fig. 4.11.b


18mt
12
10
12
10mt
18mt
16
16
18
10mt
18
5
10mt
10mt
18
3
3
5
23
6
16
23


16
18
23
16
23

N.F.D


16
+
4
6
−
18
3
−
5
6

S.F.D

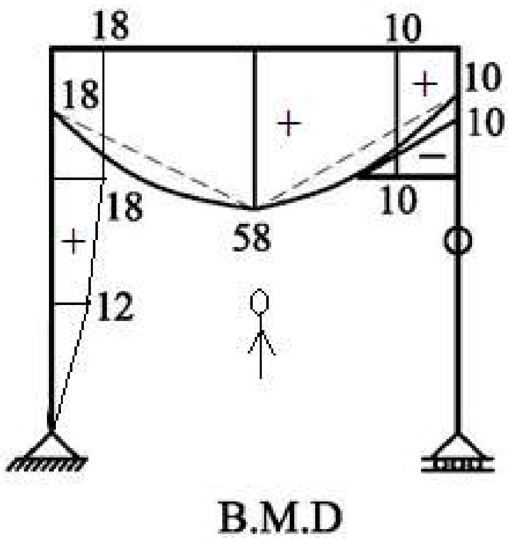


Fig. 4.11.c

Example (8)

For the given frame in Fig. 4.12, draw the normal, shear force and bending moment diagrams.

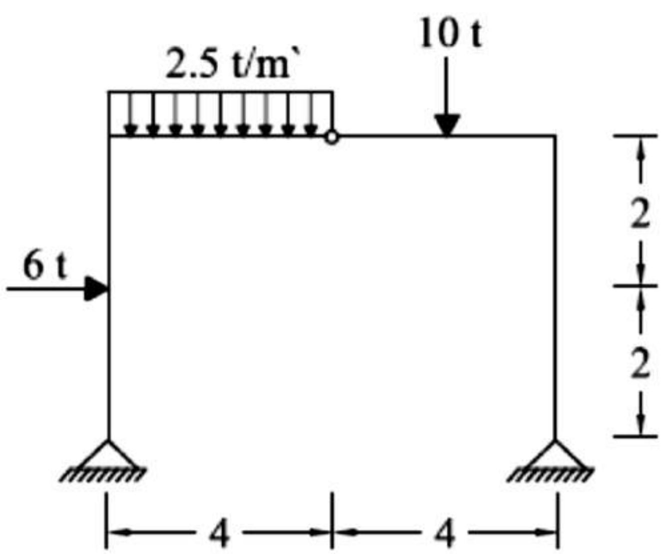


Fig. 4.12.a

<u>*Solution*</u>

- *Finding the reactions*

a) $\sum M_a = 0 \ggg\ggg 10x2 + 10x6 + 6x2 - Y_b x8 = 0$

$$\therefore Y_b = \frac{92}{8} = 11.5t \uparrow$$

b) $\sum M_b = 0 \ggg\ggg 10x2 + 10x6 - 6x2 - Y_a x8 = 0$

$$\therefore Y_a = \frac{68}{8} = 8.5t \uparrow$$

c) <u>*Check*</u> $\sum \mathrm{F_y} = 8.5 + 11.5 - 10 - 10 = 0 \ggg\ggg$ ok

d) $\sum M_c \ for\ the\ left\ side = 0$

$$10x2 + 6x2 - 8.5x4 - 4x_a = 0 \quad \ggg x_a = 0.5t \rightarrow$$

e) $\sum M_c \ for\ the\ right\ side = 0$

$$10x2 - 11.5x4 + 4x_b = 0 \quad \ggg x_b = 6.5t \leftarrow$$

f) $check \sum \mathrm{F_x} = 6 + 0.5 - 6.5 = 0 \ggg\ggg$ ok

- *For drawing B.M.D*

M_1 from left side = -6×2-0.5x4=-14 m.t

M_2 from right side = -6.5x4=-26 m.t

M_3 from equilibrium =-0.5x2=-1m.t

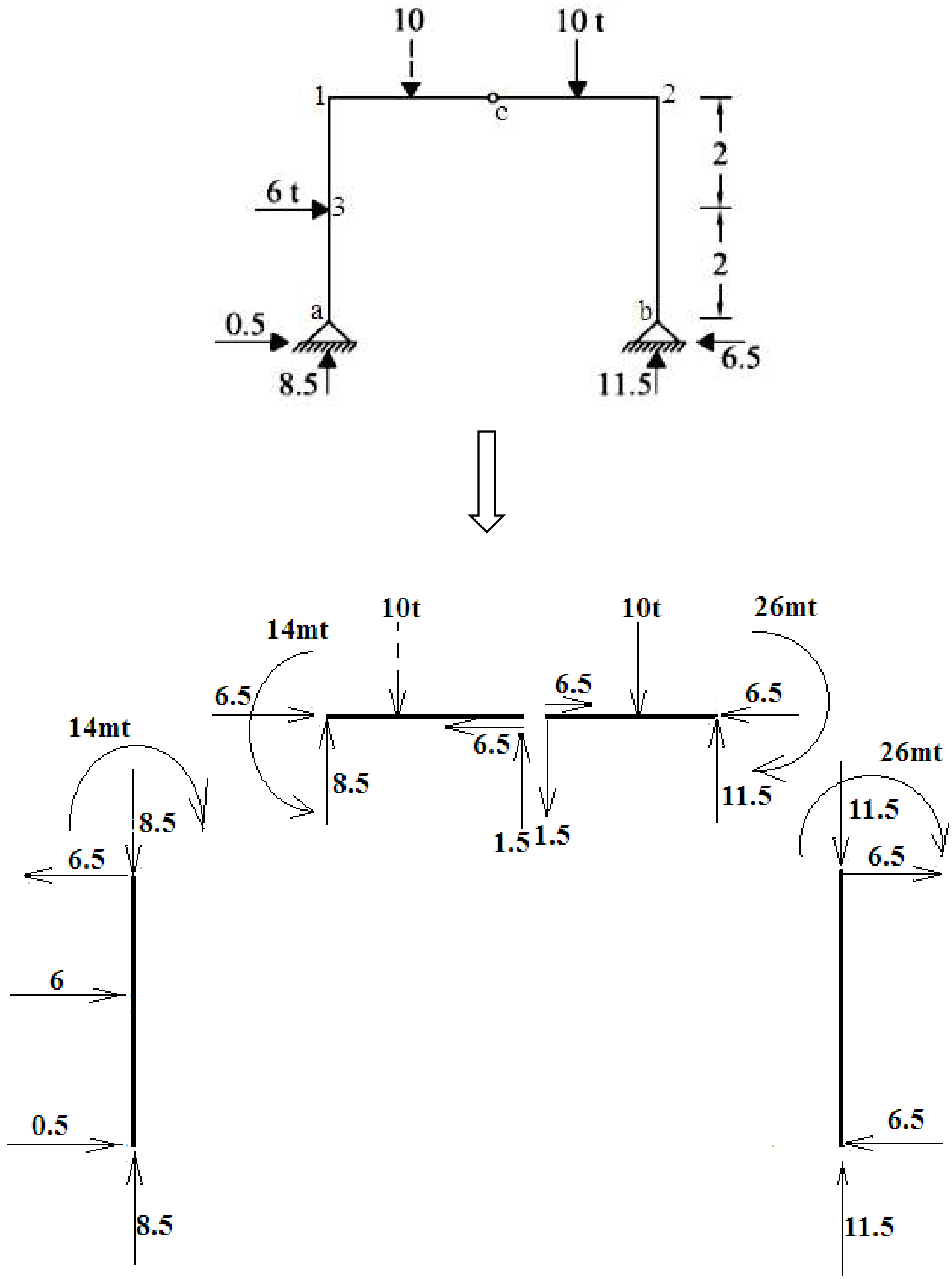
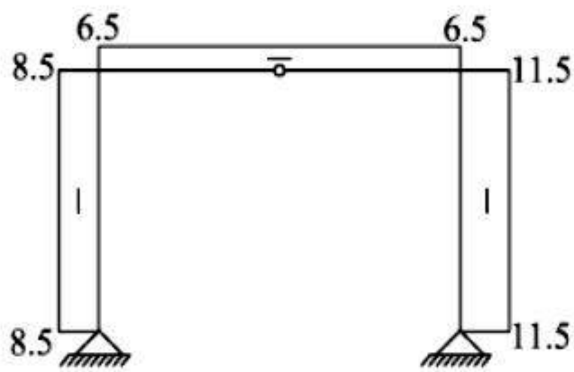
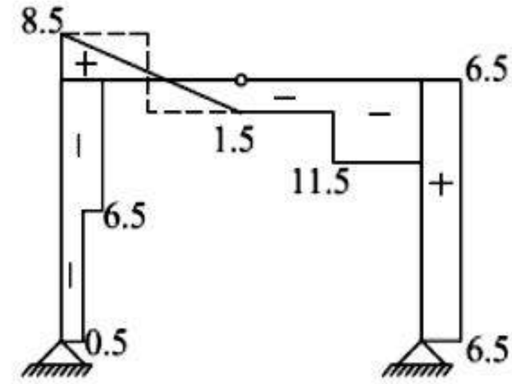

10
10 t
1
c
2
2
2
6 t
3
0.5
a
b
6.5
8.5
11.5
14mt
10t
10t
26mt
6.5
6.5
6.5
6.5
8.5
1.5
1.5
11.5
14mt
8.5
6.5
26mt
11.5
6.5
6
0.5
6.5
8.5
11.5

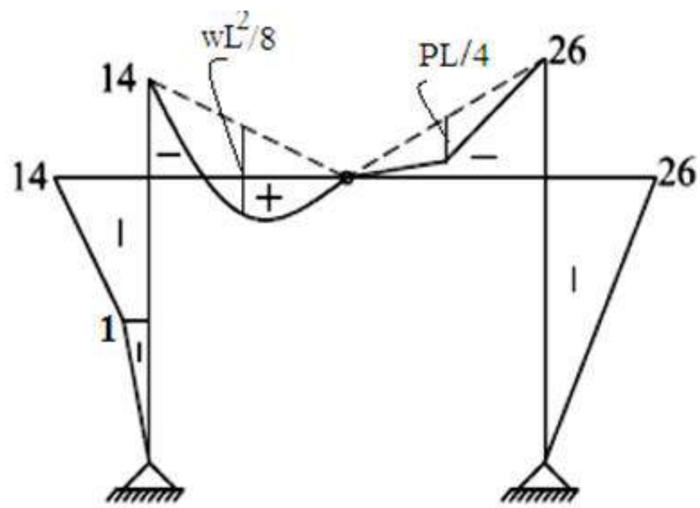
Fig. 4.12.b



N.F.D



S.F.D



B.M.D

Fig. 4.12.c

Example (9)

Draw the normal, shear force and bending moment diagrams for the shown frame in Fig. 4.13.

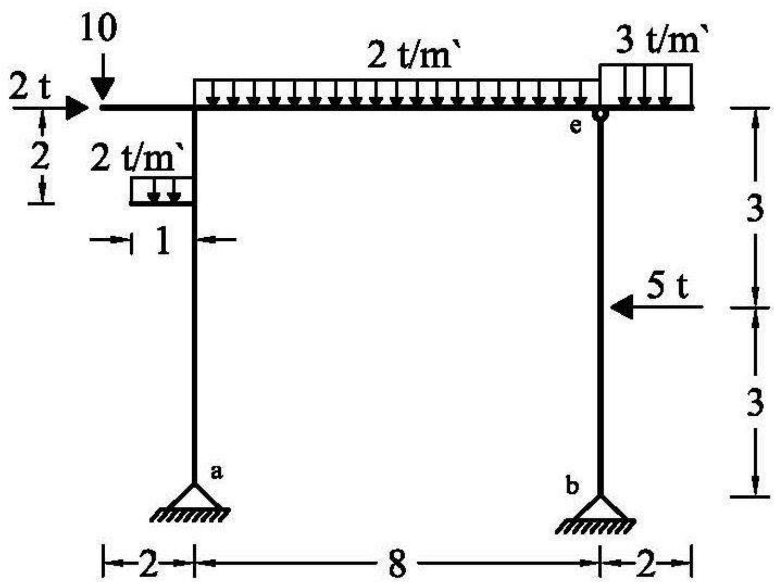


Fig. 4.13.a

Solution

- Finding the Reaction

a) $\sum M_a = 0.0$

$$=16\times4 + 6\times9 - 10\times2 + 2\times6 - 2\times0.5 - 5\times3 - Y_b\times8 = 0.0$$

$$Y_b = 11.75 \text{ t} \uparrow$$

b) $\sum M_b = 0.0$

$$=16\times4 + 2\times8.5 + 10\times10 - 2\times6 + 6\times1 + 5\times3 - Y_a\times8 = 0.0$$

$$Y_a = 22.25 \text{ t} \uparrow$$

c) Check $\sum F_y = -11.75-22.25+16+2+10+6= 0.0$ …. OK

d) $\sum M_e = 0.0$ for left part

$= 5\times3 - X_b\times6 = 0.0 \qquad X_b = 2.5\ t \rightarrow$

e) $\sum M_e = 0.0$ for right part

$= 16\times4 + 10\times10 + 2\times8.5 - 6\times1 - 22.25\times8 + X_a\times6 = 0.0$

$X_a = 0.5\ t \rightarrow$

- *For drawing B.M.D*

M_1 from left side = -0.5×4=-2m.t

M_2 from left side = -2x0.5=-1m.t

M_3 from equilibrium =2+1=-3m.t

M_4 from left side = -0.5x6-2x0.5=-4m.t

M_5 from left side = -10x2=-20 m.t

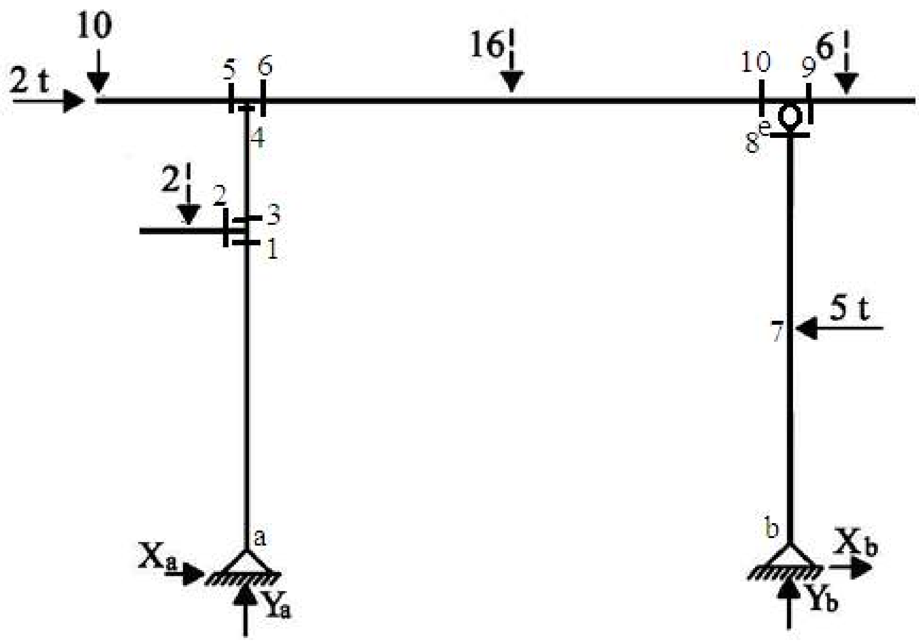
M_6 from equilibrium = 4+20=-24m.t

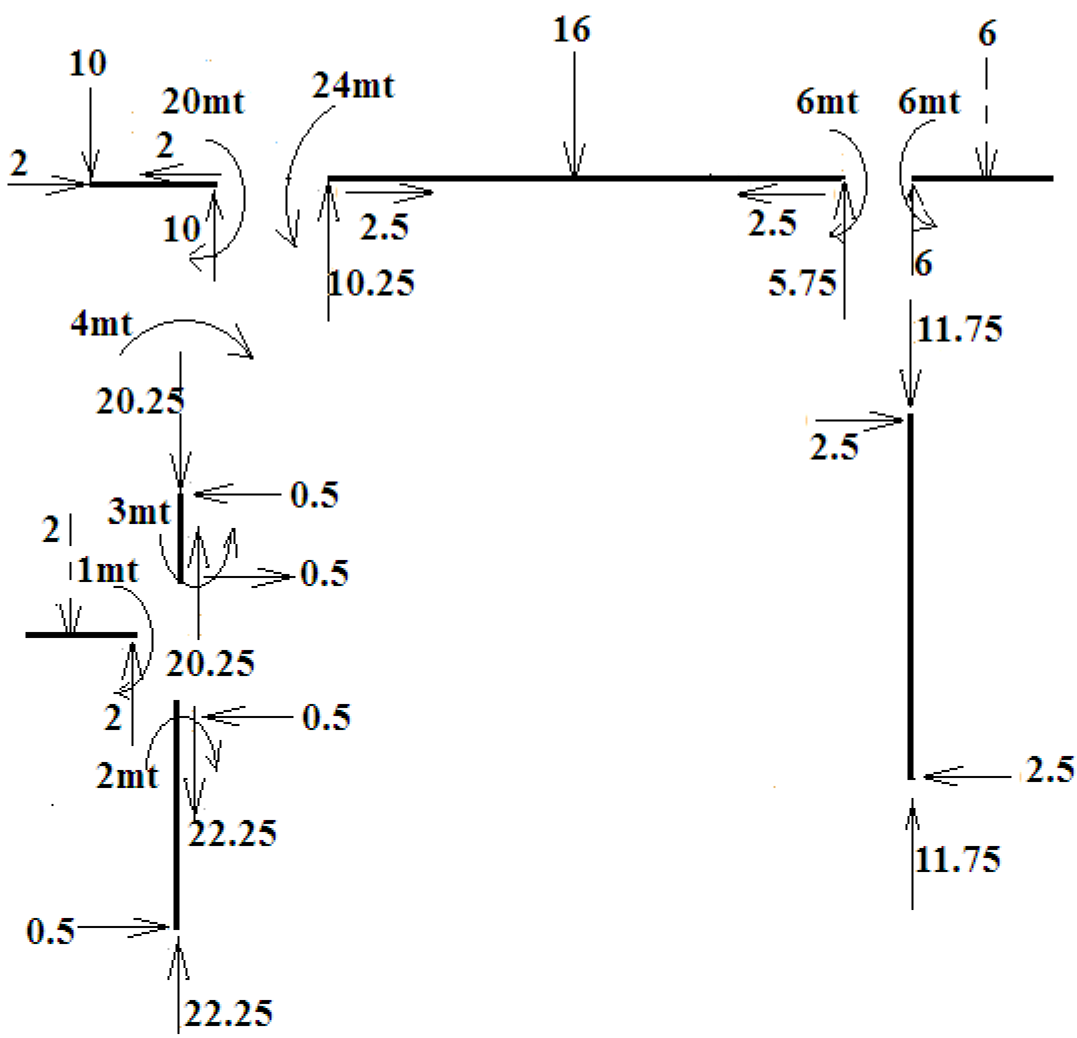
M_7 from right side = 2.5x3=7.5 m.t

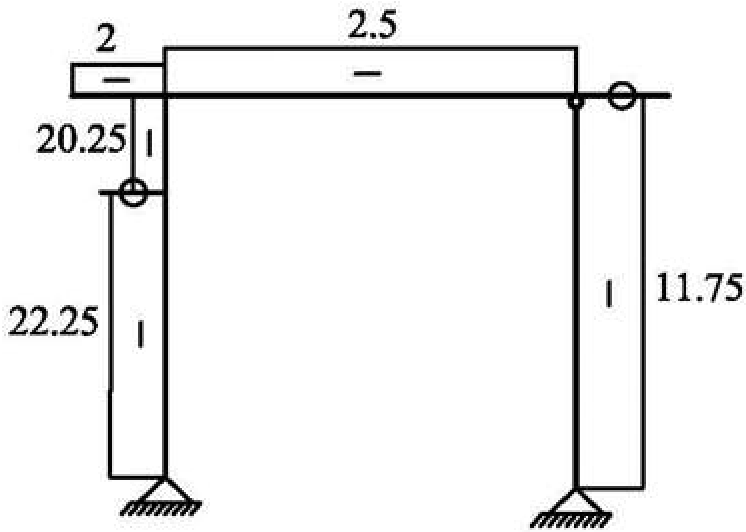
M_8 = 0

M_9 from right side = -6x1=-6 m.t

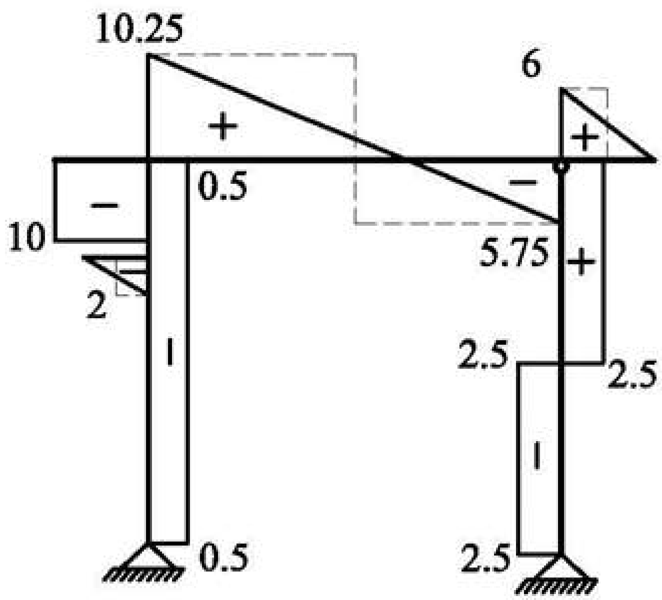
M_{10} from equilibrium = 0+6=-6 m.t


10
2 t
5
6
16
10
9
6
4
e
8
2
2
3
1
5 t
7
a
b
Xa
Ya
Xb
Yb


16
6
10
20mt
24mt
6mt
6mt
2
2
10
2.5
10.25
2.5
5.75
6
11.75
4mt
20.25
2.5
0.5
3mt
2
1mt
0.5
20.25
2
0.5
2mt
22.25
2.5
11.75
0.5
22.25



N.F.D



S.F.D

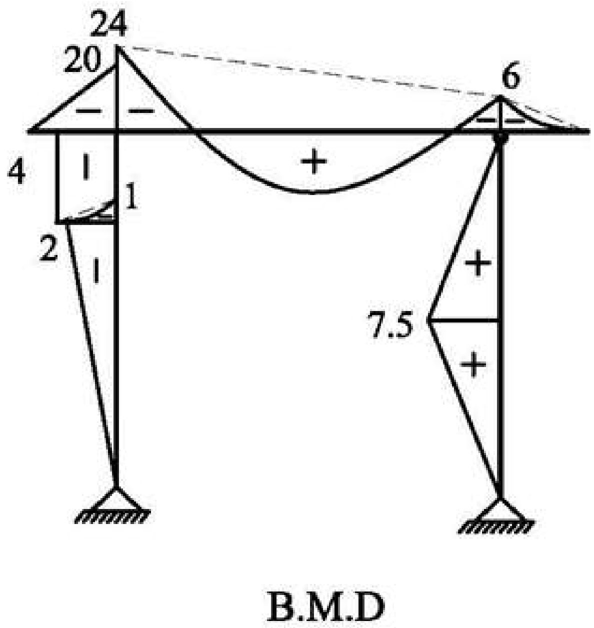


Fig. 4.13.b

Example (10)

Draw the normal, shear force and bending moment diagrams for the shown frame in Fig. 4.14.

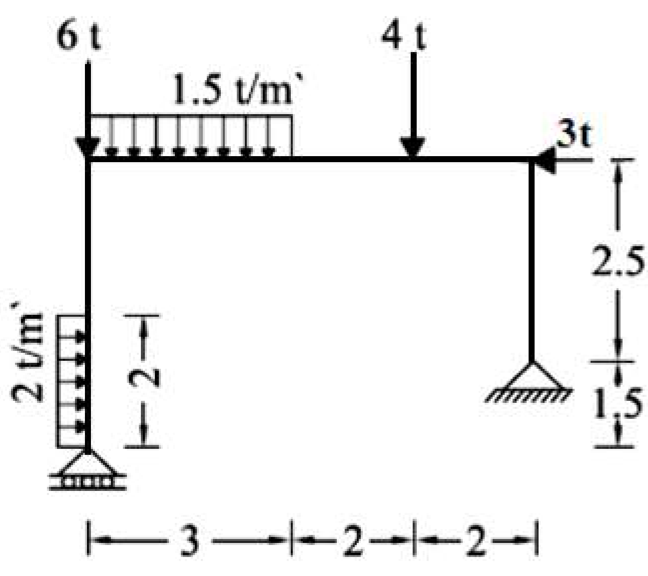


Fig. 4.14.a

Solution

- Finding the Reaction

a) $\sum F_x = 0 \ggg\ggg\ggg X_a = 1t \leftarrow$

b) $\sum M_a = 4.5\times1.5 + 4\times5 + 4\times1 - 3\times4 - 1\times1.5 - Y_b\times7 = 0$

$$Y_b = 2.46\ t \uparrow$$

c) $\sum M_b = 3\times2.5 + 4\times2 + 6\times7 + 4.5\times5.5 + 4\times0.5 - Y_a\times7 = 0.0$

$$Y_a = 12.04\ t \uparrow$$

d) Check $\sum F_y = -12.04 - 2.46 + 6 + 4 + 4.5 = 0.0$ …. OK

- *For drawing B.M.D*

M_1 from left side = -4×1=-4 m.t M_2 from left side = -4x4=-12 m.t

M_3 from right side =-1x2.5-4x2+2.46x4=0.66 m.t

M_4 from right side = -1x2.5=-2.5 m.t

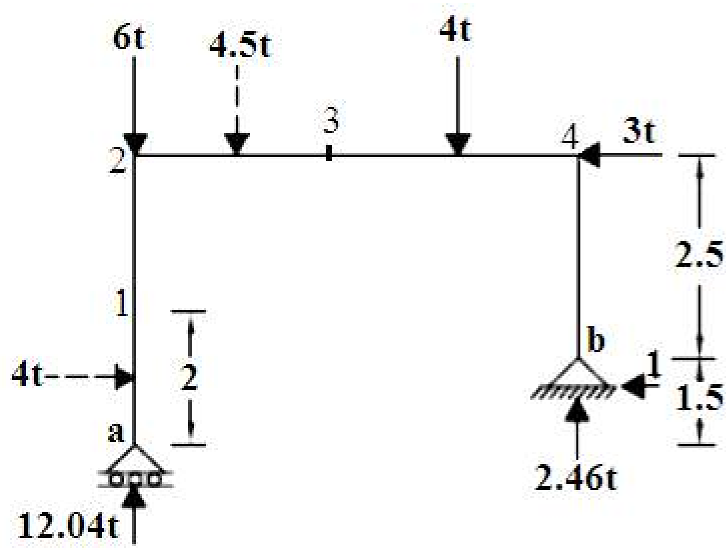


Fig. 4.14.b

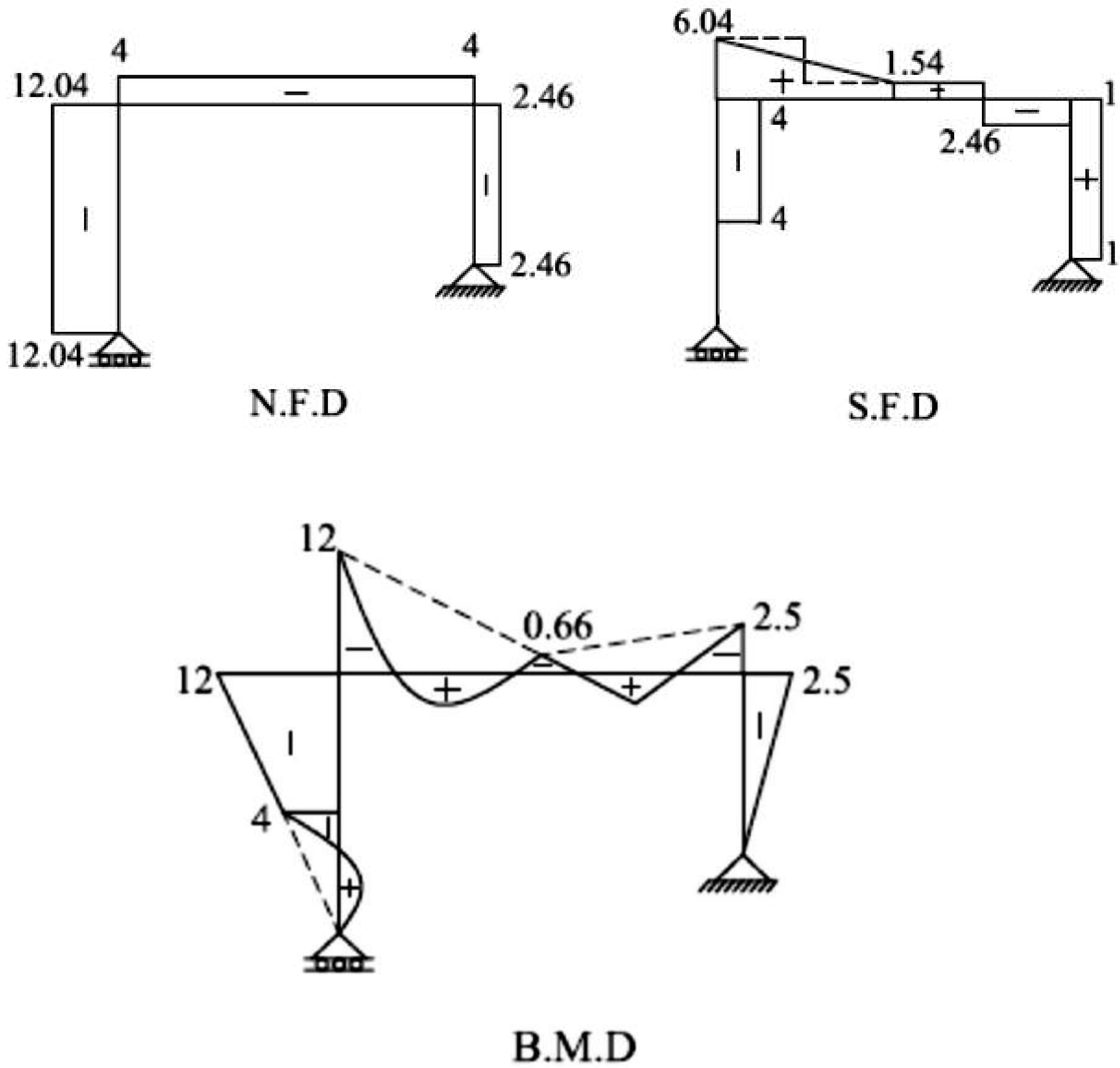

4
4
12.04
2.46
2.46
12.04
N.F.D
6.04
1.54
4
2.46
1
4
1
S.F.D
12
0.66
2.5
12
2.5
4
B.M.D

Fig. 4.14.c

Example (11)

Draw the normal, shear force and bending moment diagrams for the shown frame in Fig. 4.15.

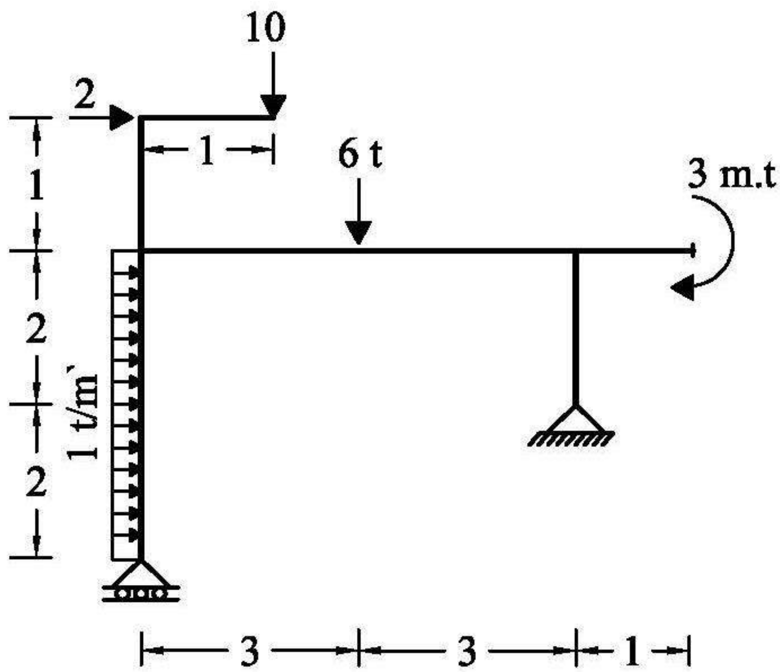


Fig. 4.15.a

Solution

$\sum F_x = 0.0 \qquad X_a = 6.0t \leftarrow$

$\sum M_a = 0.0 \qquad 4x2 + 2x5 + 10x1 + 6x3 + 3 - 6x2 + 6Y_b = 0.0$

$$Y_b = 6.27t \uparrow$$

$\sum M_b = 0.0 \qquad 6x3 + 10x5 - 2x3 - 4x0 - 3 - 6Y_a = 0.0$

$$Y_a = 9.83t \uparrow$$

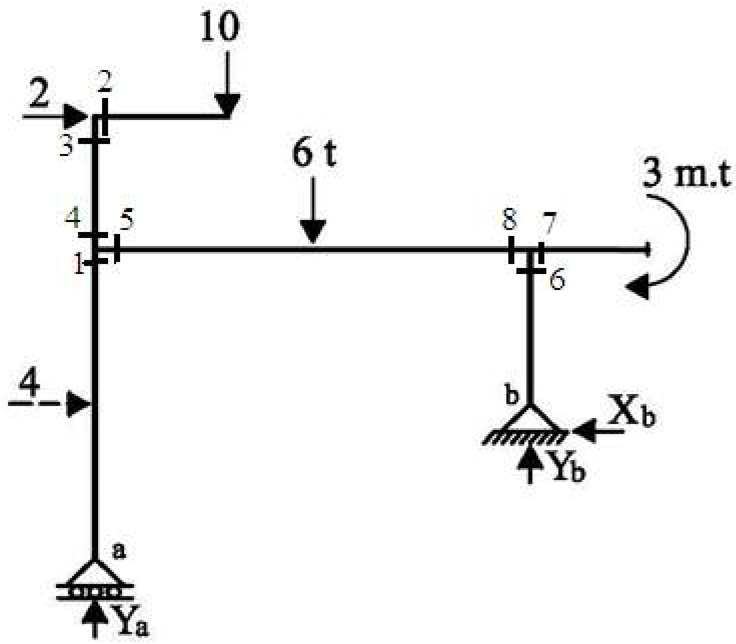


Fig. 4.15.b

➢ *For drawing B.M.D*

M_1 from left side = -4×2=-8 m.t M_2 from right side = -10x1=-10 m.t

M_3 from equilibrium = -10 m.t

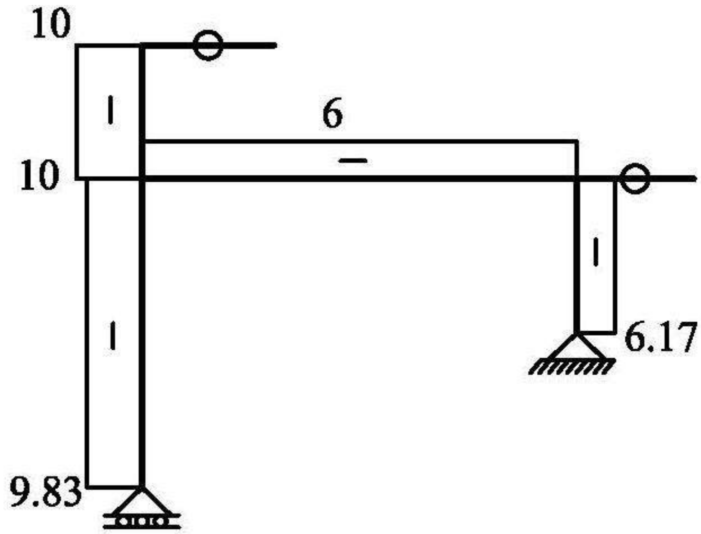
M_4 from right side =-1x10-2x1=-12 m.t

M_5 from equilibrium = 12-8=4 m.t

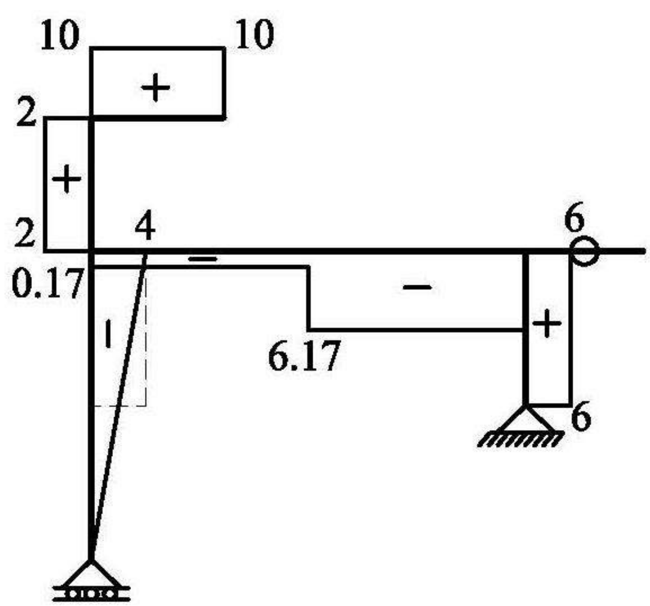
M_6 from right side = -6x2=-12 m.t

M_7 from right side = -3 m.t

M_8 from equilibrium = -12-3=-18 m.t



N.F.D



S.F.D

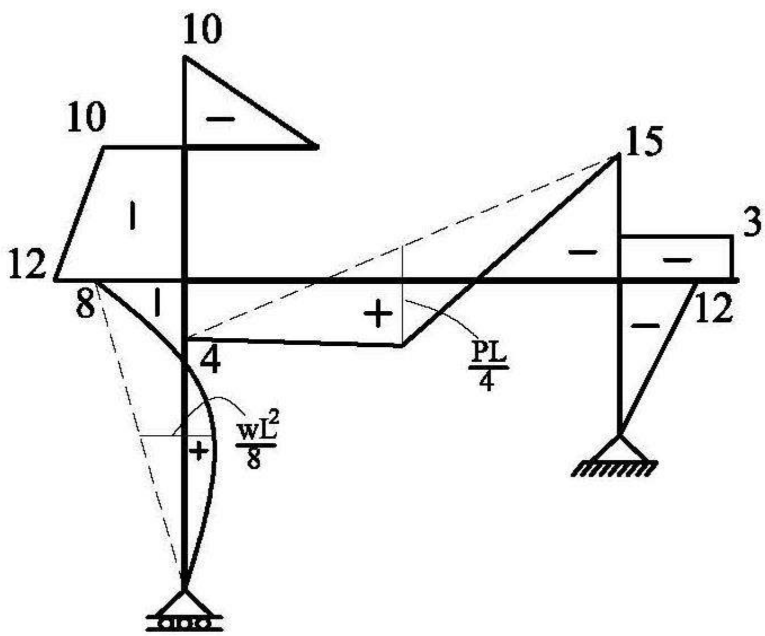


Fig. 4.15.c

<u>Example (12cas#1)</u>

For the given frame in Fig. 4.16, draw the normal, shear force and bending moment diagrams.

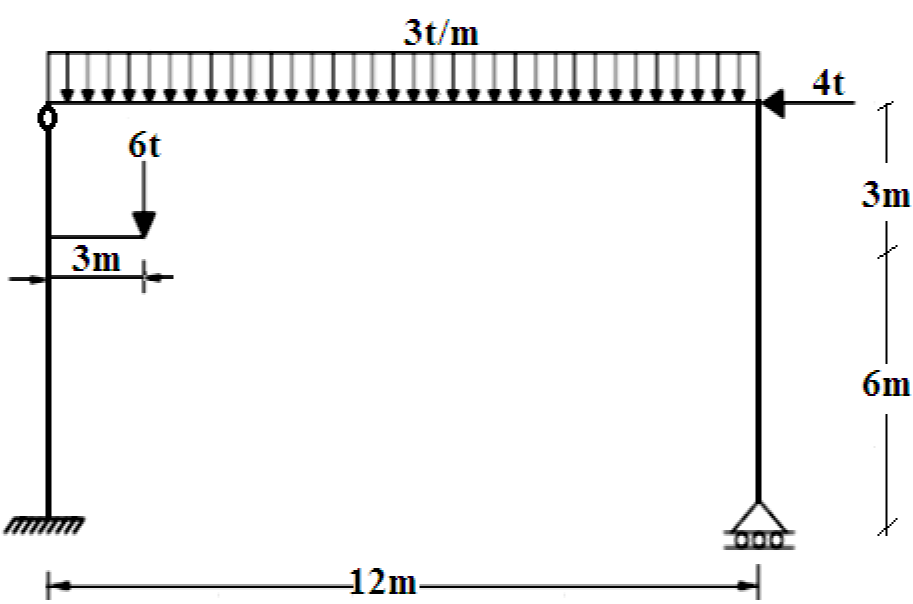


Fig. 4.16.a

<u>Solution</u>

- *Finding the reactions*
- *For part b-c*

e) $\sum F_x = 0 \gg\gg\gg X_c = 4$

f) $\sum M_c = 36x6 - Y_b x12 = 0 \therefore Y_b = 18t \uparrow$

g) $\sum M_b = 36x6 - Y_a x12 = 0 \therefore Y_a = 18t \uparrow$

<u>*Check*</u> $\sum F_y = 36 - 18 - 18 = 0 \gg\gg\gg$ ok

- *For part c-a*

h) $\sum F_x = 0 \gg\gg\gg X_a = 4 \rightarrow$

i) $\sum M_a = 0 \gg\gg\gg 6x3 - 4x9 - M_a = 0$

$\therefore M_b = 18\ mt$

<u>*Check*</u> $\sum F_y = 18 + 6 - Y_a = 0 \gg\gg\gg Y_a = 24t \uparrow$

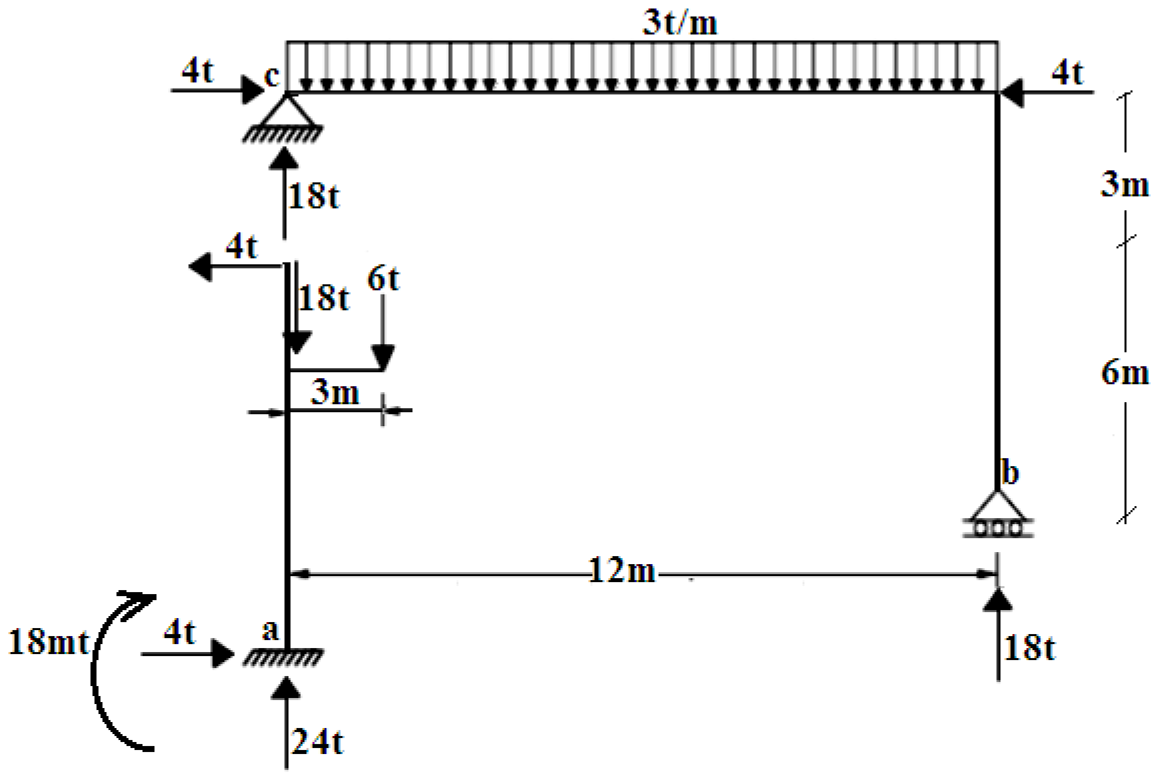
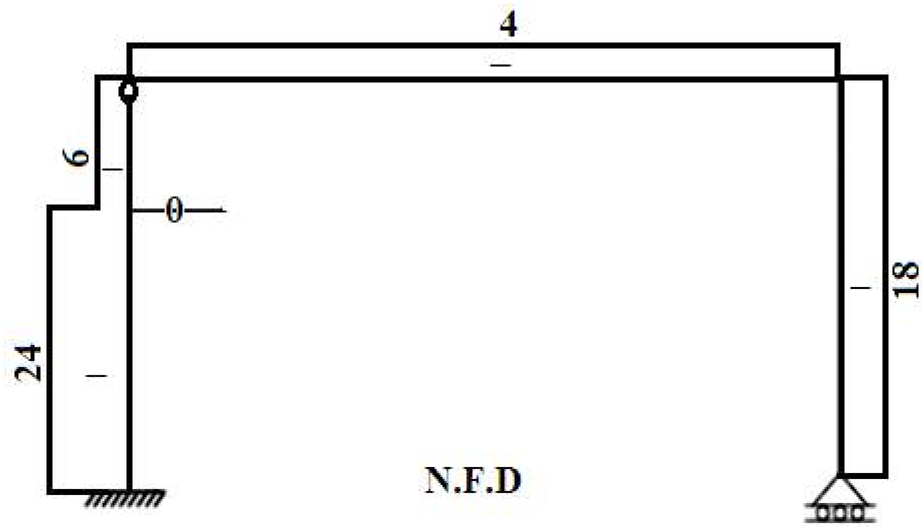
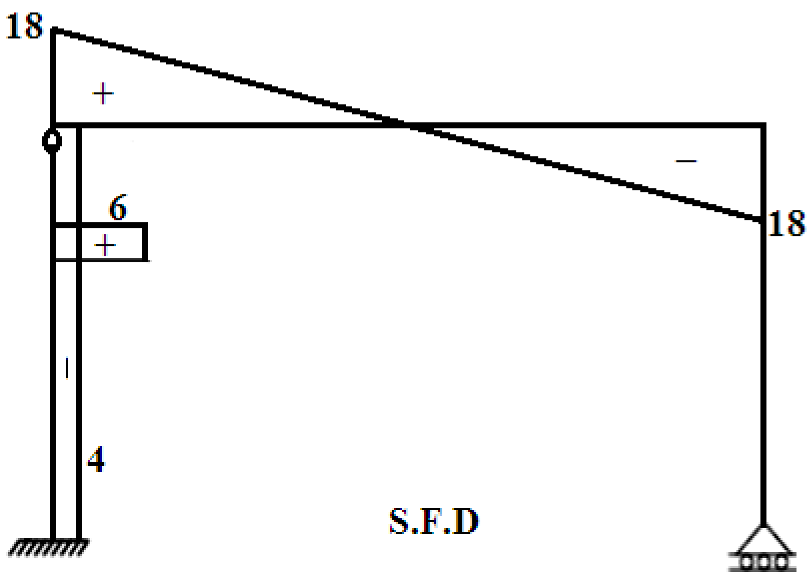

3t/m
4t
c
4t
3m
18t
4t
18t
6t
3m
6m
b
12m
18mt
4t
a
18t
24t

Fig. 4.16.b


4
6
0
18
24
N.F.D



- For the BMD

$M_1 = 18 - 4x6 = -6mt$

$M_2 = 6x3 = 18mt$

$M_3 from\ the\ equlibrium = 18 - 6 = 12mt$

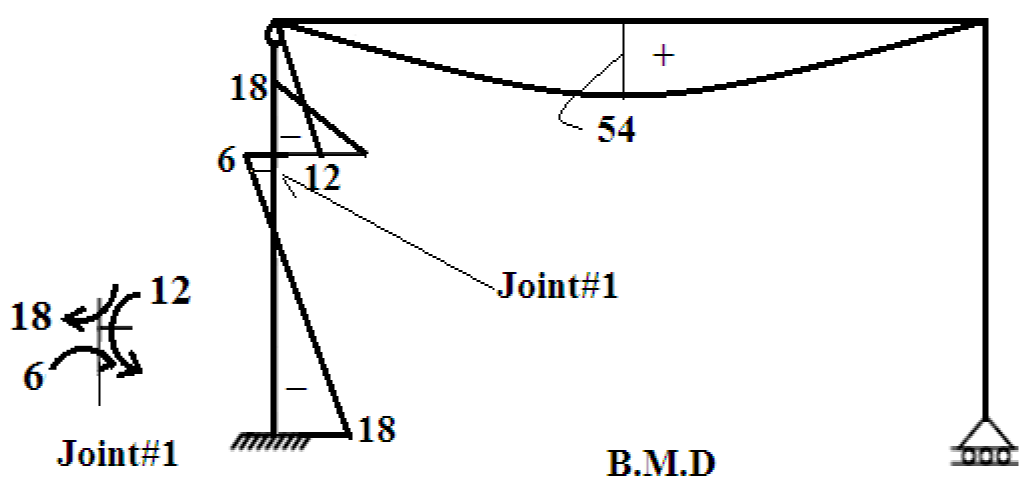


Fig. 4.16.c

Example (12cas#2)

For the given frame in Fig. 4.16.d, draw the normal, shear force and bending moment diagrams.

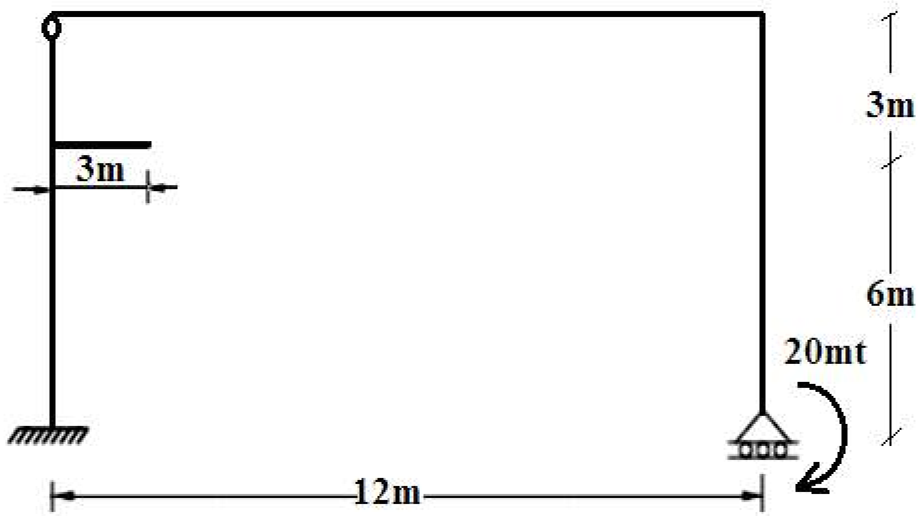
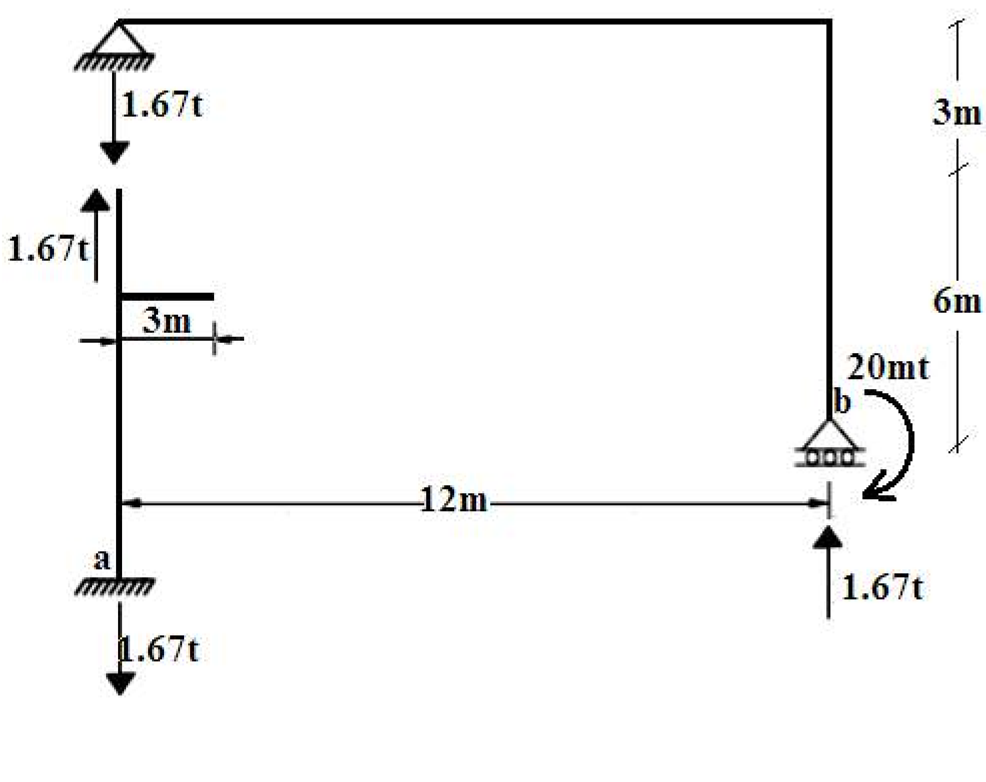


Fig. 4.16.d



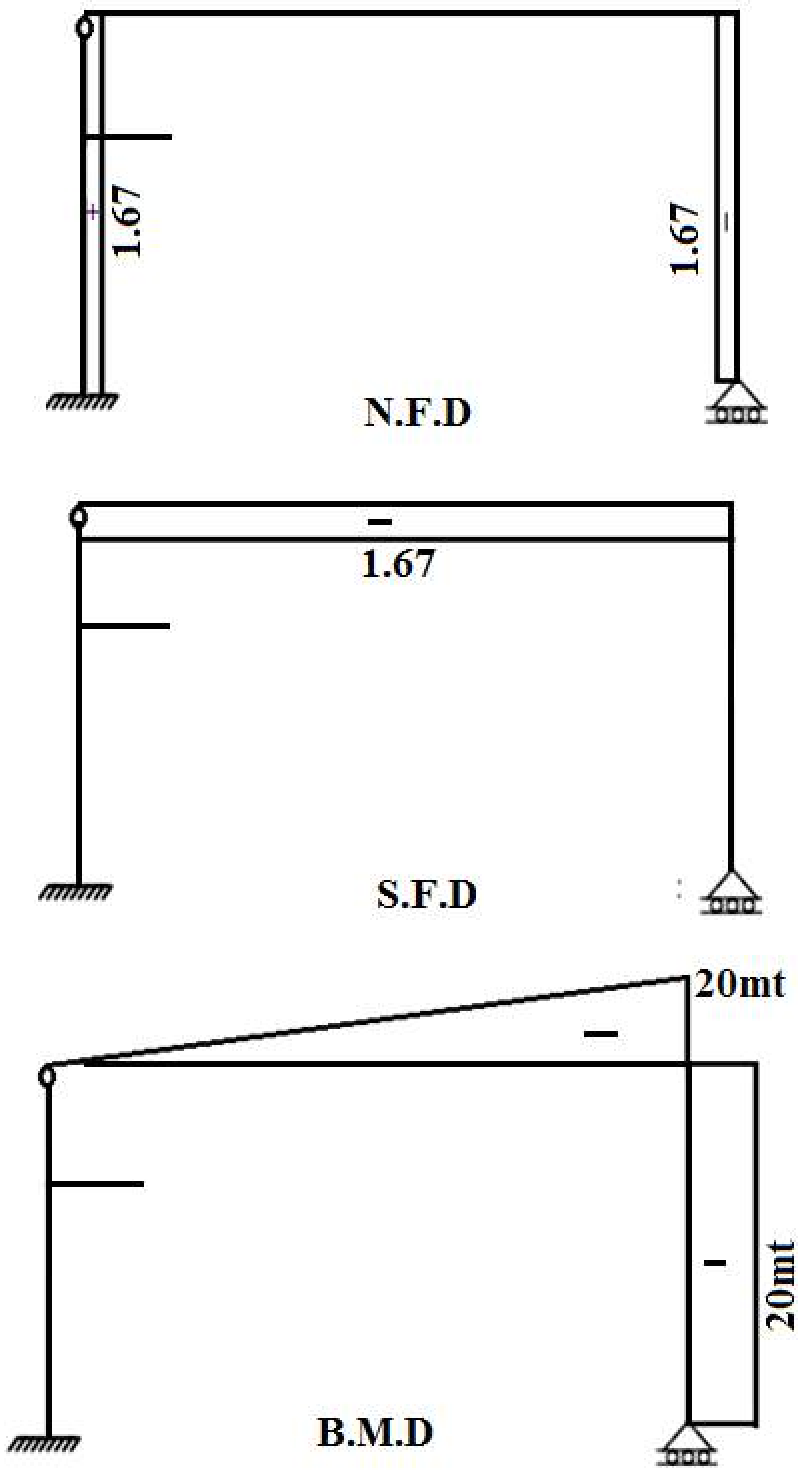


Fig. 4.16.e

Example (12cas#3)

For the given frame in Fig. 4.16.f, draw the normal, shear force and bending moment diagrams.

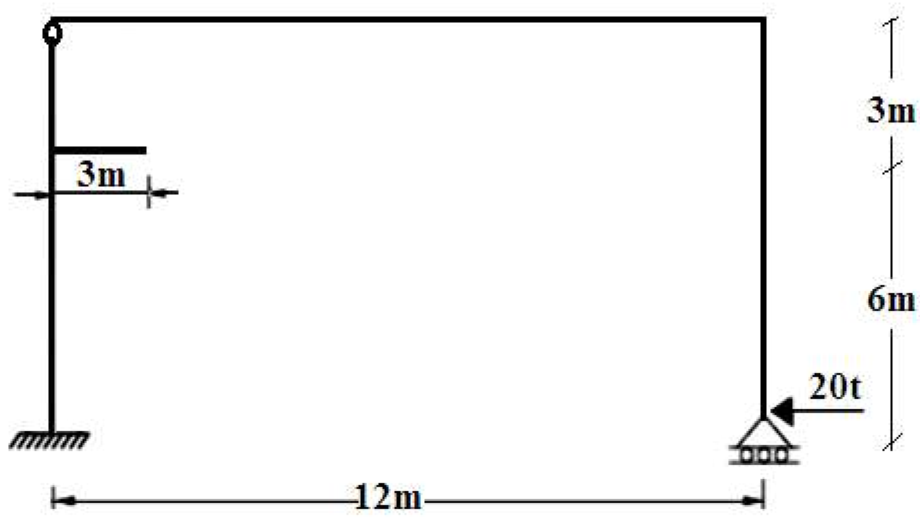
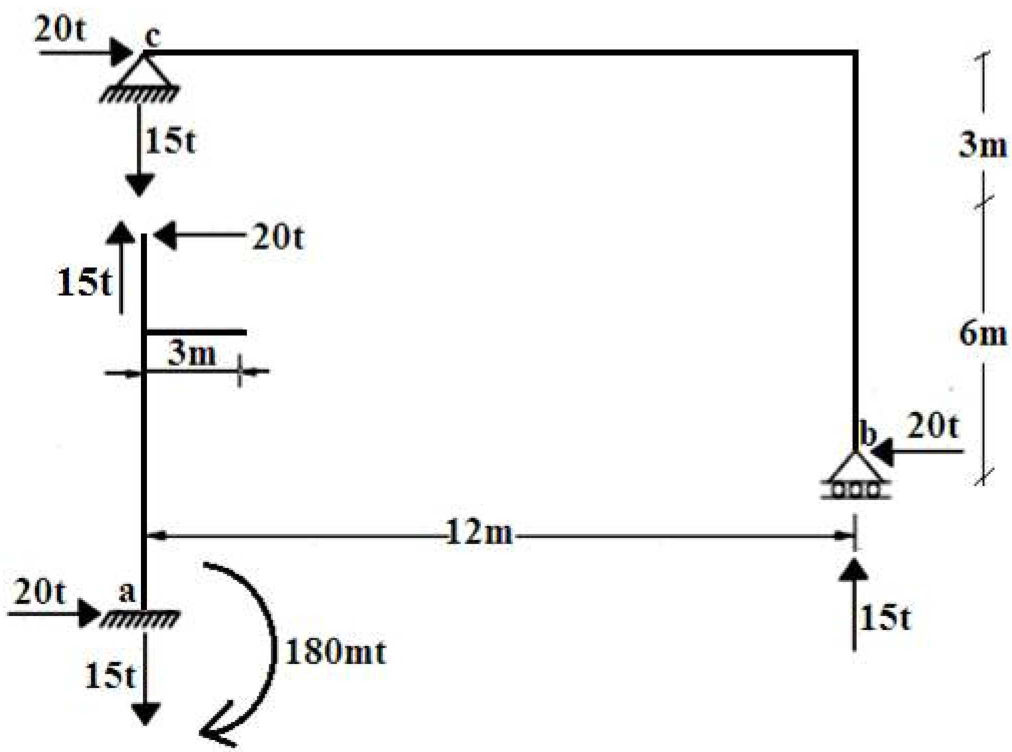
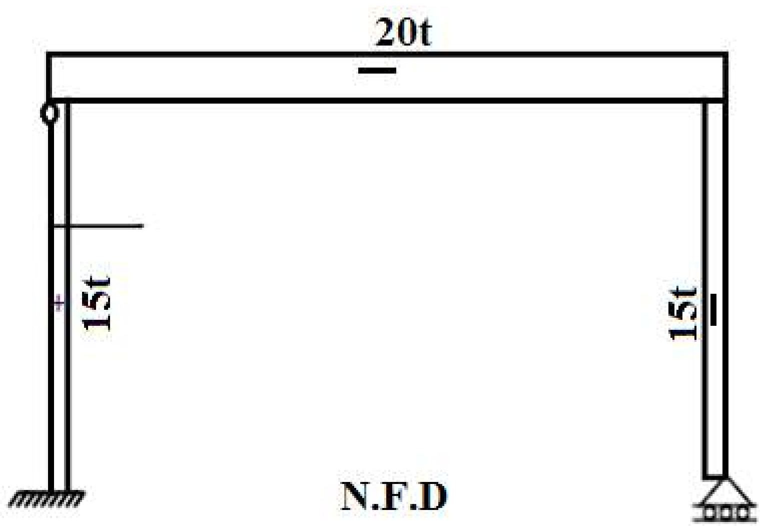
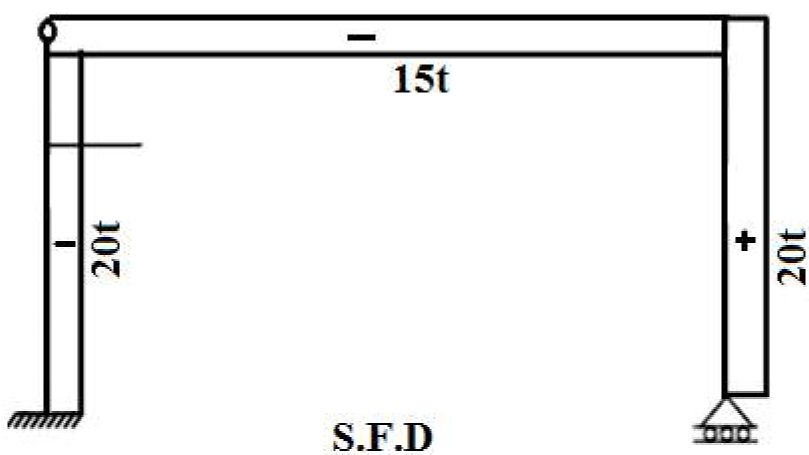


Fig. 4.16.f







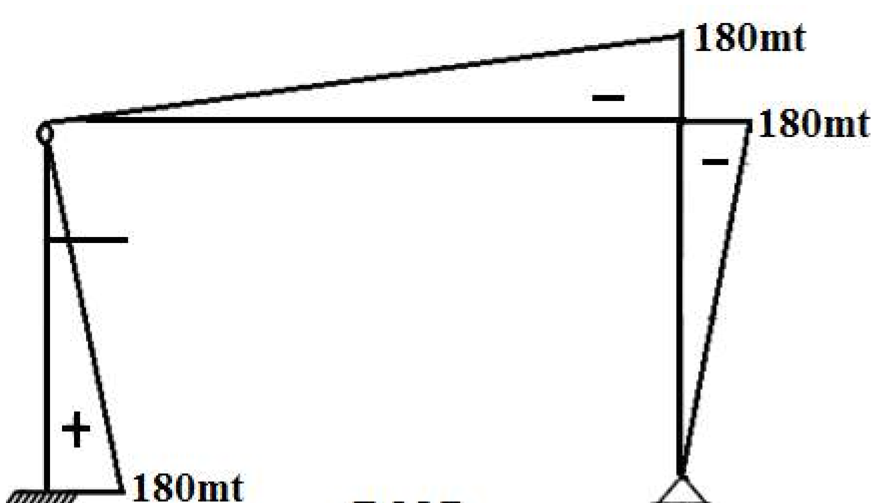


Fig. 4.16.g

Example (13)

For the given frame in Fig. 4.17.a, draw the normal, shear force and bending moment diagrams.

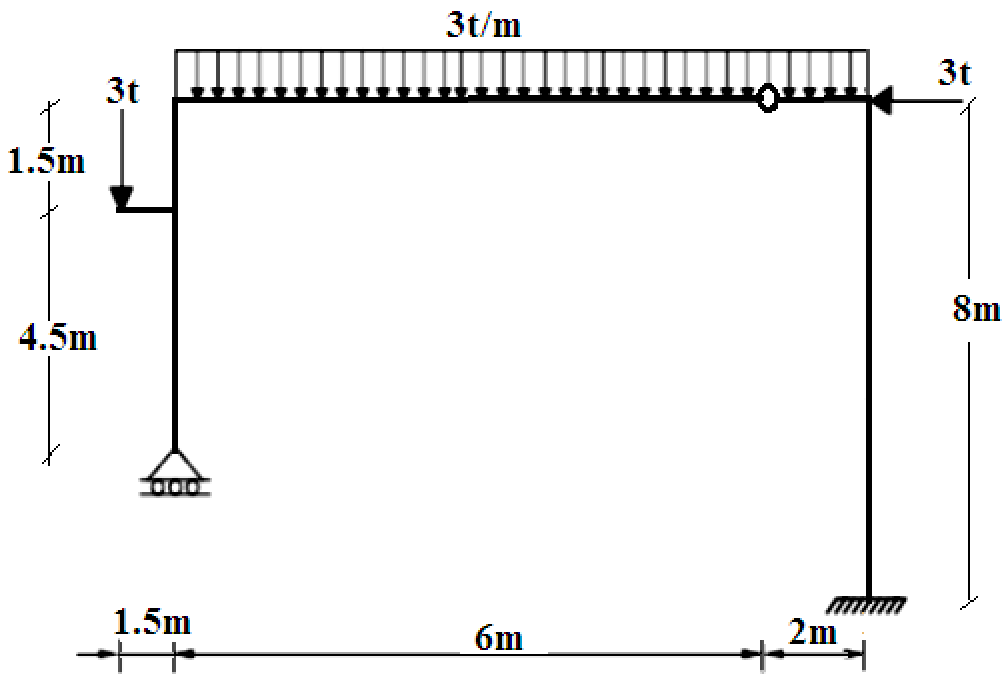


Fig. 4.17.a

Solution

- *Finding the reactions*
- *For part a-c*

j) $\sum F_x = 0 \ggg\ggg X_b = 0$

k) $\sum M_a = 0 \ggg\ggg 18x3 - 3x1.5 - Y_b x6 = 0 \therefore Y_b = 8.25t \uparrow$

l) $\sum M_b = 0 \ggg\ggg 18x3 + 3x7.5 - Y_a x6 = 0 \therefore Y_a = 12.75t \uparrow$

Check $\sum F_y = 12.75 + 8.25 - 18 - 3 = 0 \ggg\ggg$ ok

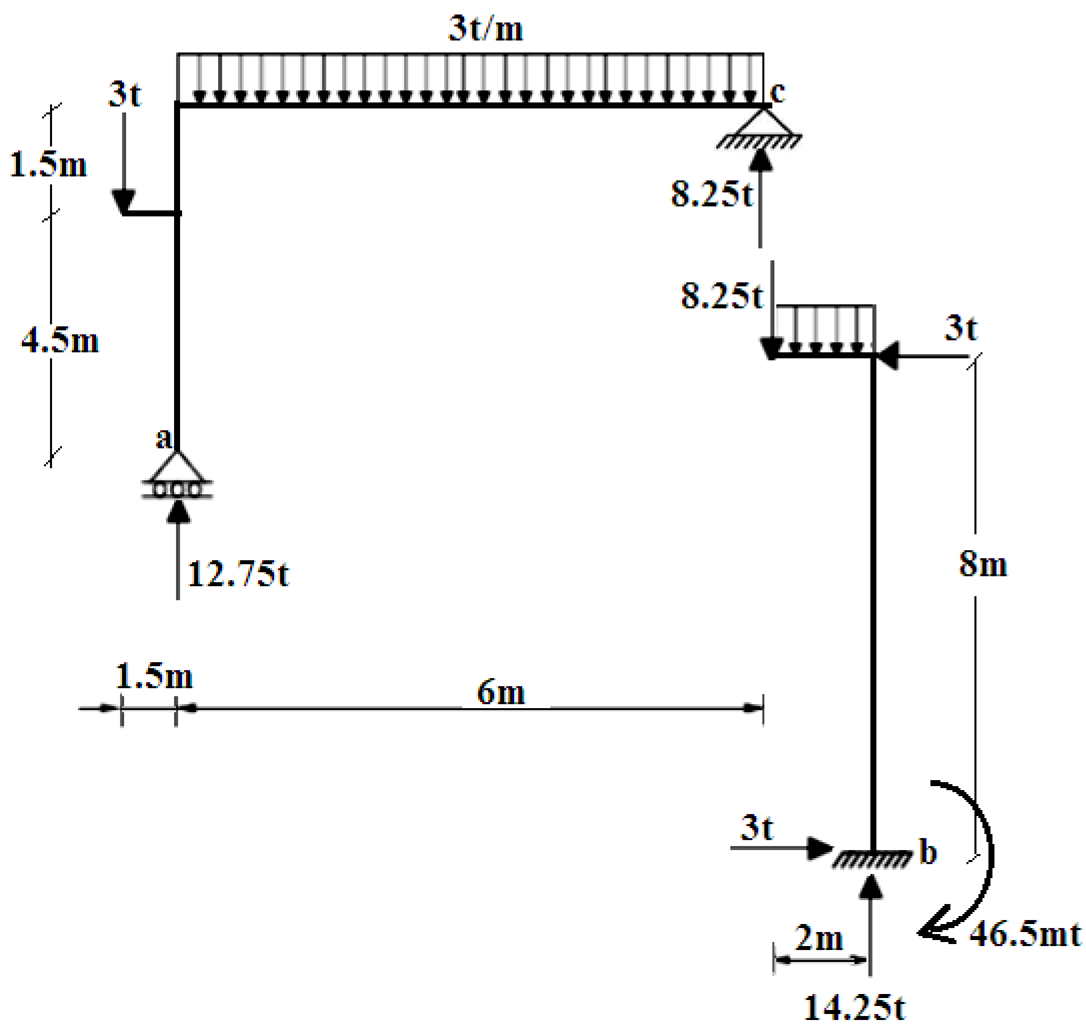


Fig. 4.17.b

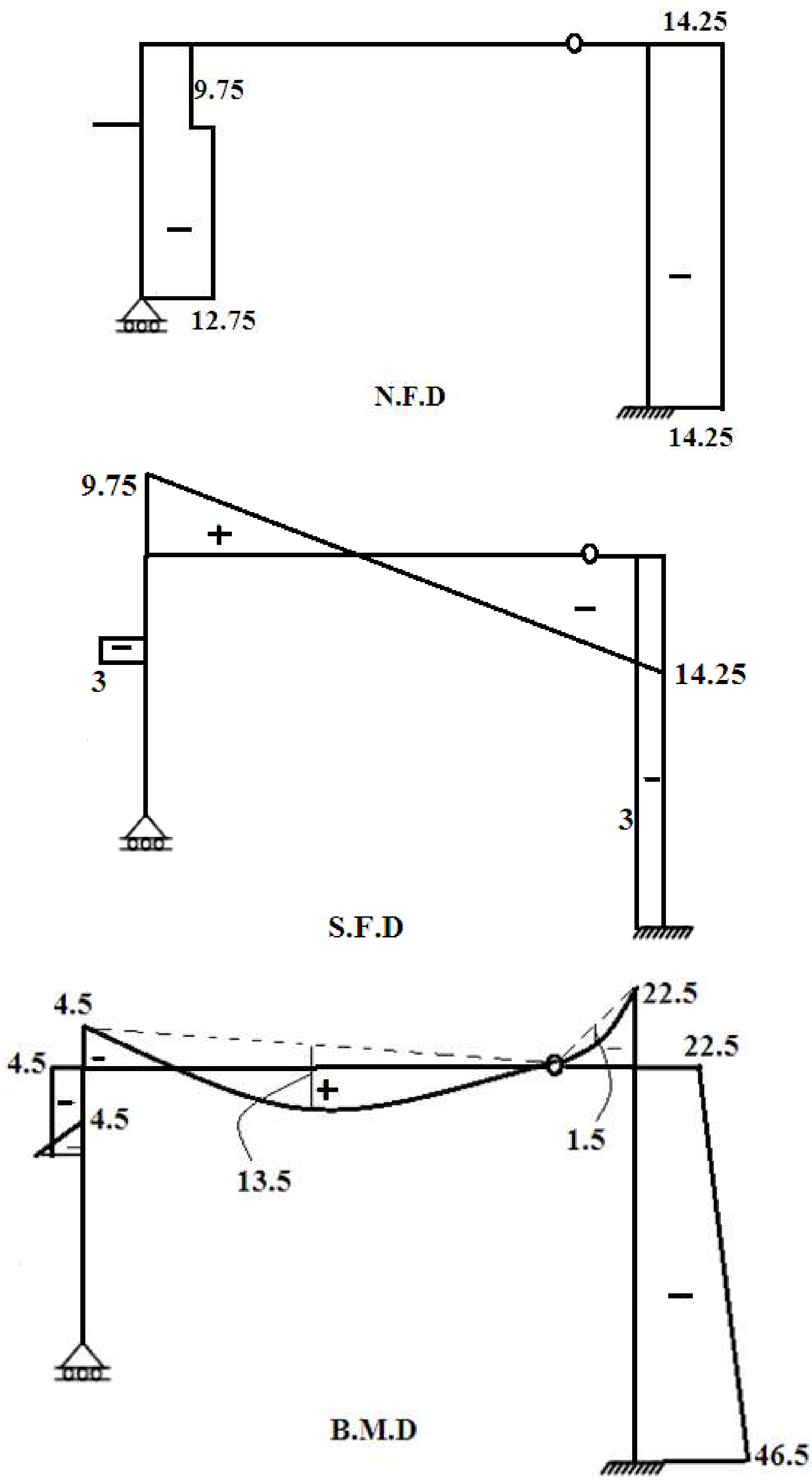
- *For part c-b*

m) $\sum F_x = 0 \ggg\ggg X_c = 3 \rightarrow$

n) $\sum M_a = 0 \ggg\ggg 8.25x2 + 3x8 + 6x1 - M_b = 0$

$\therefore M_b = 46.5\ mt$

$\underline{Check}\ \sum F_y = 8.25 + 6 - Y_b = 0 \ggg\ggg Y_b = 14.25t$



14.25
9.75
12.75
N.F.D
14.25
9.75
+
14.25
3
3
S.F.D
4.5
22.5
4.5
22.5
4.5
+
1.5
13.5
B.M.D
46.5

Fig. 4.17.c

Example (14)

For the given frame in Fig. 4.18.a, draw the normal, shear force and bending moment diagrams.

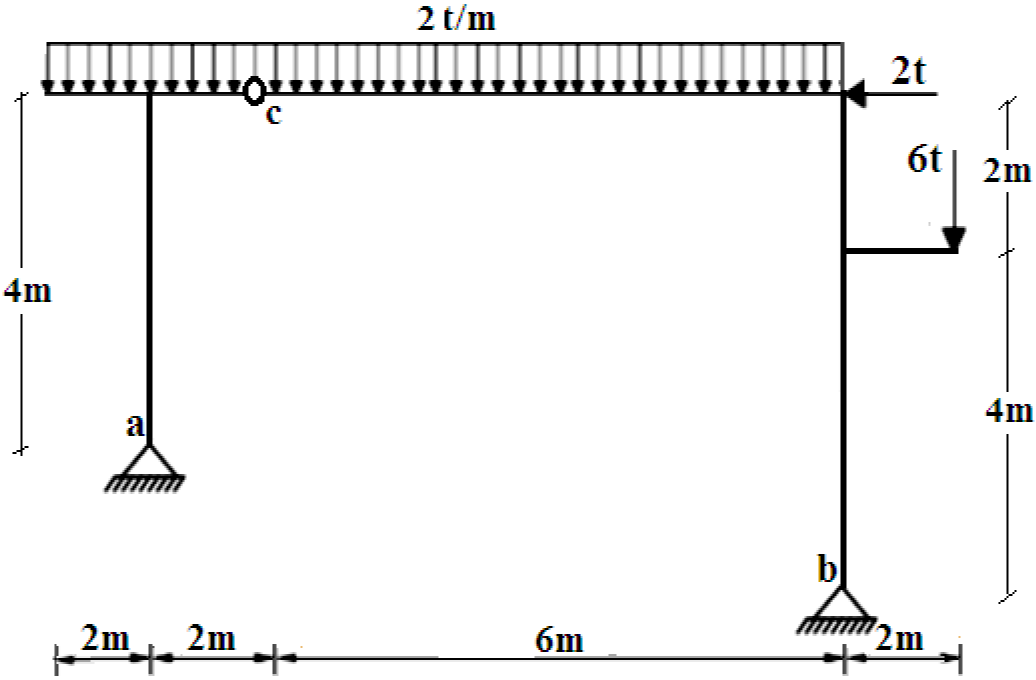


Fig. 4.18.a

Solution

➢ *Finding the reactions*

a) $\sum M_a = -4x1 + 4x1 + 12x5 - 2x4 + 6x10 + 2X_b - 8Y_b = 0$

$$\therefore 112 + 2X_b - 8Y_b = 0 \ldots \ldots (1)$$

b) $\sum M_b = -6x2 + 2x6 + 12x3 + 4x7 + 4x9 - 2X_a - 8Y_a = 0$

$$\therefore 50 - X_a - 4Y_a = 0 \ldots \ldots (2)$$

c) $\sum M_c$ for the left side $= 0$

$4x1 + 4x3 + 4x_a - 2Y_a = 0$ $\qquad \therefore 16 + 4x_a - 2Y_a = 0 \ldots \ldots (3)$

d) $\sum M_c$ for the right side $= 0$

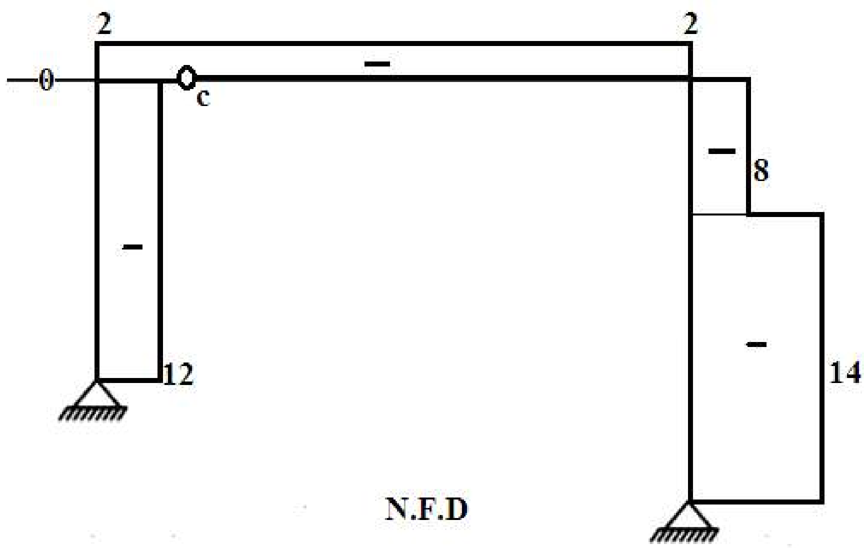
$$12x3 + 6x8 + 6x_b - 6Y_b = 0 \qquad \therefore 84 + 6x_b - 6Y_b = 0 \; \ldots\ldots (4)$$

By solving the four equations

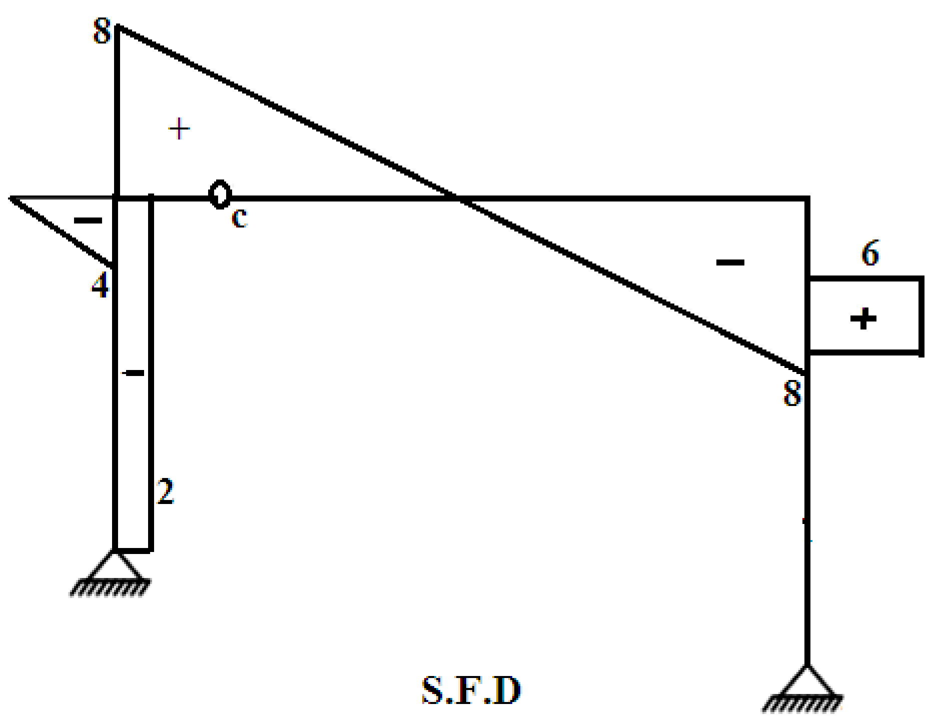
$$X_a = 2 \rightarrow \qquad Y_a = 12 \uparrow \quad X_b = 0 \qquad Y_b = 14 \uparrow$$

e) *check* $\sum F_x = 2 - 2 = 0 \ggg\ggg\ggg$ ok

f) <u>*Check*</u> $\sum F_y = 4 + 4 + 12 + 6 - 14 - 12 = 0 \ggg\ggg\ggg$ ok



N.F.D



S.F.D

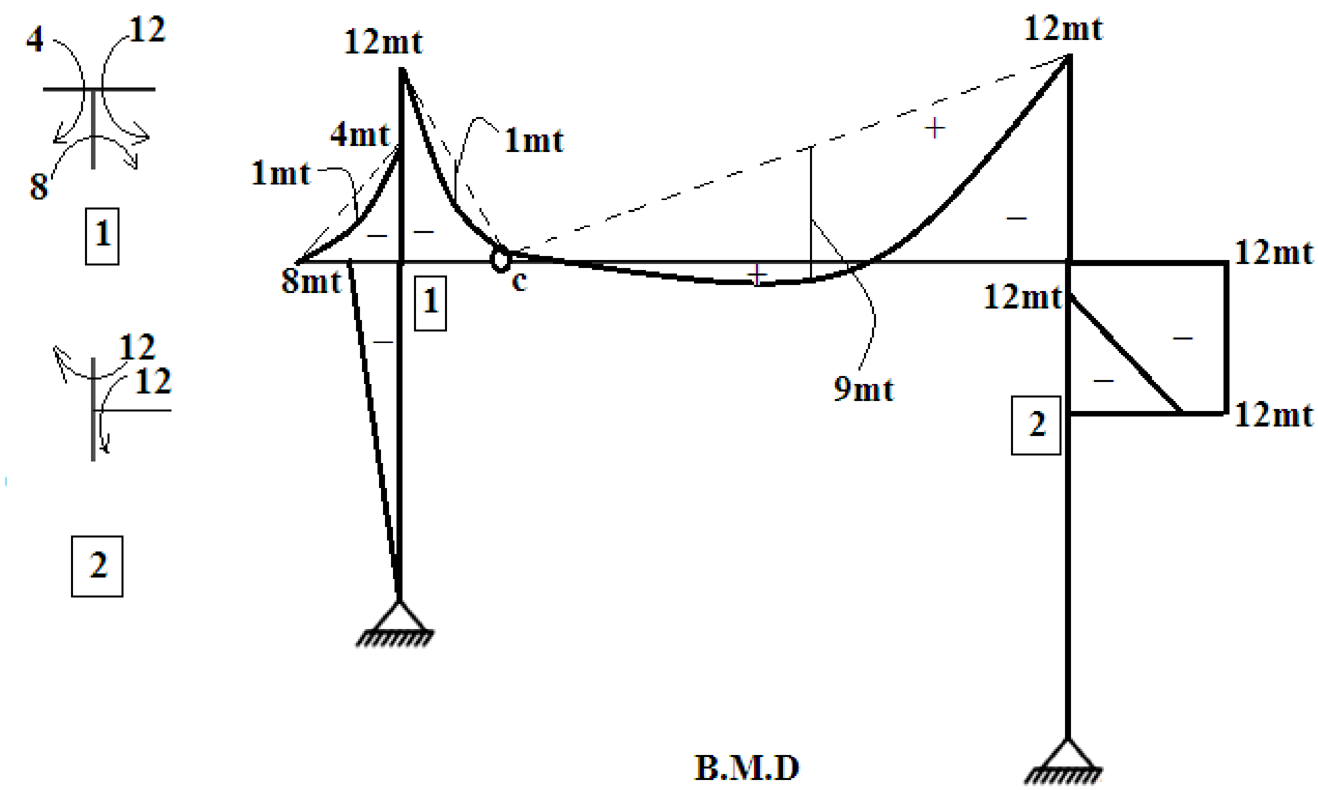


Fig. 4.18.b

Example (15)

For the given frame in Fig. 4.19.a, draw the normal, shear force and bending moment diagrams.

Solution

- *Finding the reactions*

a) $\sum M_a = 0$

$$2x4 - 6x2 + 24x2 + 84x11 + 20x20 - 3x4 - 2X_b - 18Y_b = 0$$

$$\therefore 1356 - 2X_b - 18Y_b = 0 \ldots\ldots(1)$$

b) $\sum M_b = 3x2 - 20x2 + 84x7 + 24x16 + 6x20 - 2x2 + 2X_a - 18Y_a = 0$

$$\therefore 1054 + 2X_a - 18Y_a = 0 \ldots\ldots(2)$$

c) $\sum M_c$ for the left side $= 0$

$$24x2 + 6x6 + 2x2 + 6x_a - 4Y_a = 0$$

$$\therefore 88 + 6x_a - 4Y_a = 0 \ldots\ldots(3)$$

d) $\sum M_c$ for the right side $= 0$

$$84x7 + 20x16 + 3x2 + 4x_b - 14Y_b = 0$$

$$\therefore 914 + 4x_b - 14Y_b = 0 \ \ldots\ldots(4)$$

By solving the four equations

$X_a = 26.32 \rightarrow \quad Y_a = 61.48 \uparrow \quad X_b = 25.32 \leftarrow \quad Y_b = 72.52 \uparrow$

e) *check* $\sum F_x = 26.32 + 2 - 3 - 25.32 = 0 \ggg\ggg$ ok

f) <u>*Check*</u> $\sum F_y = 20 + 24 + 84 - 61.48 - 72.52 = 0 \ggg\ggg$ ok

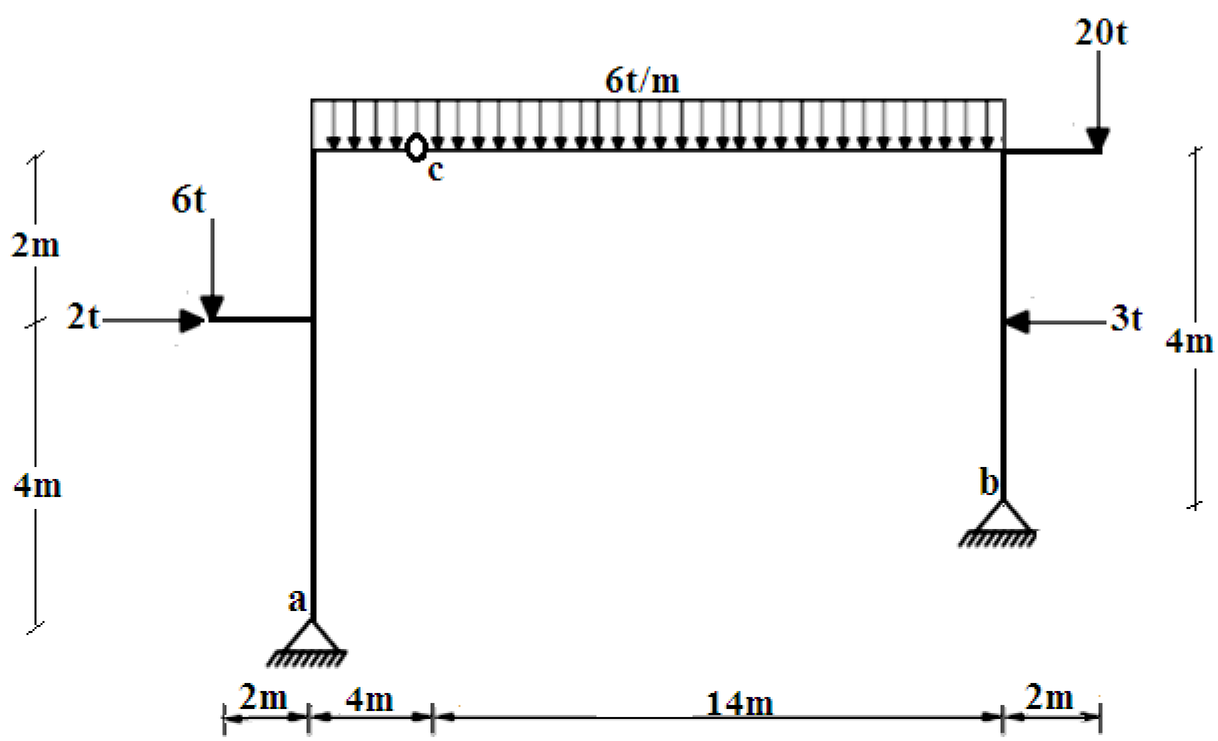
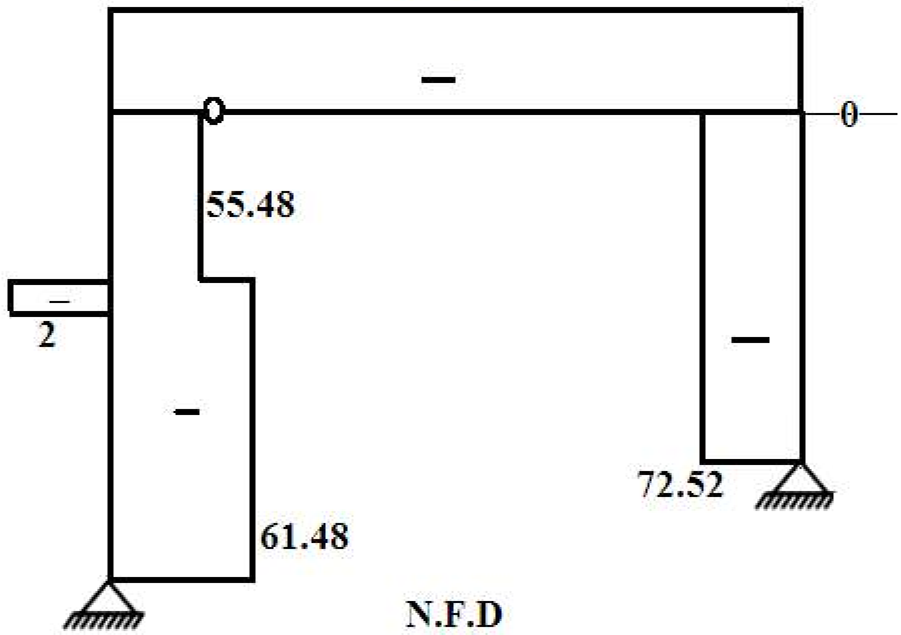
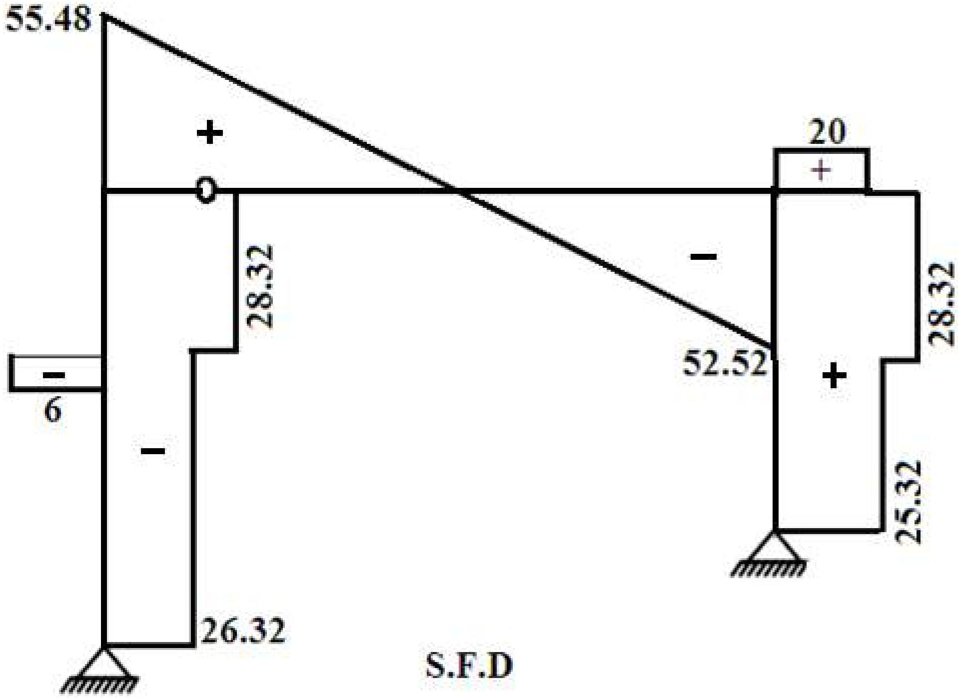


Fig. 4.19.a



N.F.D



S.F.D

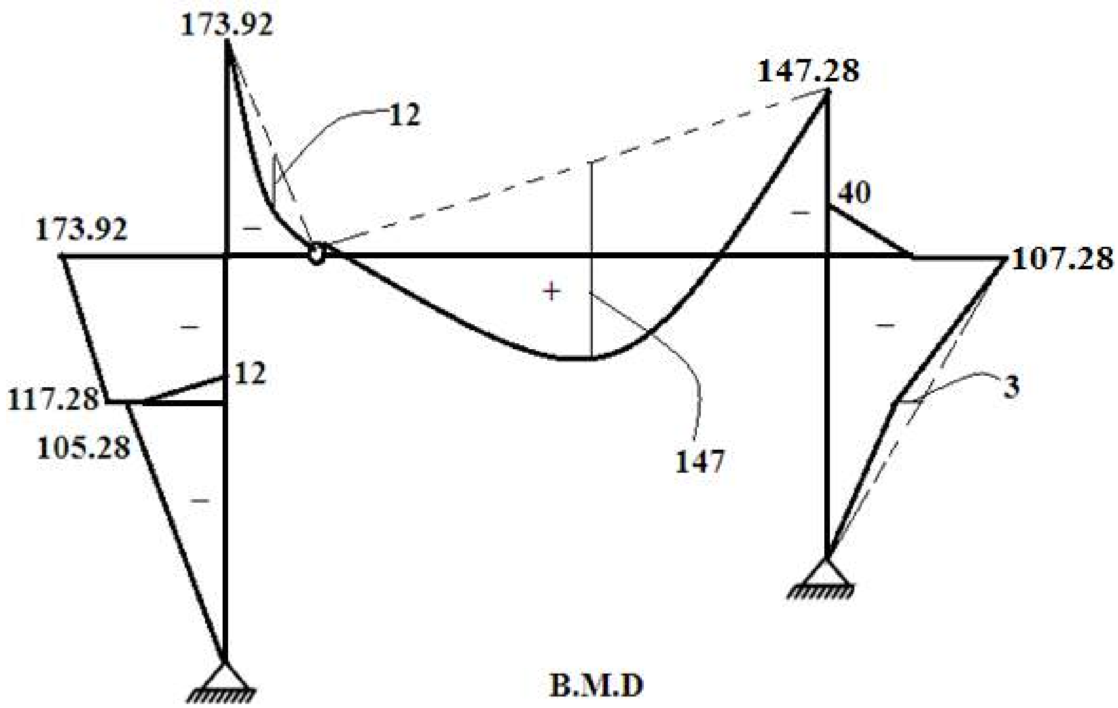


Fig. 4.19.b

Example (16)

For the given frame in Fig. 4.20.a, draw the normal, shear force and bending moment diagrams.

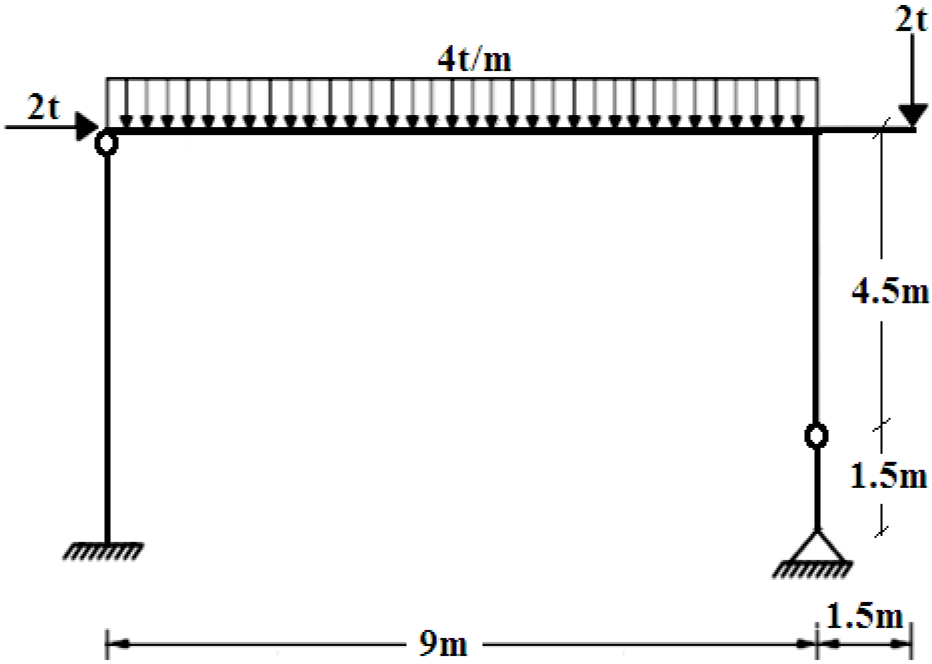


Fig. 4.20.a

Solution

➢ *Finding the reactions*

The hinged support at right side and the intermediate hinge are link support and it was considered as roller support as presented in Fig. 4.20.b

- For part c-b

$\sum F_x = 2 - X_c = 0 \ggg X_c = 2t \leftarrow$

a) $\sum M_c = 36x4.5 + 2x10.5 - 9Y_b = 0 \qquad \therefore Y_b = 20.33 \uparrow$

b) $\sum M_b = 36x4.5 - 2x1.5 - 9Y_c = 0 \quad \therefore Y_c = 17.67t \uparrow$

c) <u>*Check*</u> $\sum F_y = 36 + 2 - 17.67 - 20.33 = 0 \ggg\ggg$ ok

For part a-c

d) $\sum F_y = 0 \ggg Y_a = 17.67t \uparrow \quad \sum F_x = 0 \ggg X_a = 2t \leftarrow$

e) $\sum M_a = 0 \ggg 2x6 - M_a = 0 \gg M_a = 12mt$

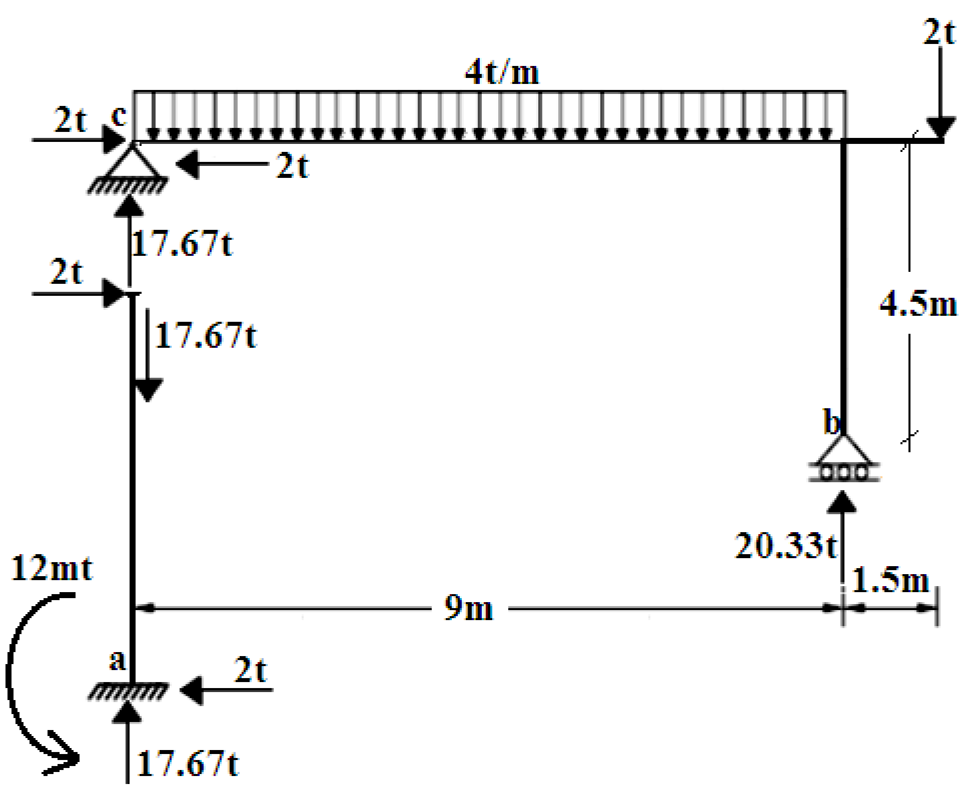


Fig. 4.20.b

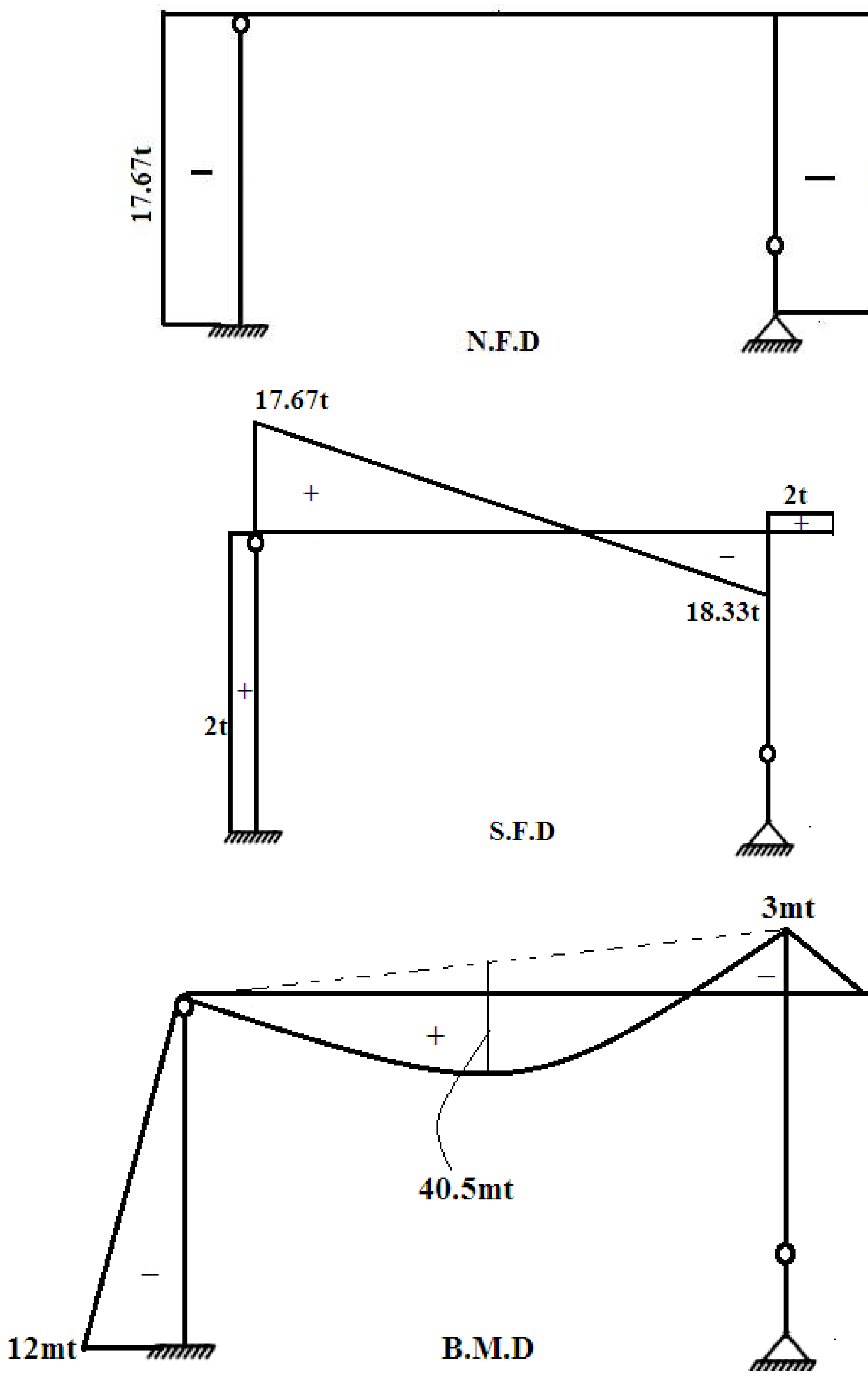


Fig. 4.20.c

Example (17)

For the given frame in Fig. 4.21.a, draw the normal, shear force and bending moment diagrams.

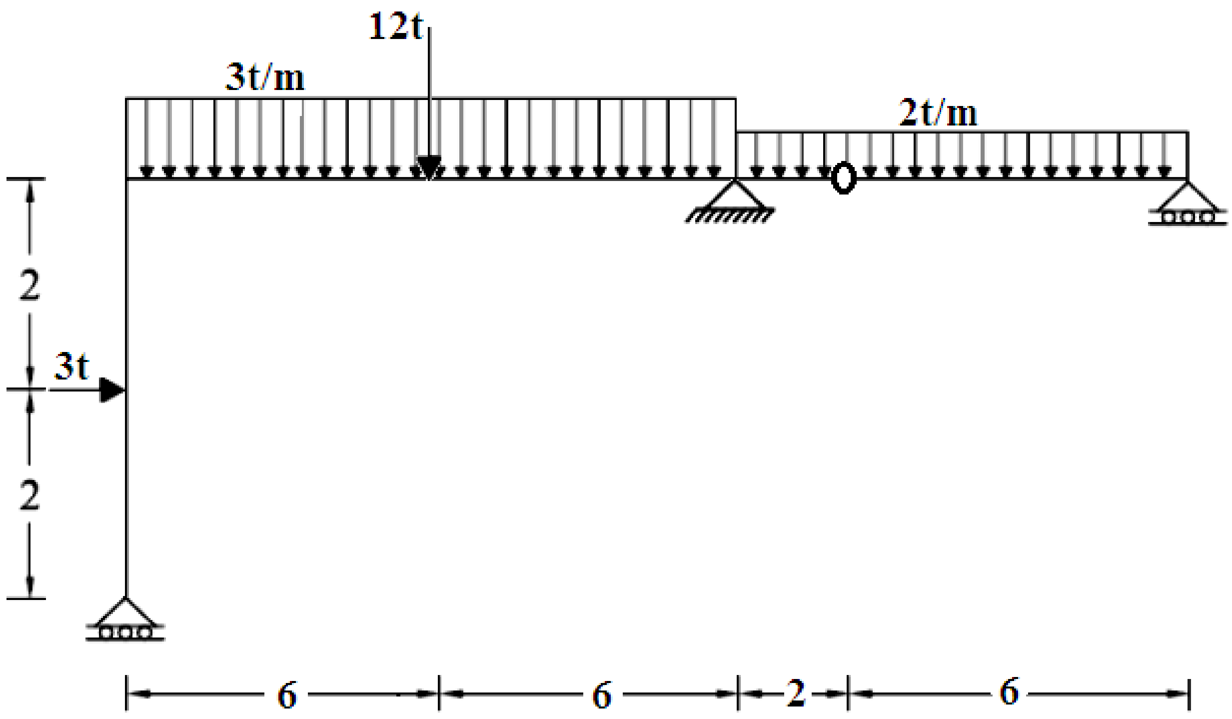


Fig. 4.21.a

Solution

❖ *Finding the reactions*

➢ *For part d-c*

e) $\sum F_x = 0 \gg\gg\gg\gg X_c = 0 \rightarrow$

f) $\sum M_c = 0 \gg\gg\gg\gg \therefore Y_d = 6 \uparrow$

g) $\sum M_d = 0 \gg\gg\gg\gg \therefore Y_c = 6 \uparrow$

➢ *For part a-b*

a) $\sum F_x = 0 \gg\gg\gg\gg X_b = 3t \leftarrow$

b) $\sum M_a = 0 \ggg\ggg 3x2 + 12x6 + 36x6 + 4x13 + 6x14 - 3x4 - Y_b x12 = 0$

$$\therefore Y_b = \frac{418}{12} = 34.833t \uparrow$$

c) $\sum M_b = 0 \ggg\ggg 36x6 + 12x6 + 3x2 - 4x1 - 6x2 - Y_a x12 = 0$

$$\therefore Y_a = \frac{287}{12} = 23.17t \uparrow$$

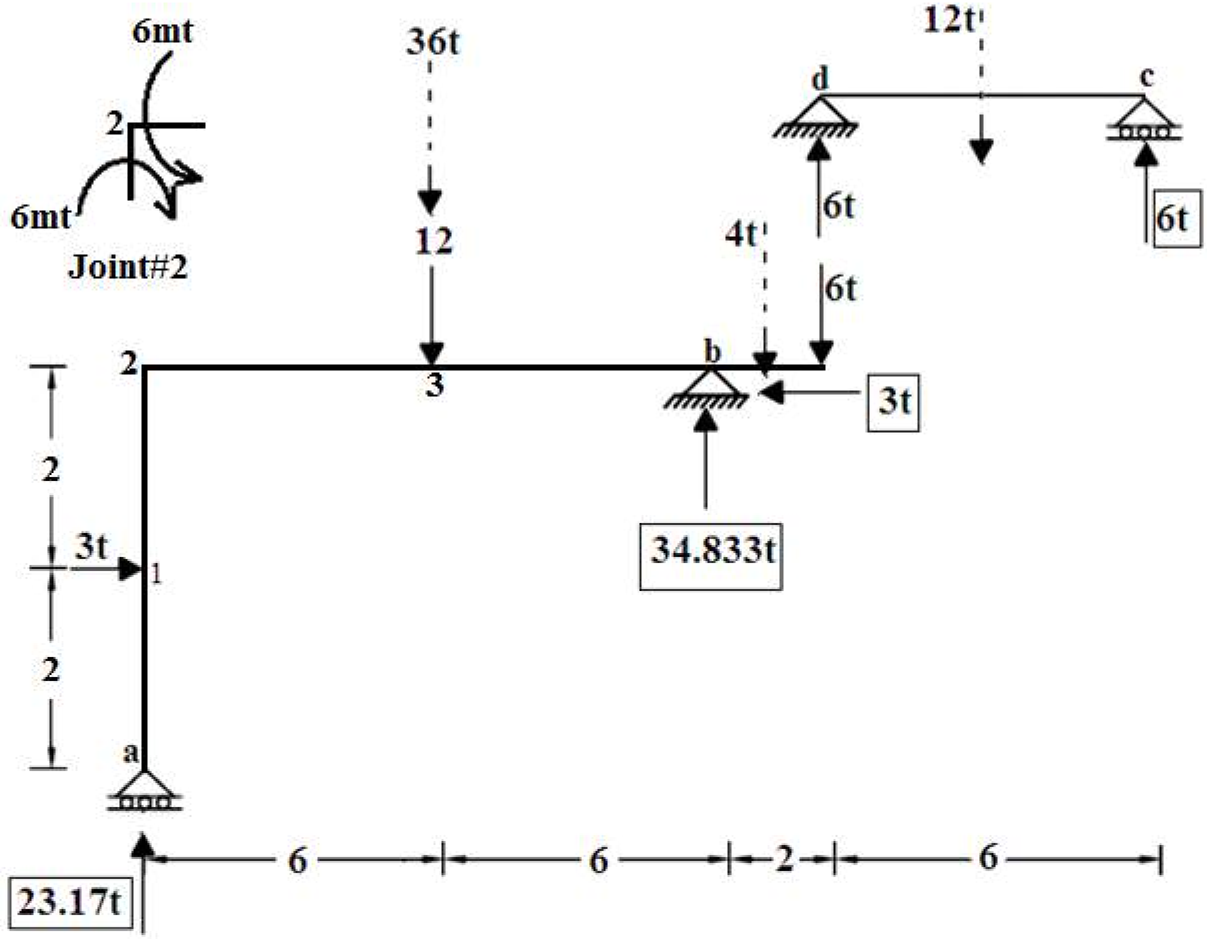
<u>*Check*</u> $\sum F_y = 23.17 + 34.833 + 6 - 36 - 12 - 4 - 12 = 0 \ggg$ ok

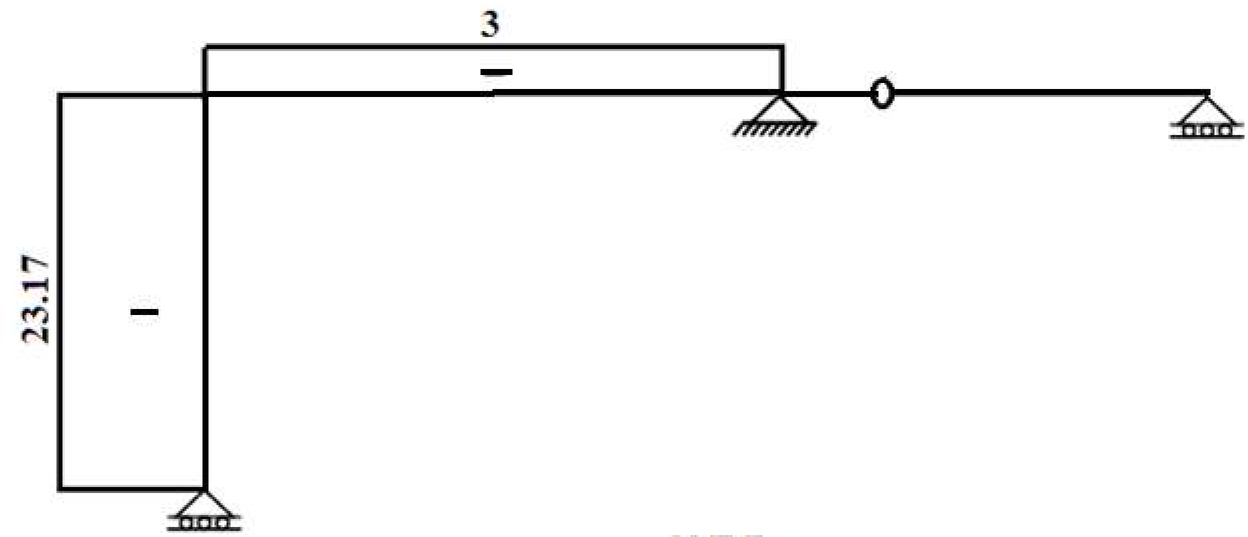
- *For drawing B.M.D*

M_1 from left side = 0 M_2 from left side = -3x2= -6 m.t

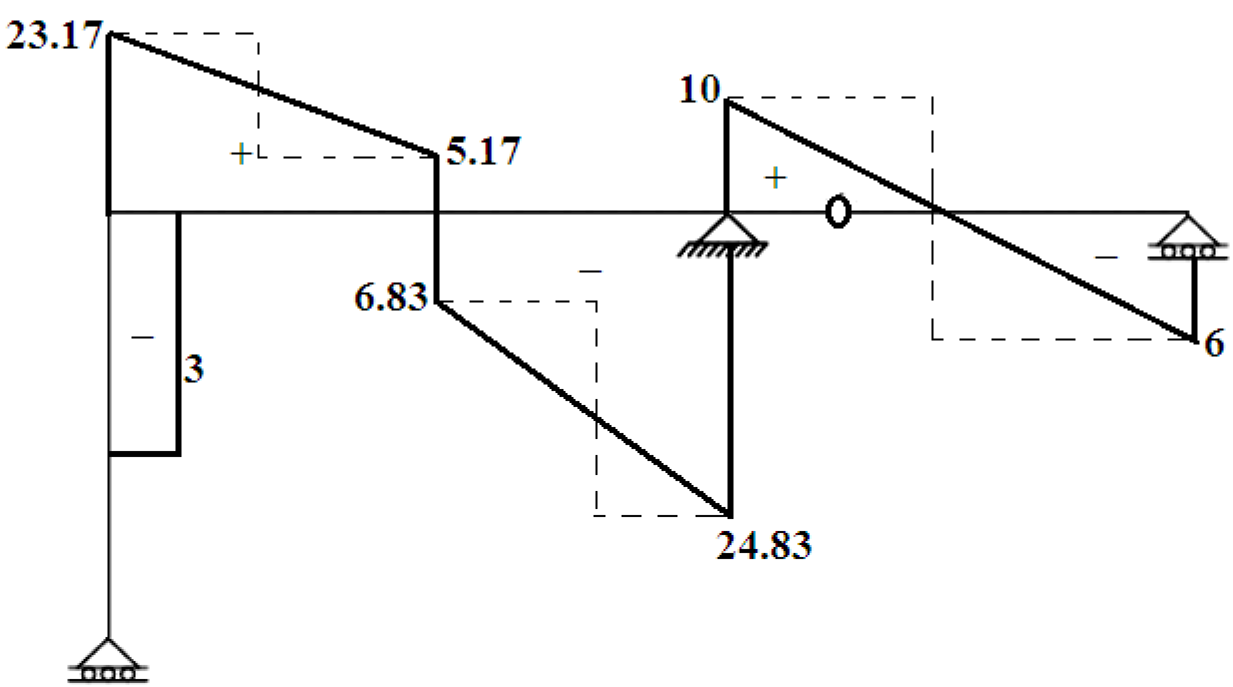
M_3 from right side = -3x2-18x3+23.17x6=79 m.t

M_4 from right side = -4x1-6x2=16 m.t





N.F.D



S.F.D

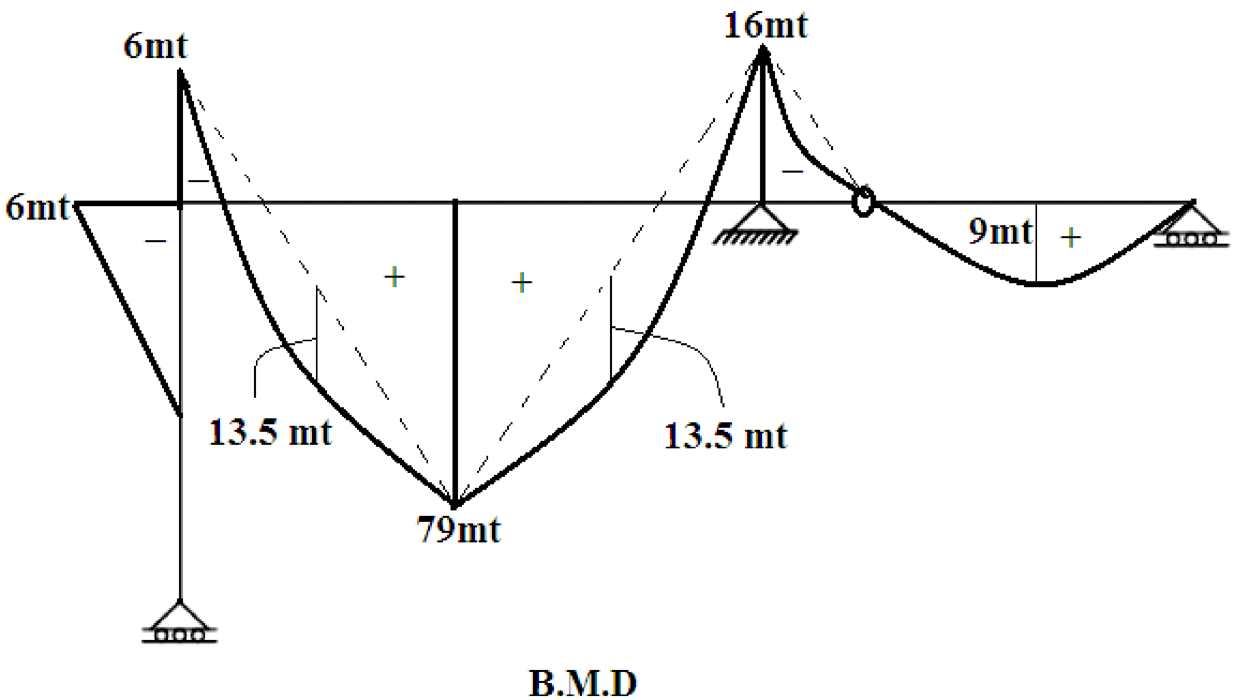


Fig. 4.21.b

Example (18)

For the given frame in Fig. 4.22.a, draw the normal, shear force and bending moment diagrams.

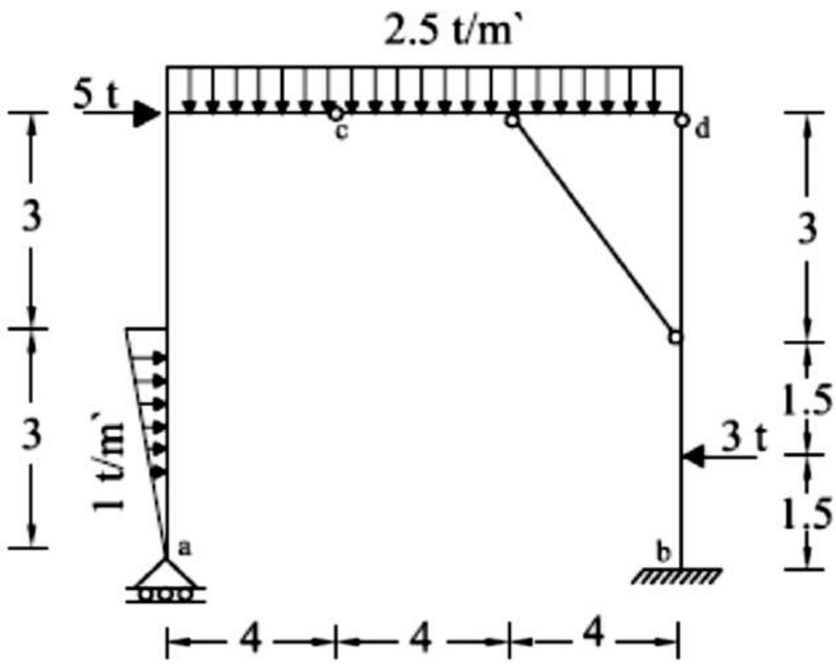


Fig. 4.22.a

Solution

❖ *Finding the reactions*

➢ *For part a-c*

a) $\sum F_x = 0 \ggg\ggg X_c = 6.5t \leftarrow$

b) $\sum M_a = 0 \ggg\ggg 1.5x2 + 5x6 + 10x2 - 6.5x6 - Y_c x4 = 0$

$$\therefore Y_c = \frac{14}{4} = 3.5t \uparrow$$

c) $\sum M_c = 0 \ggg\ggg 1.5x4 + 10x2 - Y_a x4 = 0 \therefore Y_a = \frac{26}{4} = 6.5t \uparrow$

d) <u>*Check*</u> $\sum F_y = 10 - 6.5 - 3.5 = 0 \ggg\ggg$ ok

➢ *For part c-b*

a) $\sum F_x = 0 \ggg\ggg X_b = 3.5t \leftarrow$

b) $\sum F_y = 0 \ggg\ggg 10 + 10 - 3.5 - Y_b = 0 \therefore Y_b = 23.5t \uparrow$

c) $\sum M_b = 0 \ggg\ggg 10x2 + 10x6 + 3.5x8 + 3x1.5 - 6.5x6 - M_b = 0$

$$\therefore M_b = 73.5\ mt$$

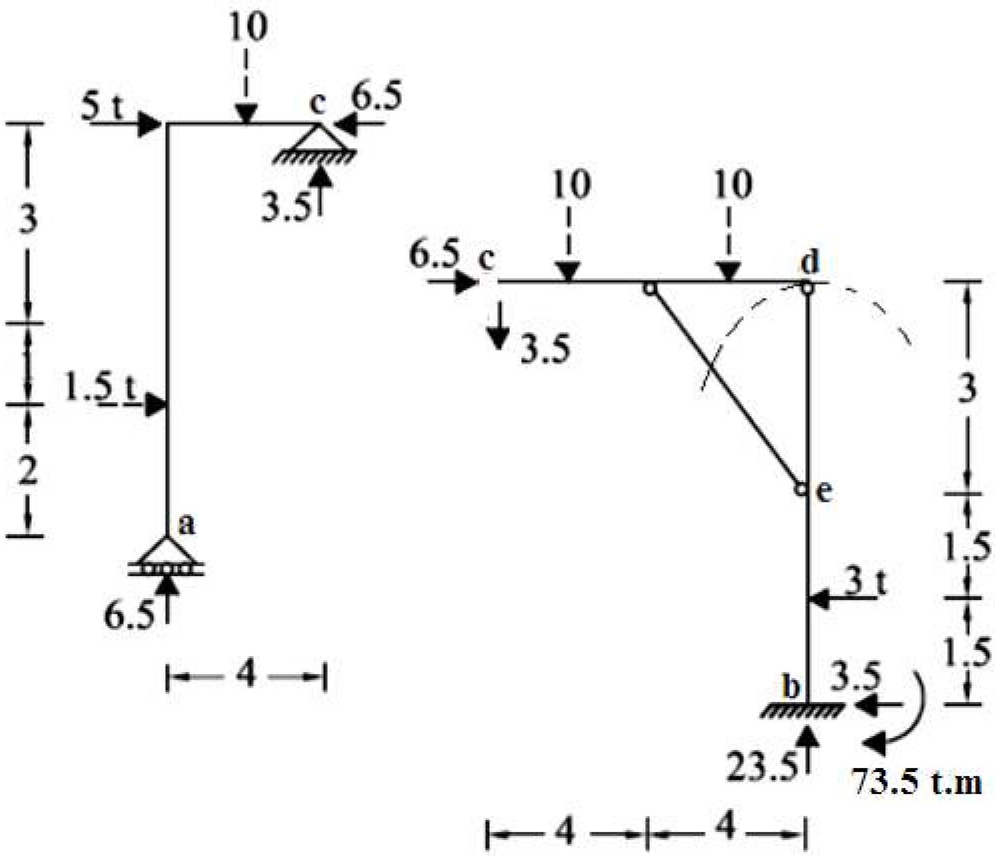
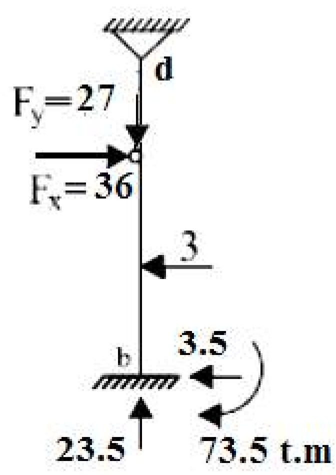


Fig. 4.22.b

For part d-e-b



$\sum M_d = 0.0$

$73.5+3.5x6+3x4.5- 3F_1x=0$

$F_1x = 36$ t

$F_1y = 36x3/4=27t$

$F_1 = 45t$ (compression)

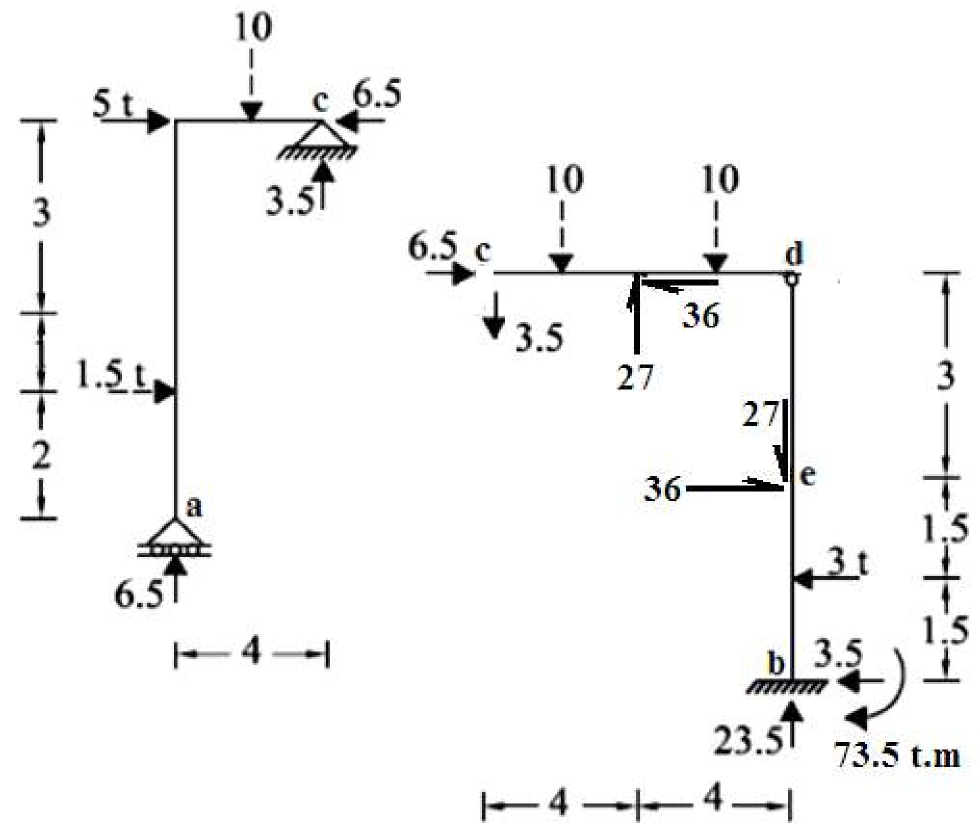
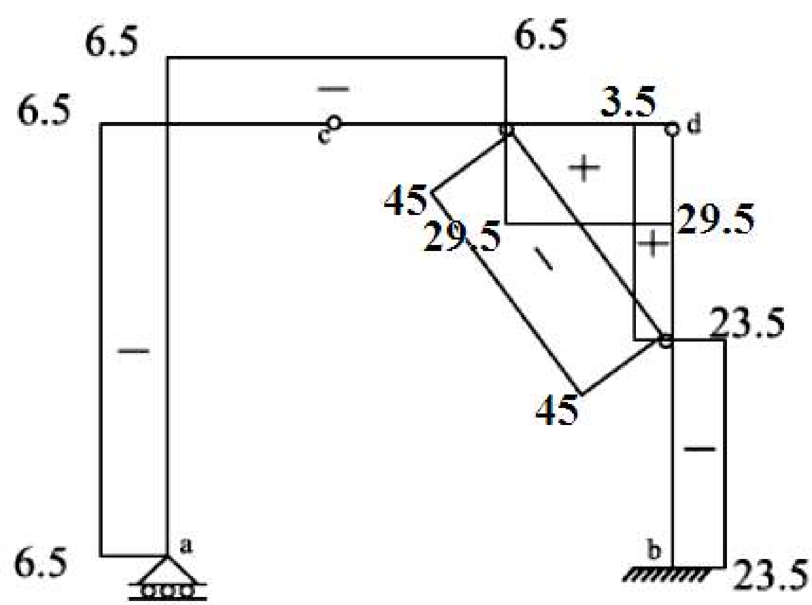
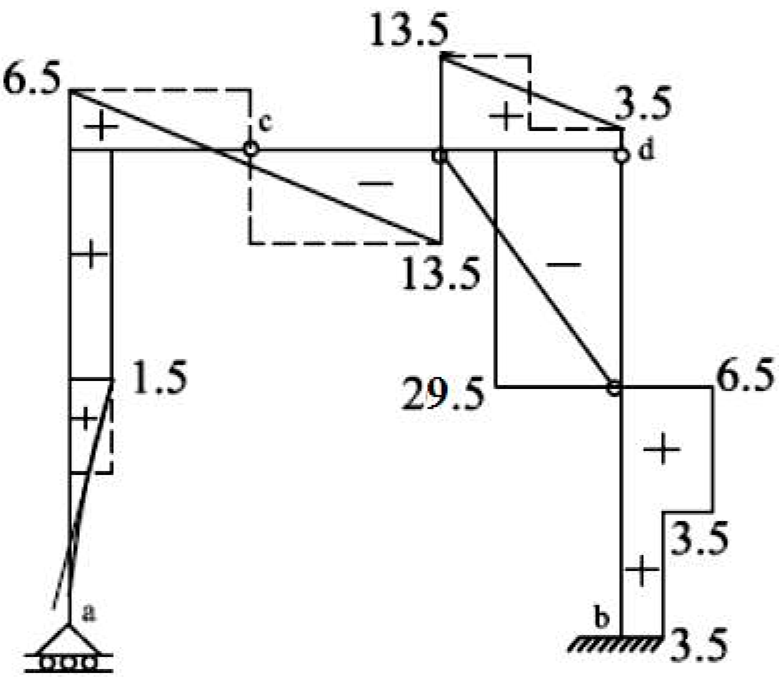

10
5 t
c
6.5
3.5
3
1.5 t
2
a
6.5
4
10
10
6.5
c
d
3.5
36
27
27
e
36
3
1.5
3 t
1.5
b
3.5
23.5
73.5 t.m
4
4

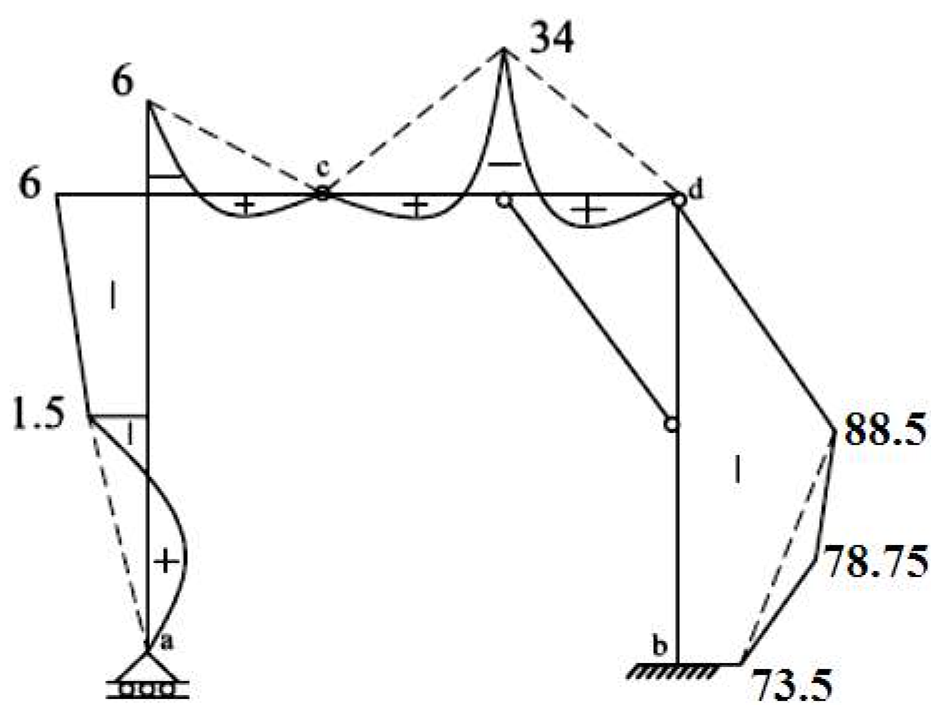
Fig. 4.10.c


6.5
6.5
6.5
3.5
c
d
45
29.5
29.5
23.5
45
6.5
a
b
23.5

N.F.D



S.F.D



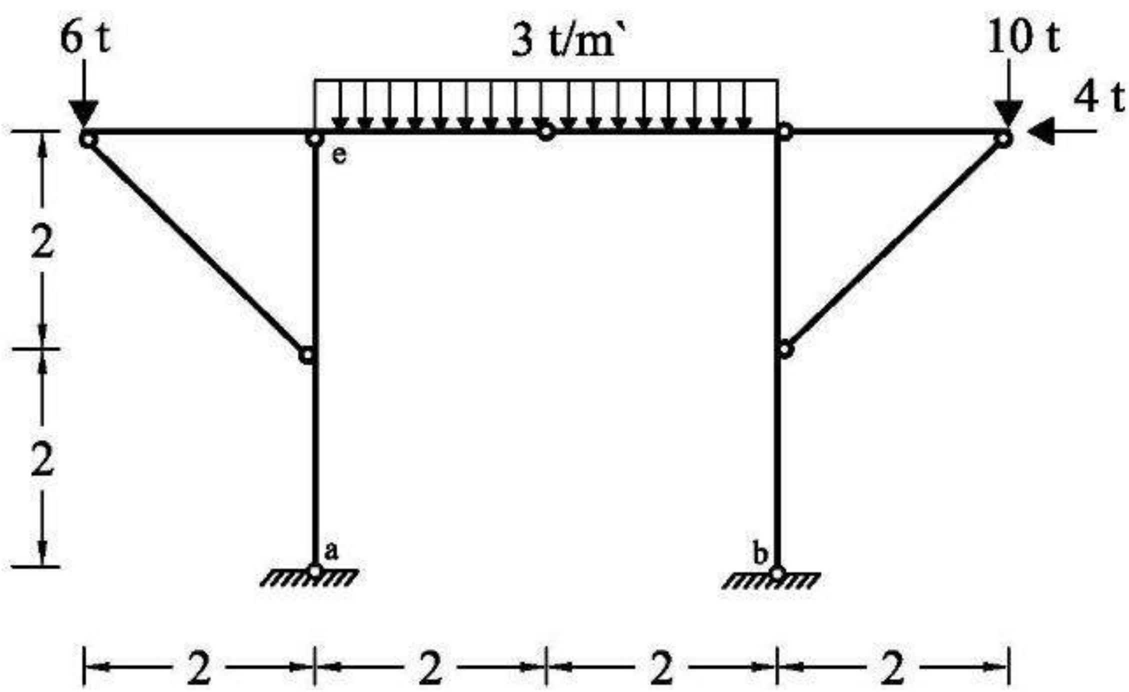
B.M.D

Fig. 4.22.b

Example (18)

For the given frame in Fig. 4.22.a, draw the normal, shear force and bending moment diagrams.

Fig. 4.23.a



Solution

➢ *Finding the reactions*

f) $\sum M_a = 0 \ggg\ggg -6x2 + 12x2 + 10x6 - 4x4 - Y_b x4 = 0$

$$\therefore Y_b = \frac{56}{4} = 14t \uparrow$$

g) $\sum M_b = 0 \ggg\ggg 6x6 + 12x2 + 4x4 - 10x2 - Y_a x6 = 0$

$$\therefore Y_a = \frac{56}{4} = 14t \uparrow$$

h) Check $\sum F_y = 6 + 12 + 10 - 14 - 14 = 0 \ggg\ggg$ ok

i) $\sum M_c\, for\ the\ left\ side = 0$

$$6x4 + 6x1 - 14x2 - 4x_a = 0 \quad \gg\gg x_a = 0.5t \leftarrow$$

j) $\sum M_c \; for \; the \; right \; side = 0$

$$6x1 + 10x4 + 4x_b = 0 \quad \ggg x_b = 4.5t \rightarrow$$

k) $check \sum F_x = 0.5 + 4 - 4.5 = 0 \ggg\ggg$ ok

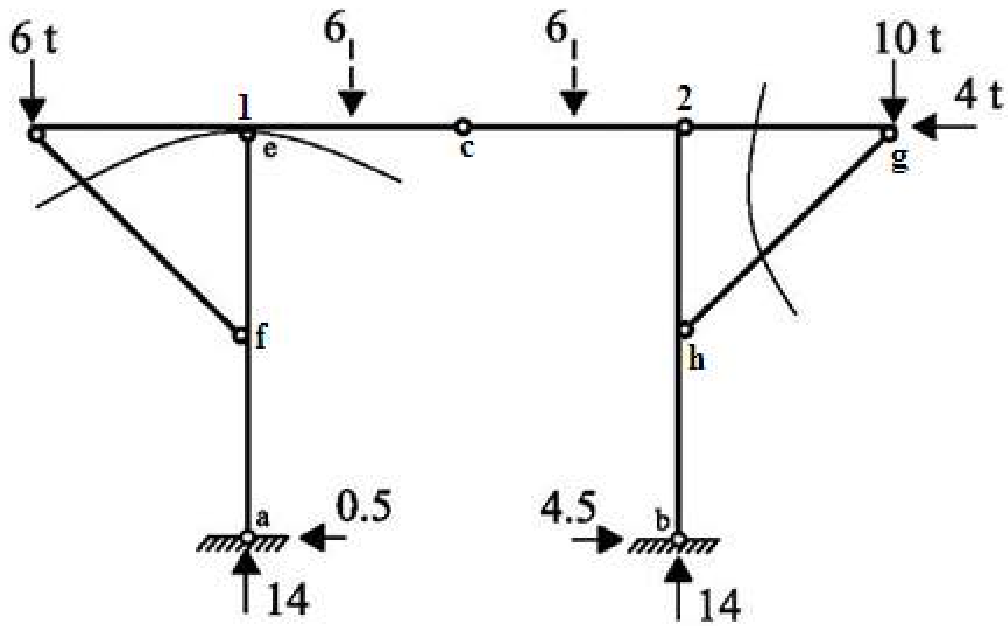
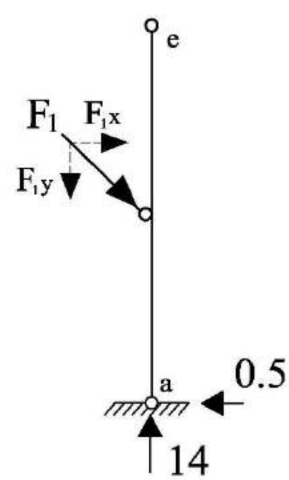


Fig. 4.23.b



➢ For part e-f-a

$\sum M_e = 0.0 \quad 0.5\times 4 = F_1x\times 2 \; F_1x = 1.0$ t $\quad F_1y = 1.0$ t

$F_1 = 1\sqrt{2}$ t (compression)

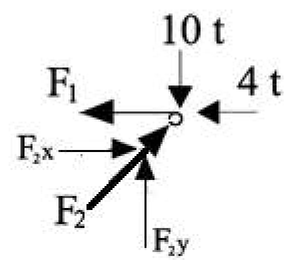
➢ For part 2-g-h

$\sum F_y = 0.0$

$F_2y = 10$ t

$F_2x = 10$ t

$F_2 = 10\sqrt{2}$ t (compression)



$\sum F_x = 0.0$

$4 - 10 = F_1$ $\qquad$ $F_1 = 6$ t ← (Tension)

- *For drawing B.M.D*

M_f from left side = -0.5×2= -1 m.t

M_1 for the cantilever from left side = -6x2= -12m.t

M_1 for beam from equilibrium = -12m.t $\quad M_h$ from right side = 4.5x2=9mt

M_2 from right side = 4.5x4-10x2= -2m.t

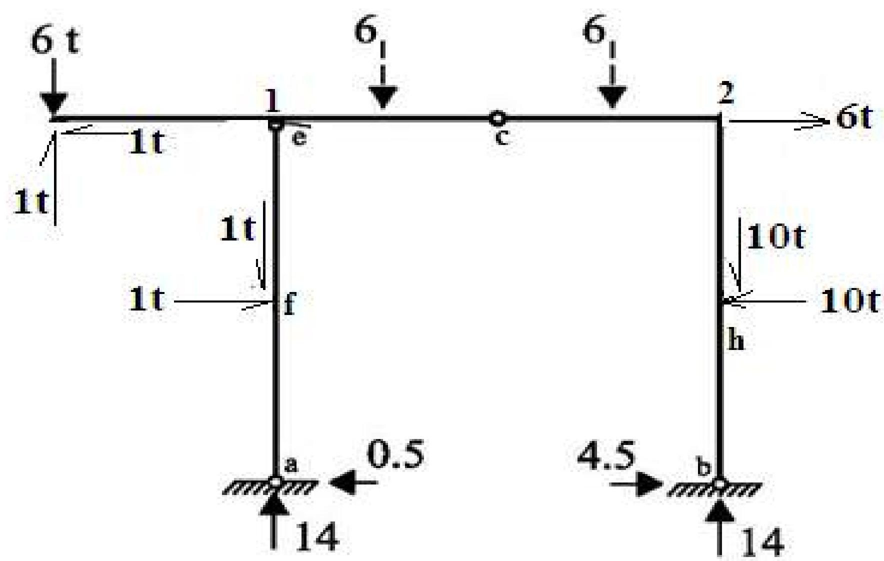
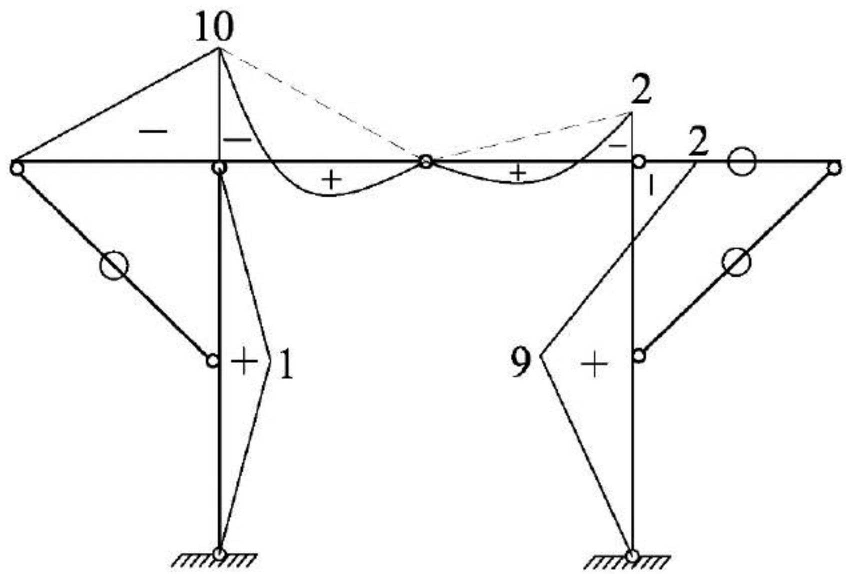
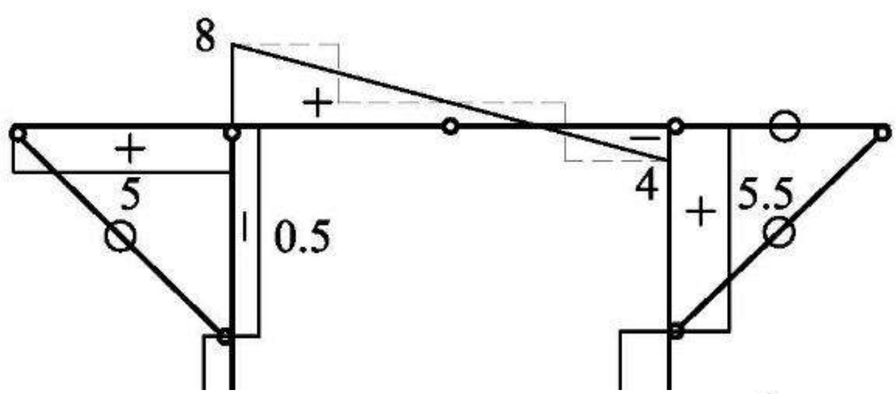
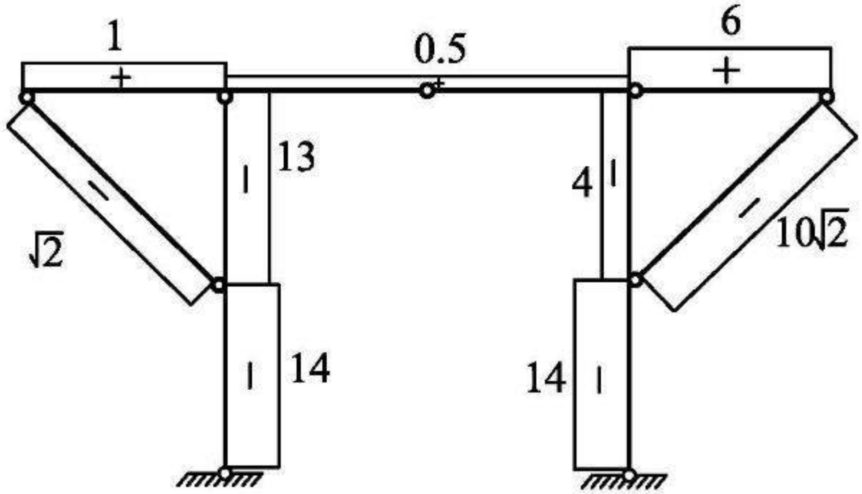


Fig. 4.23.c



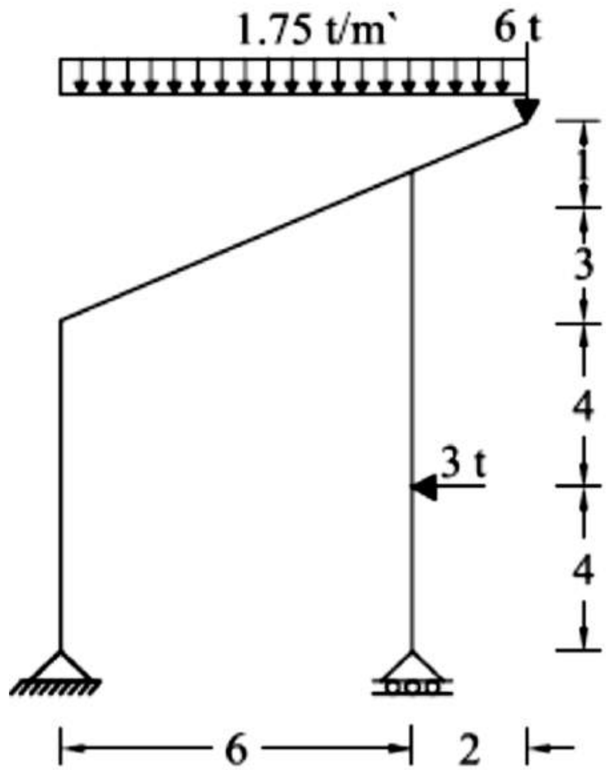
B.M.D





N.F.D

Fig. 4.23.d



Example (19)

For the given frame in Fig. 4.24.a, draw the normal, shear force and bending moment diagrams.

Solution

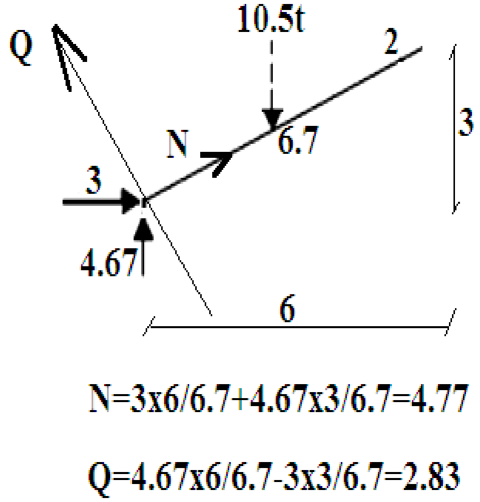
a. Finding the reactions

a. $\sum F_x = 0 \gg\gg\gg\gg X_a = 3t \leftarrow$

a) $\sum M_a = 10.5x3 + 6x8 + 3.5x7 - 3x4 - Y_b x6 = 0 \quad \therefore$

$Y_b = 15.33t \uparrow$

b) $\sum M_b 10.5x3 + 3x4 - 6x2 - 3.5x1 - Y_a x6 = 0 \qquad \therefore Y_a = 4.67t \uparrow$



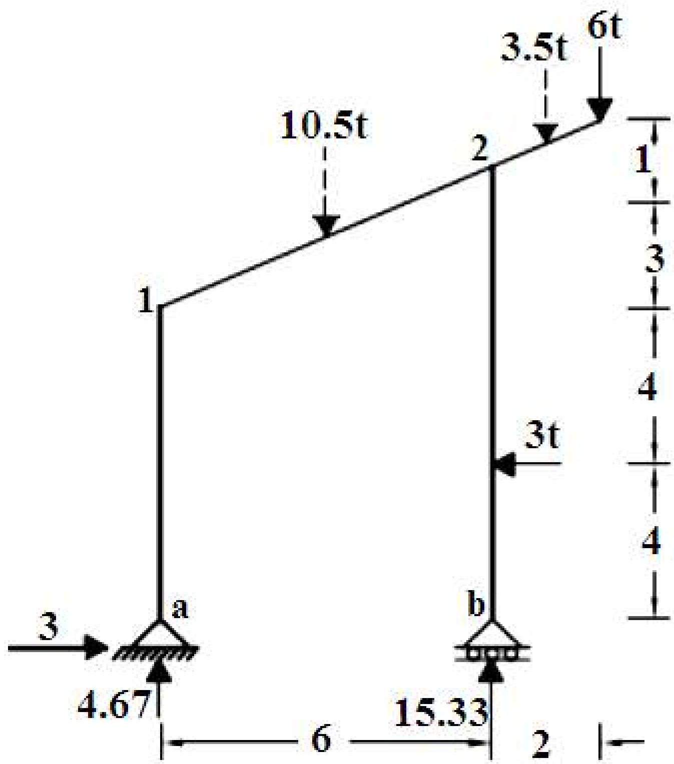
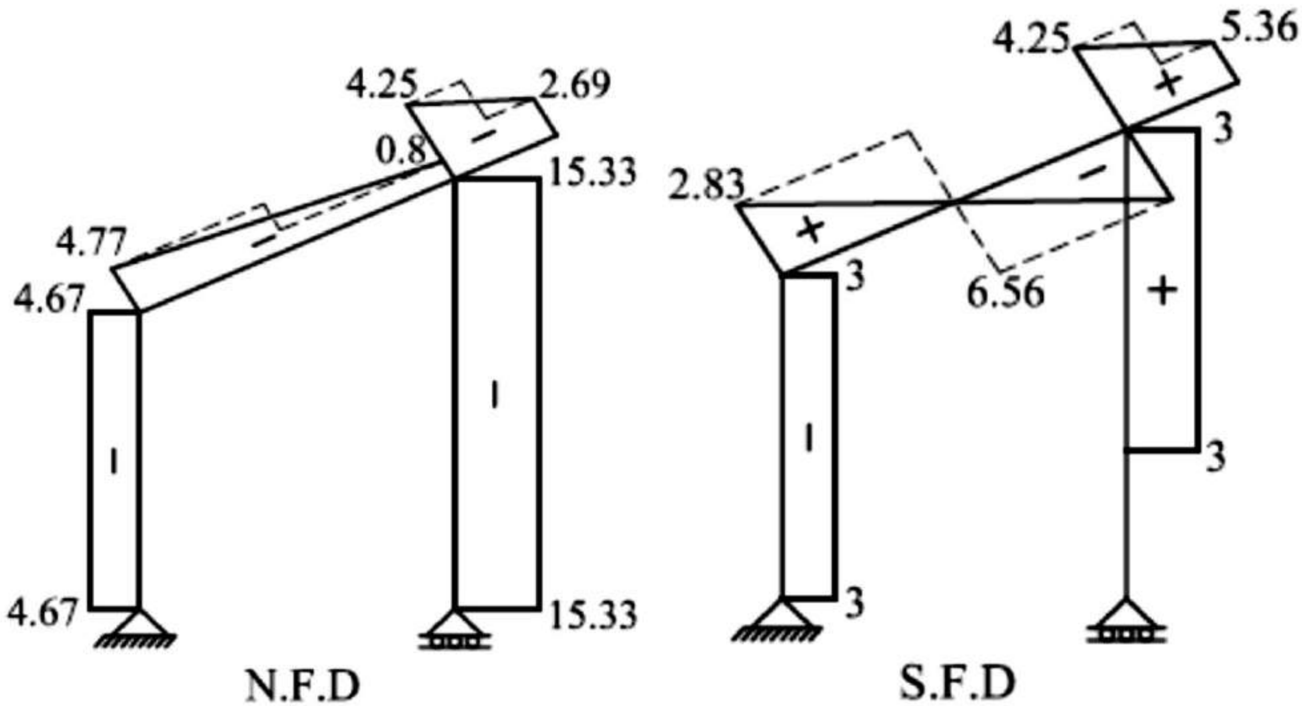
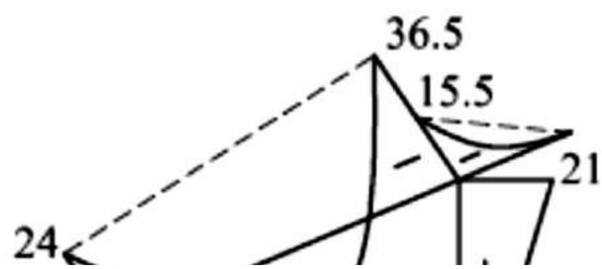


Fig. 4.24.b



Example (20)

For the given frame in Fig. 4.25, draw the normal, shear force and bending moment diagrams



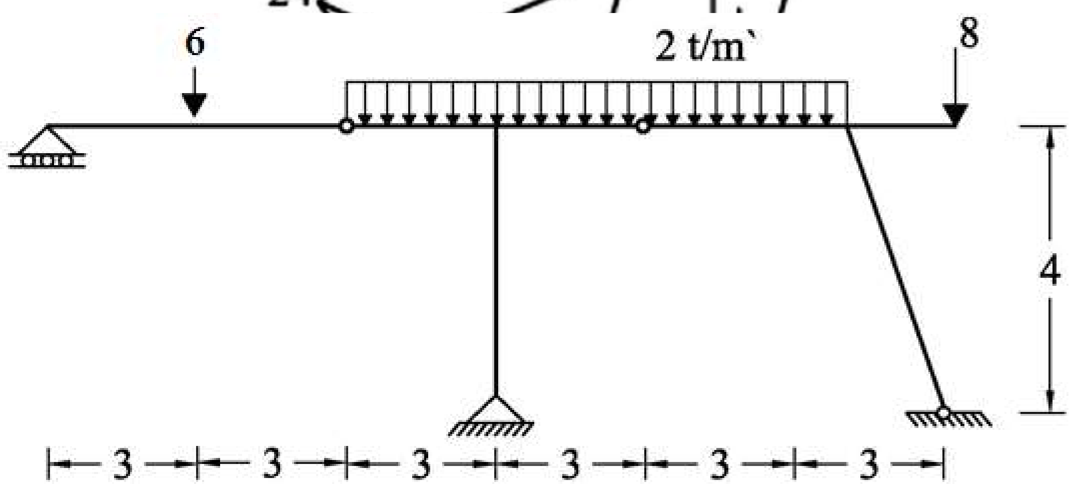


Fig. 4.25.a

Solution

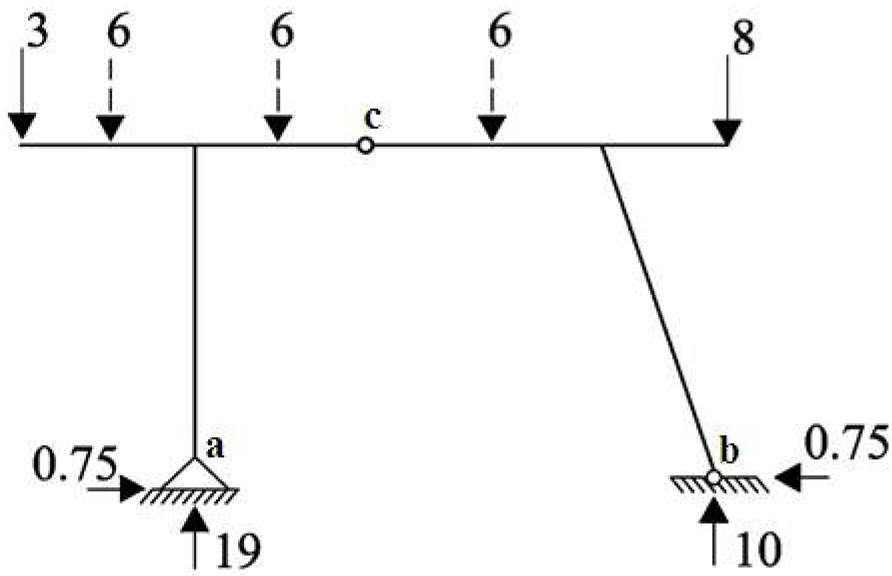


Fig. 4.25.b

- *Finding the reactions*

$$\sum M_a = -3x3 - 6x1.5 + 6x1.5 + 6x4.5 + 8x9 - Y_b x9 = 0$$

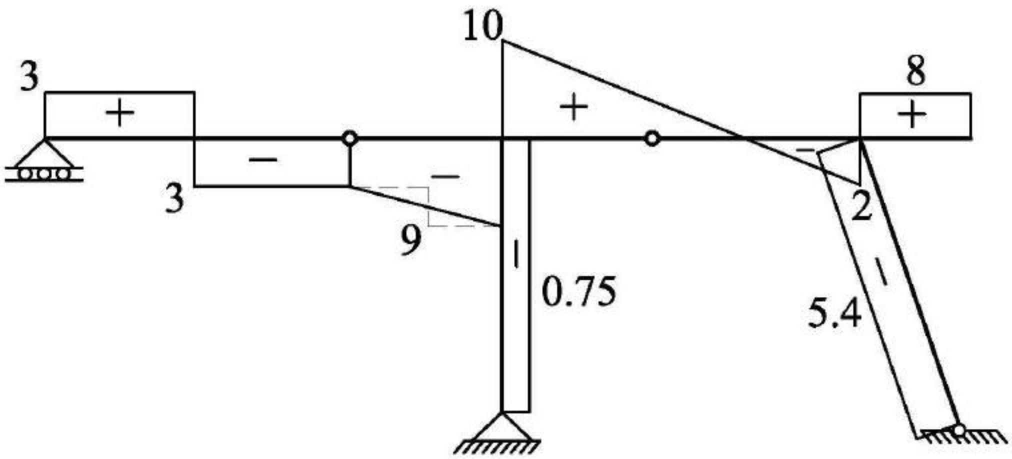
$$\therefore Y_b = 10t \uparrow$$

$$\sum M_b = 3x12 + 6x10.5 + 6x7.5 + 6x4.5 - Y_a x9 = 0$$

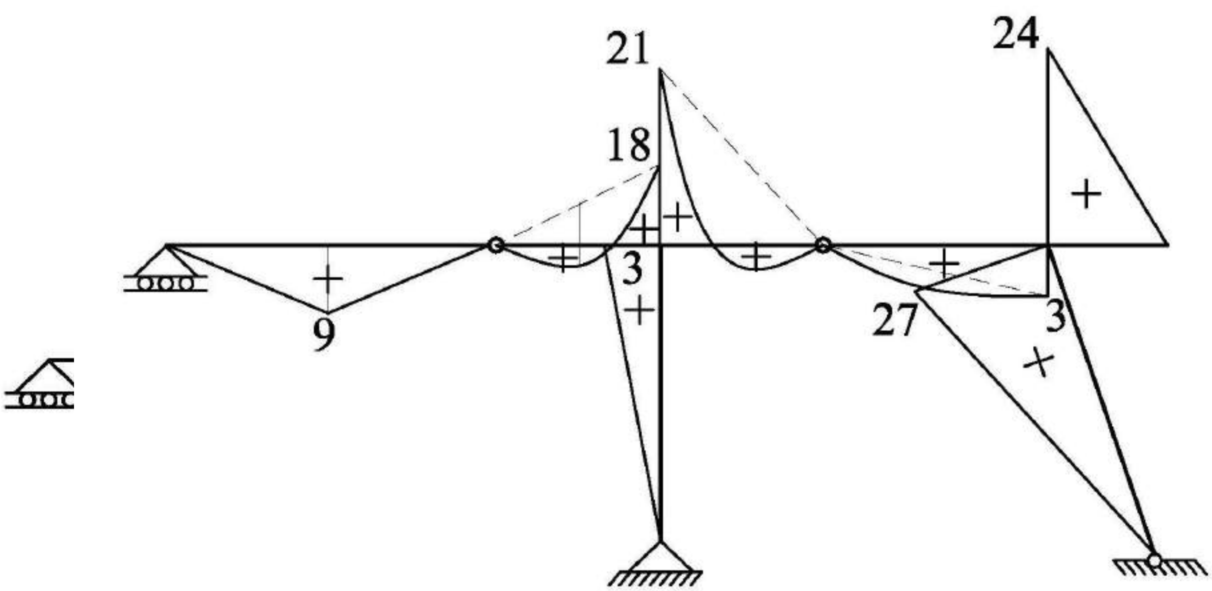
$$\therefore Y_a = 19t \uparrow$$

$$\sum M_c \; for \; the \; left \; part \; = 3x6 + 6x4.5 + 6x1.5 - 19x3 - x_a x4 = 0$$

$$\therefore x_a = 0.75t \rightarrow$$



S.F.D



B.M.D

N.F.D

$$\sum M_c \; for \; the \; right \; part \; = 6x1.5 + 8x6 - 10x6 + x_b x4 = 0$$

$$\therefore x_b = 0.75t \leftarrow$$

Fig. 4.25.c

Example (21)

For the given frame in Fig. 4.26.a, draw the normal, shear force and bending moment diagrams.

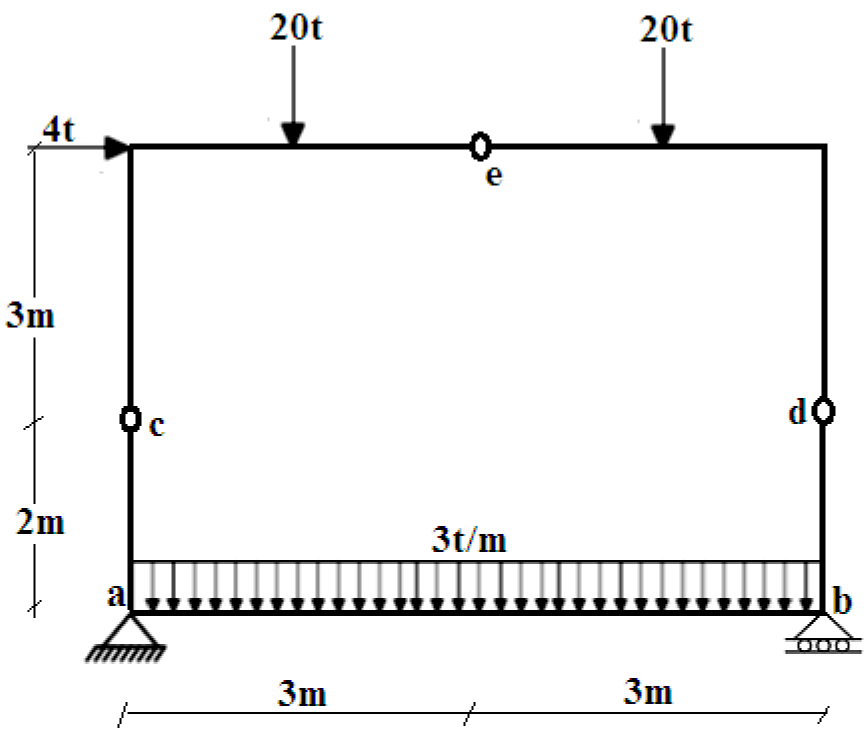


Fig. 4.26.a

Solution

- *Finding the reactions*

$\sum F_x = 0 \ggg\ggg\ggg X_a = 4t \leftarrow$

$\sum M_a = 20x1.5 + 20x4.5 + 18x3 + 4x5 - Y_b x6 = 0 \qquad \therefore Y_b = 32.33t \uparrow$

$\sum M_b = 20x1.5 + 20x4.5 + 18x3 - 4x5 - Y_a x6 = 0 \qquad \therefore Y_a = 25.67t \uparrow$

Check $\sum F_y = 20 + 20 + 18 - 32.33 - 25.67 = 0 \ggg$ ok

- *To finding the internal forces*

The structures must be separate into two parts (see Fig. 4. 26.b) as below

For part c-e-d

$$\sum M_c = 20x1.5 + 20x4.5 + 4x3 - 6Y_d = 0 \qquad \therefore Y_d = 22t \uparrow$$

$$\sum M_d = 20x1.5 + 20x4.5 - 4x3 - 6Y_c = 0 \qquad \therefore Y_c = 18t \uparrow$$

$$\sum M_e \text{ for the left side} = 20x1.5 - 18x3 - 3X_c = 0 \quad \therefore X_c = 8t \rightarrow$$

$$\sum M_e \text{ for the right side} = 20x1.5 - 22x3 - 3X_d = 0 \therefore X_d = 12t \leftarrow$$

For part c-a-b-d *This part is equilibrium*

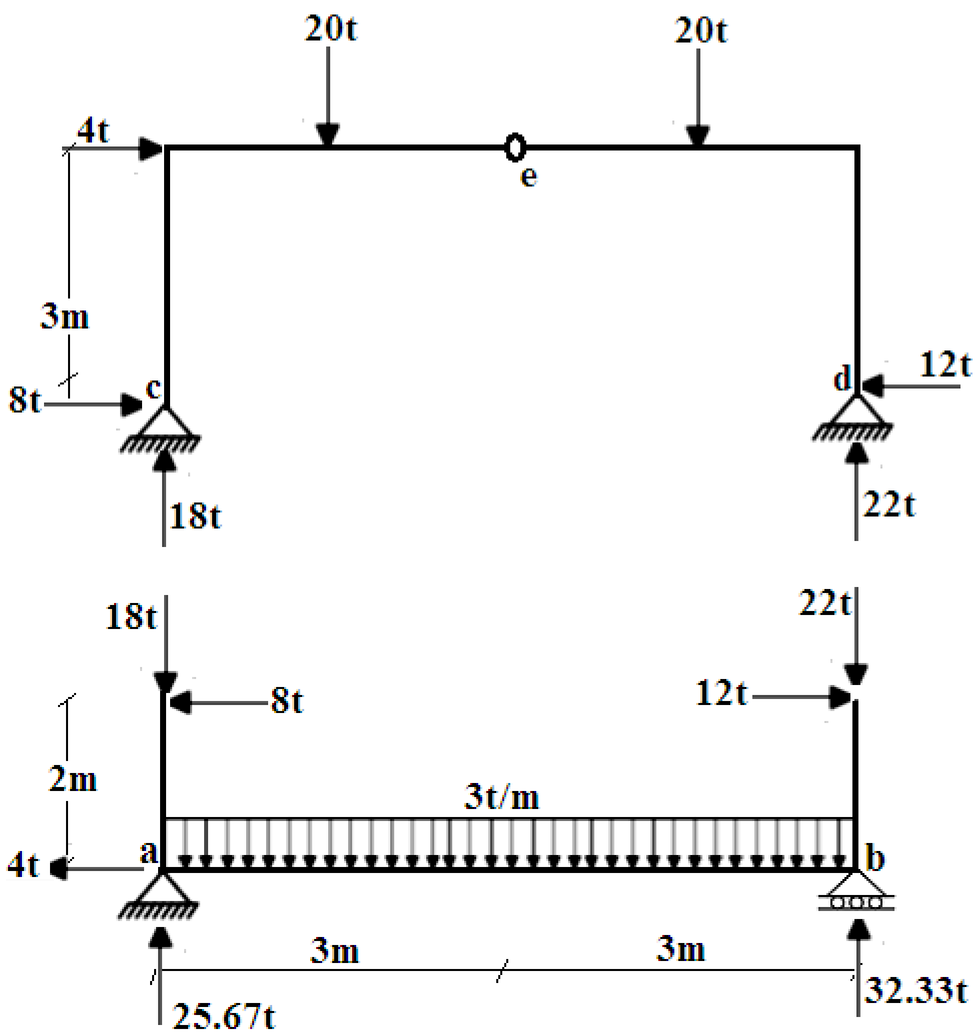
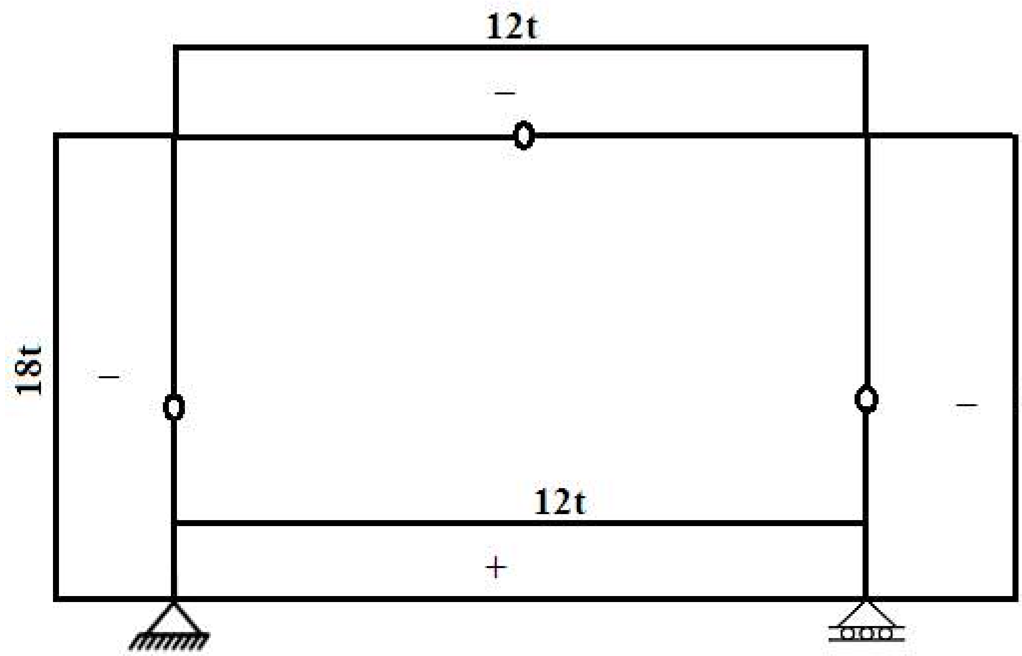
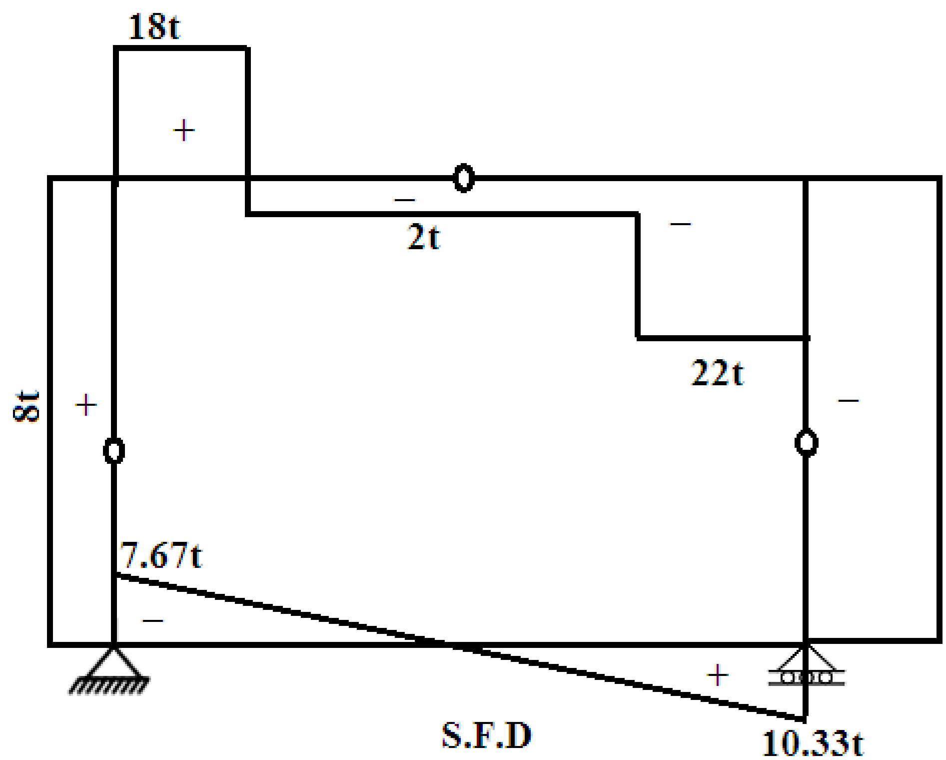


Fig. 4. 26.b



N.F.D



S.F.D

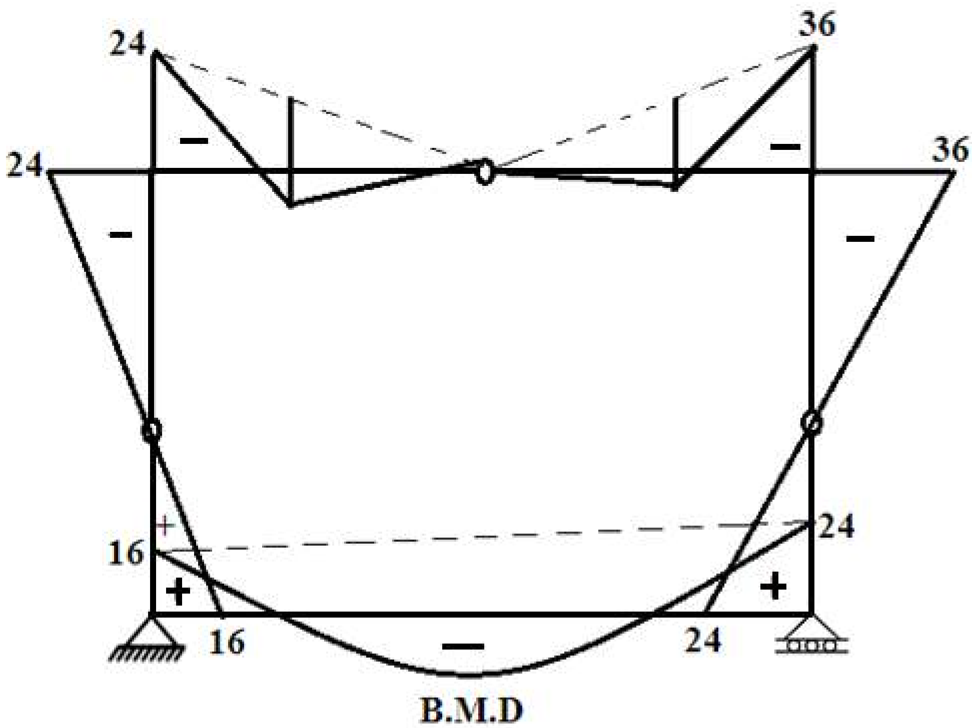


Fig. 4. 26.c

Example (22)

For the given frame in Fig. 4.27, draw the normal, shear force and bending moment diagrams.

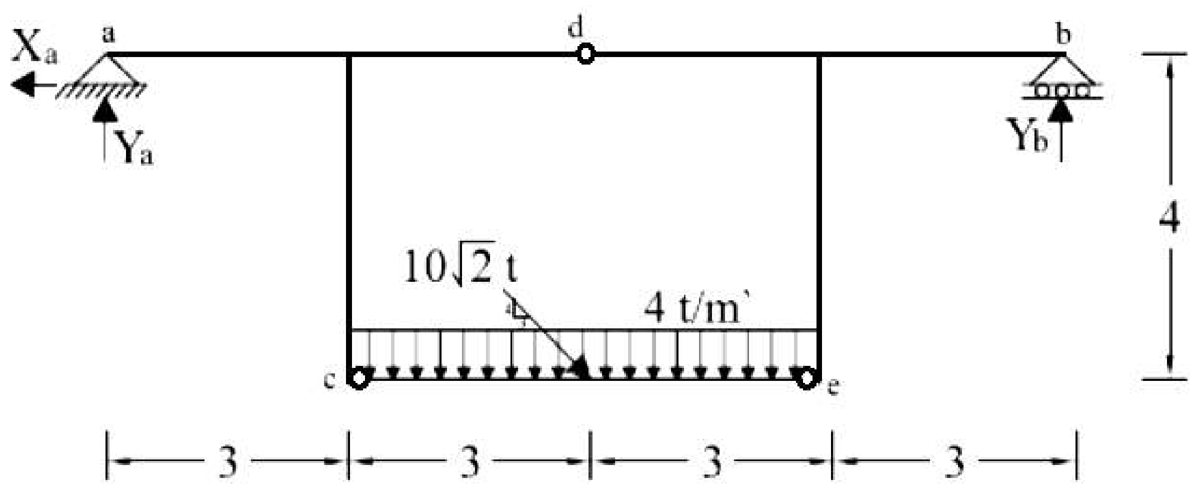


Fig. 4.27.a

Solution

➢ *Finding the reactions*

b. $\sum F_x = 0 \ggg\ggg\ggg X_a = 6t \leftarrow$

c) $\sum M_a = 24x6 + 8x6 - 6x4 - Y_b x12 = 0 \quad \therefore Y_b = 14t \uparrow$

$$\sum M_b = 24x6 + 8x6 + 6x4 - Y_a x12 = 0 \qquad \therefore Y_a = 18t \uparrow$$

b. Finding the internal forces at the intermediate hinges as shown in Fig. 4.27.b

$\sum M_e = 0.0$ for right part$=12\times1.5 +8\times3 +12\times4.5 - Y_c\times6 = 0.0 \quad Y_c = 16.0$ t $\uparrow$

$\sum \mathbf{M_d} = 12\times1.5 -6\times4 -14\times6 - X_c\times4 +16\times3 -12\times1.5 = 0.0 \qquad X_c = 15.0$ t $\leftarrow$

$\sum \mathbf{M_c} = 12\times1.5 +12\times4.5 +8\times3 -14\times9 + X_d\times4 - Y_d\times3 = 0.0$

$-30 + X_d\times4 - Y_d\times3 = 0.0$ ……………………… (1)

$\sum \mathbf{M_e} = 14\times3 - Y_d\times3 - X_d\times4 = 0.0$ ……………… (2)

By **Solving (1), (2)** $12 - Y_d\times6 = 0.0 \quad Y_d = 2.0$ t $\uparrow \qquad X_d = 9.0$ t $\rightarrow$

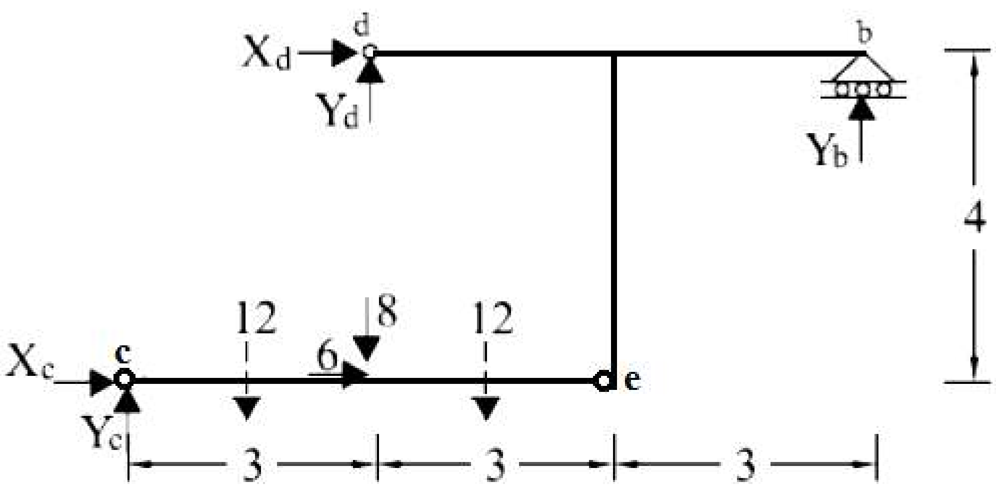
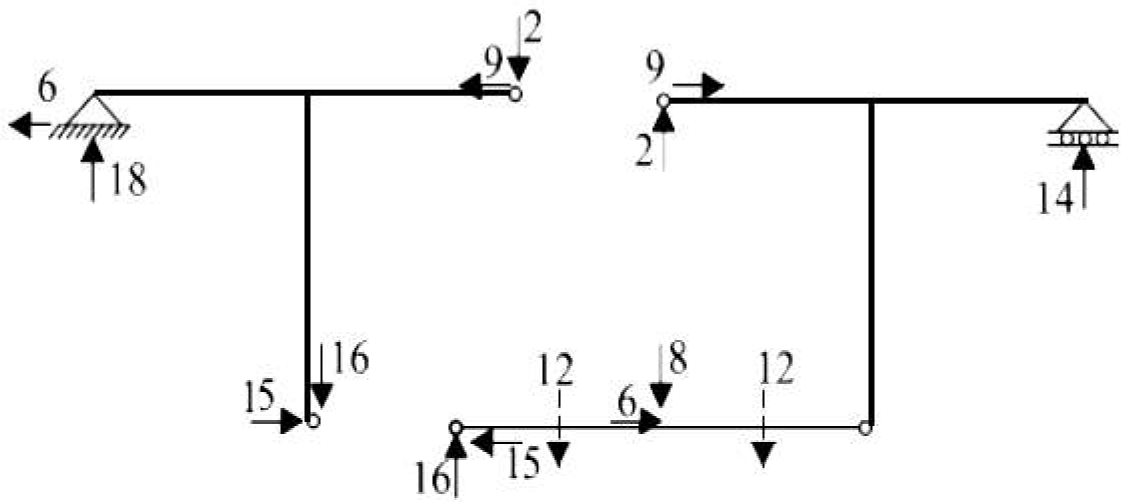
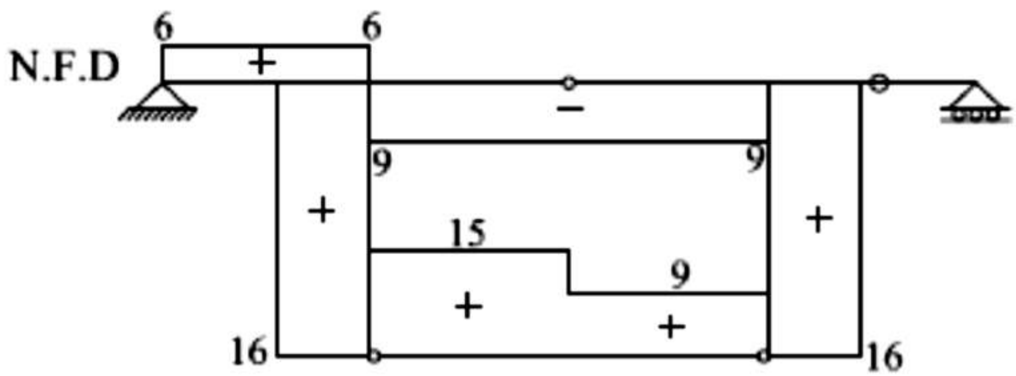
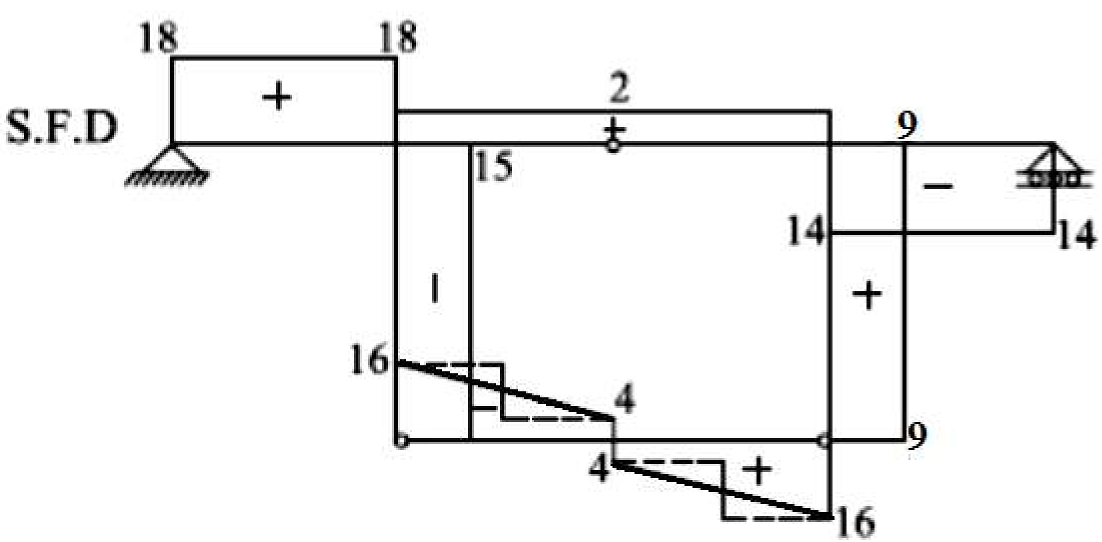


Fig. 4.27.b


6
18
9
2
16
15
9
2
14
12
8
6
12
16
15


N.F.D
6
6
+
−
9
9
+
+
15
9
+
+
16
16


S.F.D
18
18
+
2
+
9
15
−
14
14
+
16
4
4
+
9
16

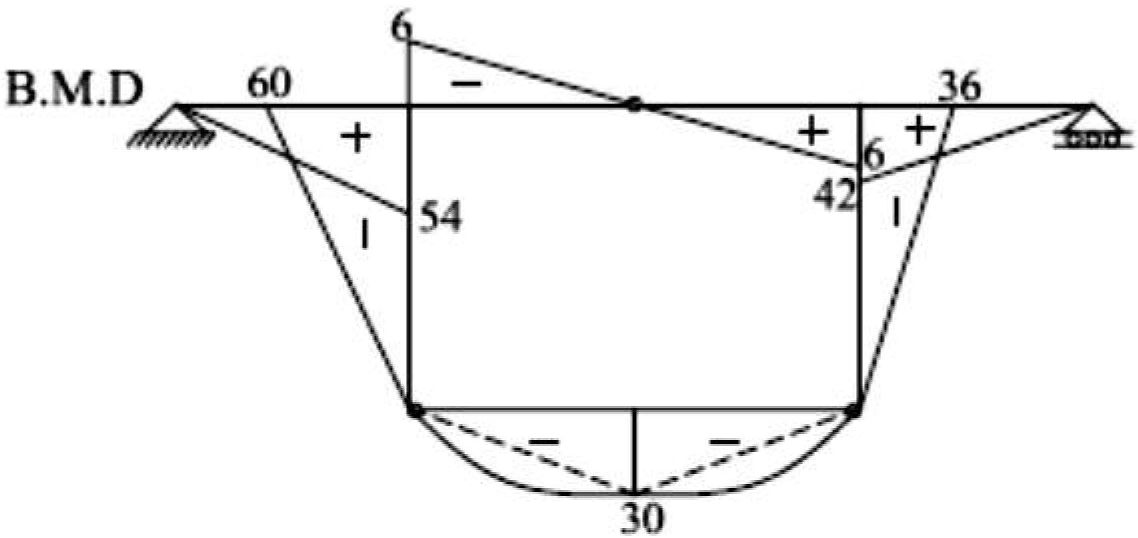


Fig. 4. 27.b

Example (23)

For the given frame in Fig. 4.28.a, draw the normal, shear force and bending moment diagrams.

Solution

- *For part d-e-f*

$\sum M_d = 10x4 + 16x4 - 8Y_e = 0 \qquad \therefore Y_e = 13t \uparrow$

$\sum M_e = 16x4 - 10x4 - 8Y_d = 0 \qquad \therefore Y_d = 3t \uparrow$

$\sum M_f$ for the left side $= 8x2 - 3x4 - 4X_d = 0 \quad \therefore X_d = 1t \leftarrow$

$\sum M_f$ for the right side $= 8x2 - 13x4 - 4X_e = 0 \therefore X_e = 9t \leftarrow$

Check $\sum F_y = 16 - 13 - 3 = 0 \ggg$ ok

Check $\sum F_x = 10 - 13 - 3 = 0 \ggg$ ok

- *For part a-c-b*

$\sum M_a = 24x4 + 13x8 + (1 + 9)x6 - 8Y_b = 0 \qquad \therefore Y_b = 32.5t \uparrow$

$\sum M_b = 24x4 + 3x8 - (1 + 9)x6 - 8Y_a = 0 \qquad \therefore Y_a = 7.5t \uparrow$

$\sum M_c$ for the left side $= 12x2 + 3x4 - 1x2 - 7.5x4 - 4X_a = 0$

$$\therefore X_a = 1t \leftarrow$$

$$\sum M_f \text{ for the right side} = 12x2 + 13x4 + 9x2 - 32.5x4 - 4X_b = 0$$

$$\therefore X_b = 9t \leftarrow$$

Check $\sum F_y = 24 + 13 + 3 - 7.5 - 32.5 = 0 \ggg$ ok

Check $\sum F_x = 1 - 1 - 9 + 9 = 0 \ggg$ ok

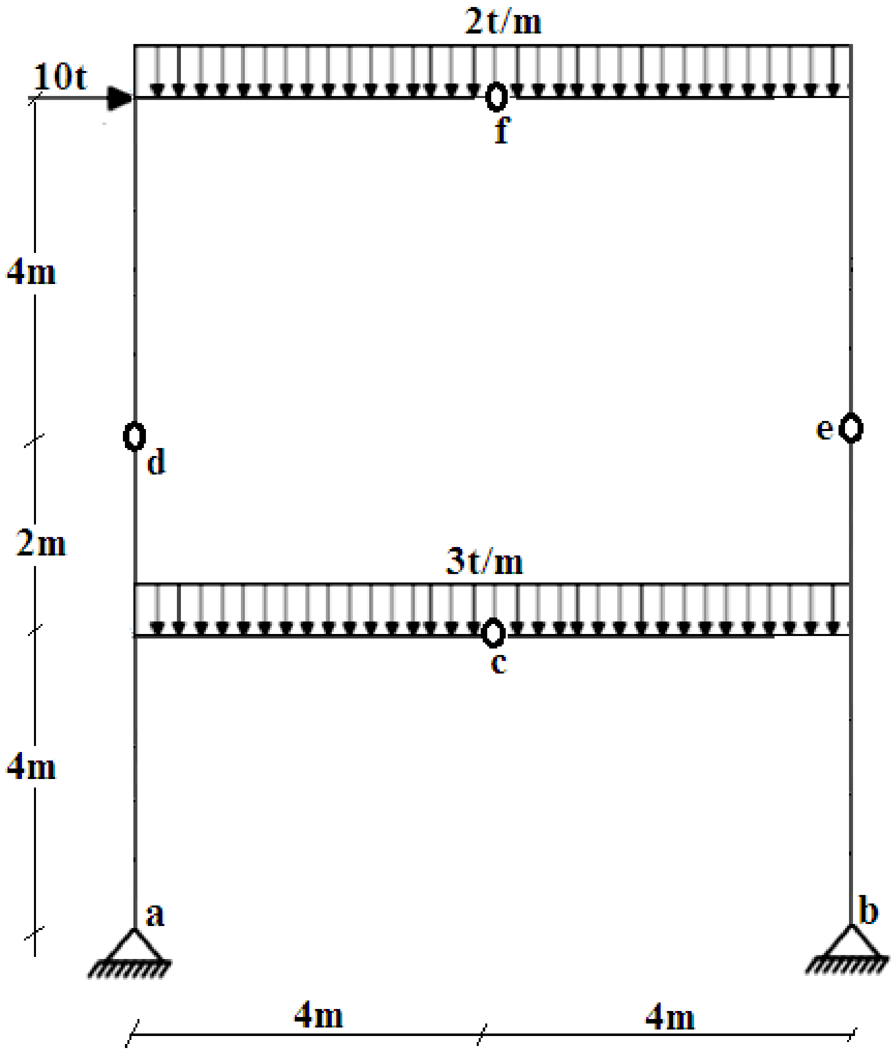
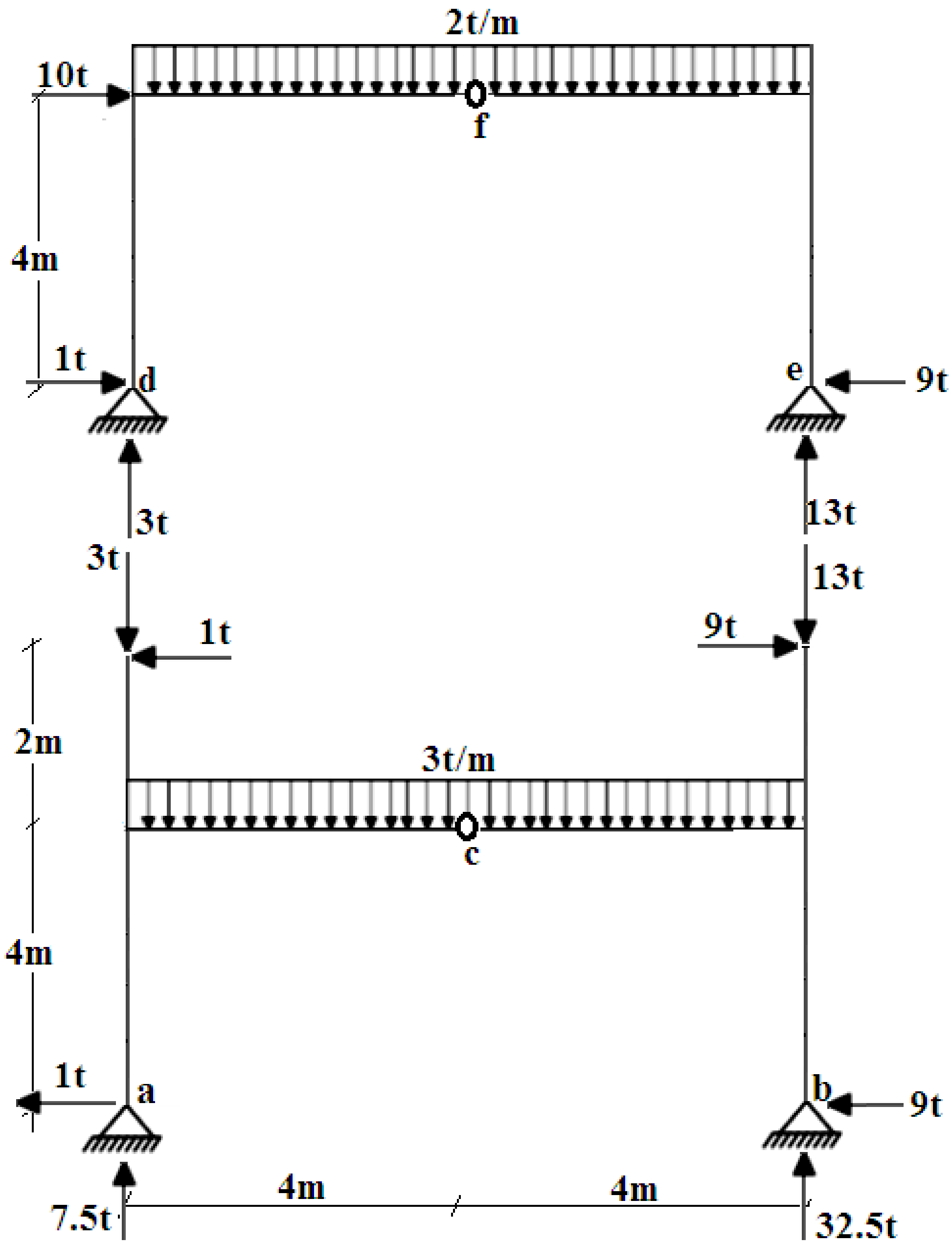
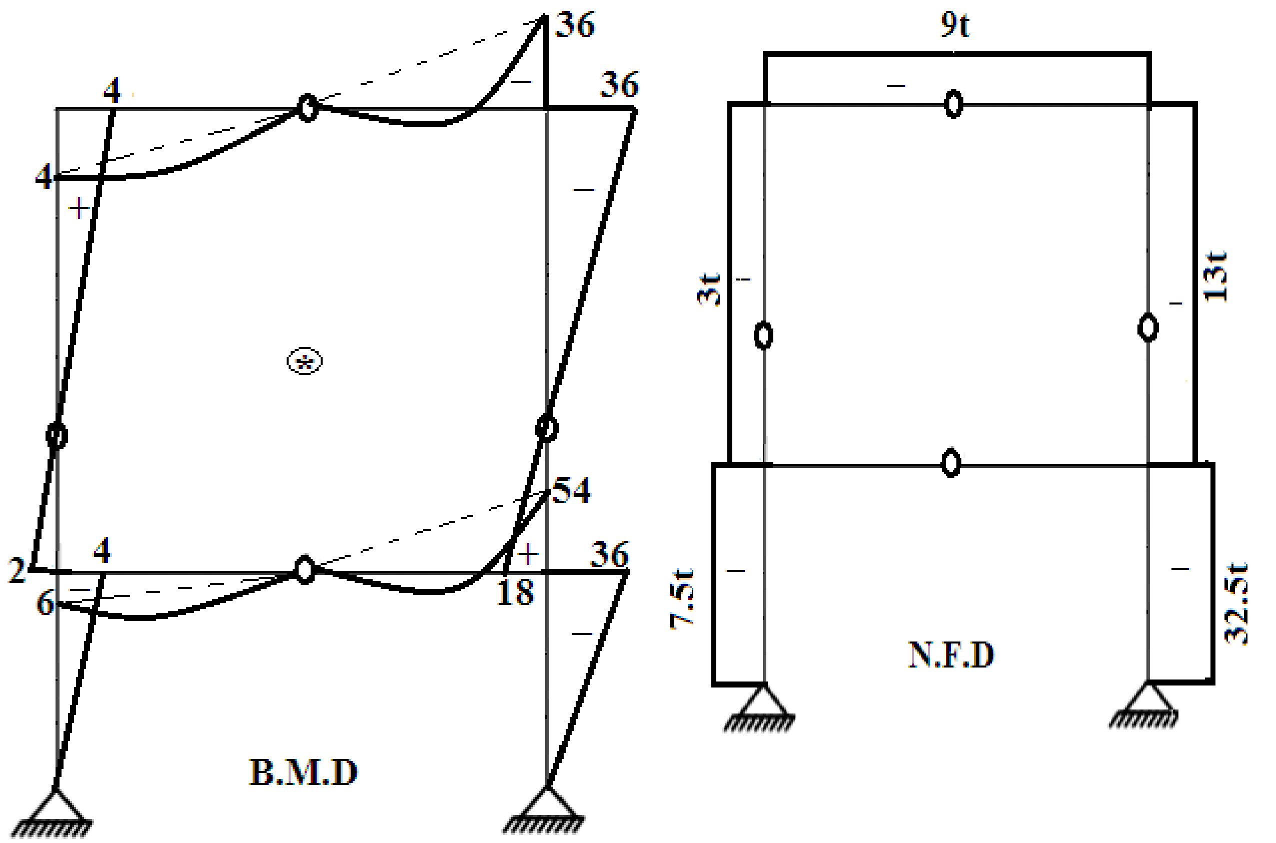


Fig. 4.28.a



2t/m
10t
f
4m
1t
d
e
9t
3t
3t
13t
13t
1t
9t
2m
3t/m
c
4m
1t
a
b
9t
7.5t
4m
4m
32.5t

Fig. 4.28.b



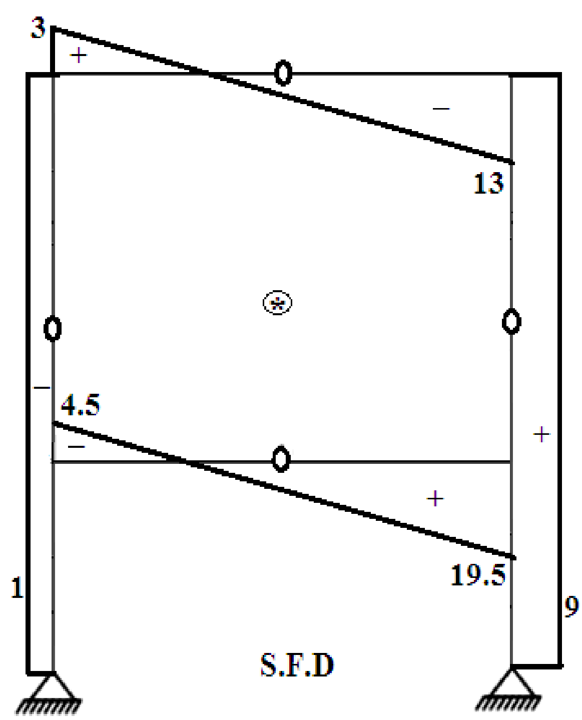


Fig. 4.28

<u>Example (24)</u>

For the given frame in Fig. 4.29.a, draw the normal, shear force and bending moment diagrams.

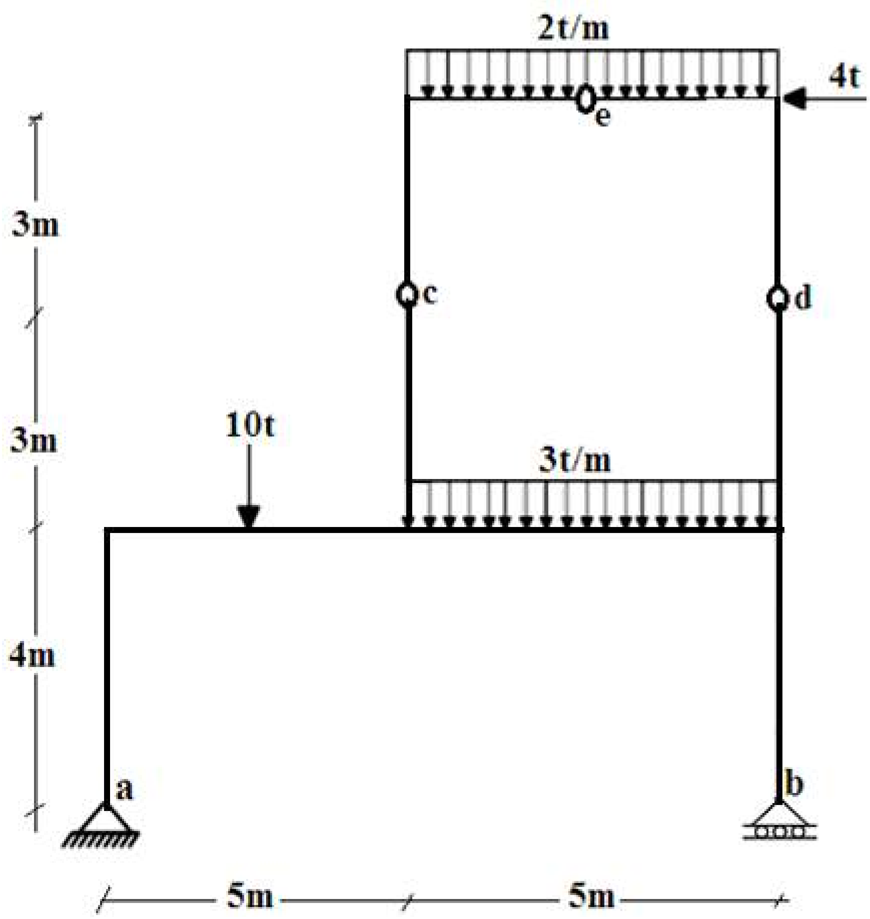


Fig. 4.29.a

Solution

- *Finding the reactions*

$\sum F_x = X_a - 4 = 0 \ggg X_a = 4t \rightarrow$

$\sum M_a = 10x2.5 + 15x7.5 + 10x7.5 - 4x10 - 10Y_b = 0 \therefore Y_b = 17.25t \uparrow$

$\sum M_b = 10x2.5 + 15x2.5 + 10x7.5 + 4x10 - 10Y_a = 0 \quad \therefore Y_a = 17.75t \uparrow$

Check $\sum F_y = 17.75 + 17.25 - 10 - 10 - 15 = 0 \ggg$ ok

- *For part c-e-d*

$\sum M_c = 10x2.5 - 4x3 - 5Y_d = 0 \qquad \therefore Y_b = 2.6t \uparrow$

$\sum M_d = 10x2.5 + 4x3 - 5Y_c = 0 \qquad \therefore Y_c = 7.4t \uparrow$

$\sum M_e$ for the left side $= 5x1.25 - 7.4x2.5 - 3X_c = 0 \therefore X_c = 4.08t \leftarrow$

$\sum M_e$ for the right side $= 5x1.25 - 2.6x2.5 - 3X_d = 0 \therefore X_d = 0.08t \leftarrow$

Check $\sum F_y = 10 - 7.4 - 2.6 = 0$ ok $\sum F_x = 4.08 - 4 - 0.08 = 0$ ok

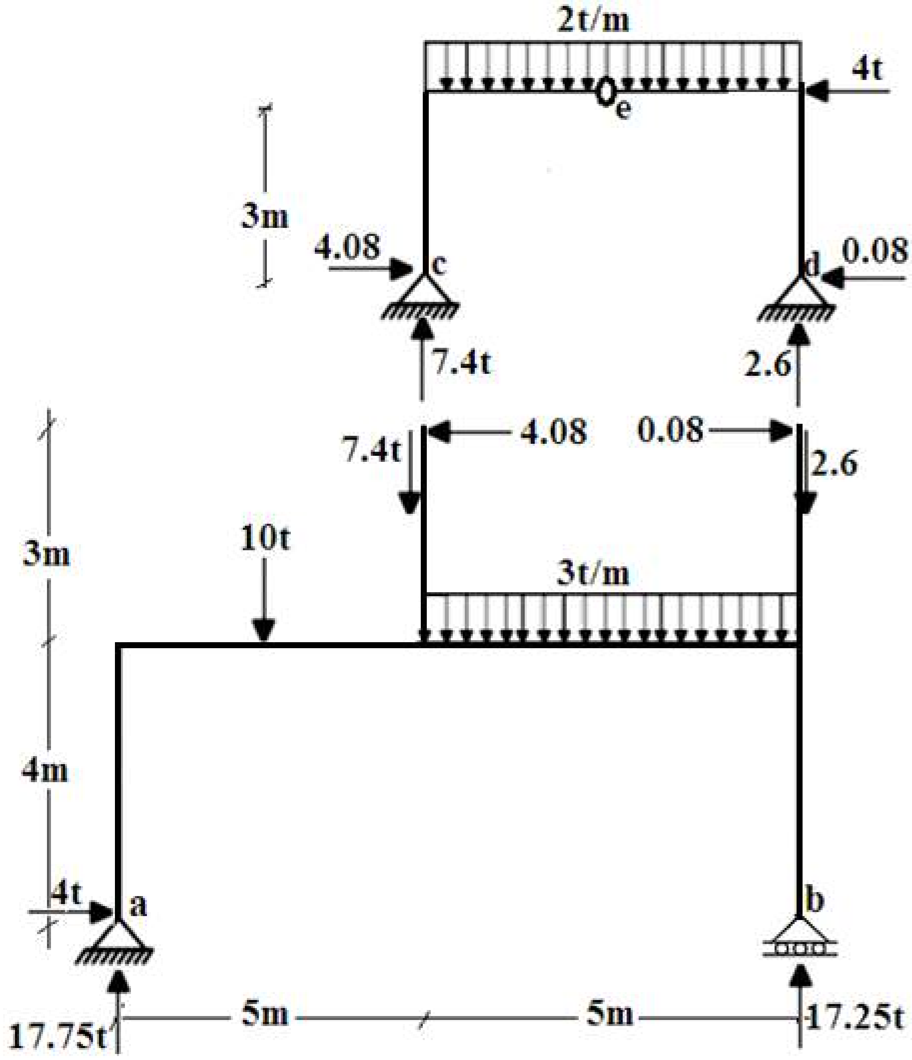
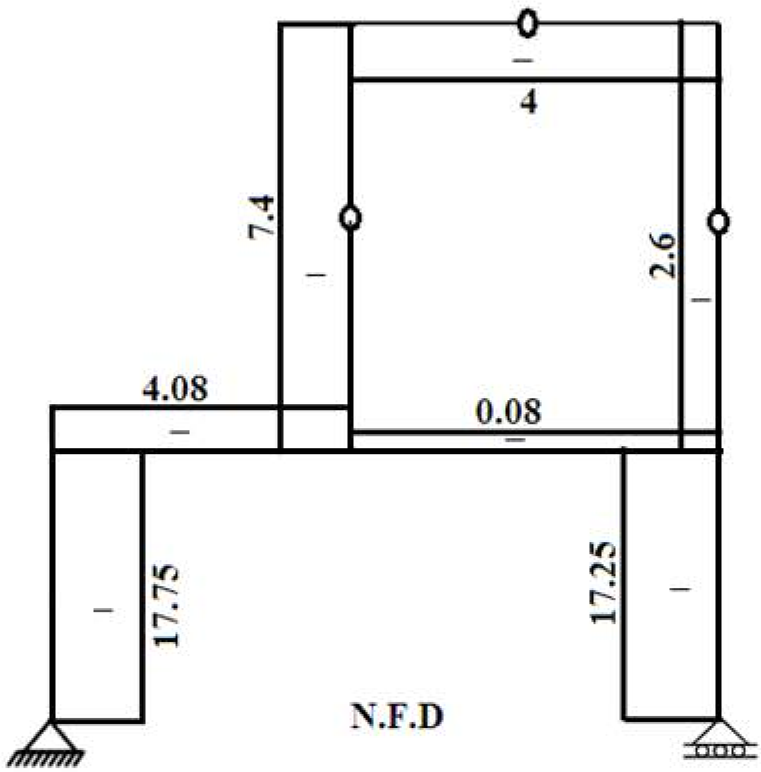
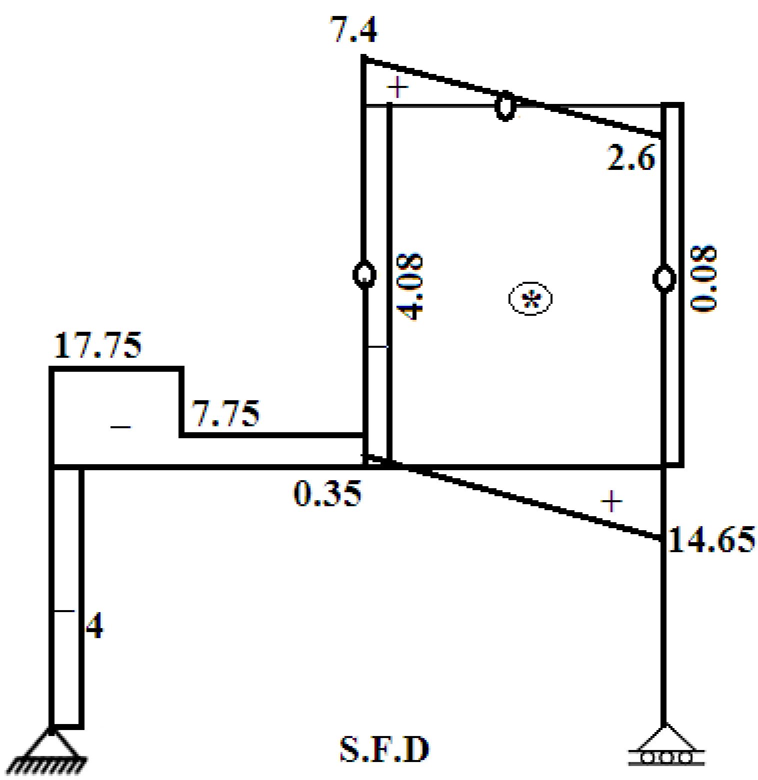


Fig. 4.29.b


4
7.4
2.6
4.08
0.08
17.75
17.25
N.F.D


7.4
2.6
4.08
0.08
17.75
7.75
0.35
14.65
4
S.F.D

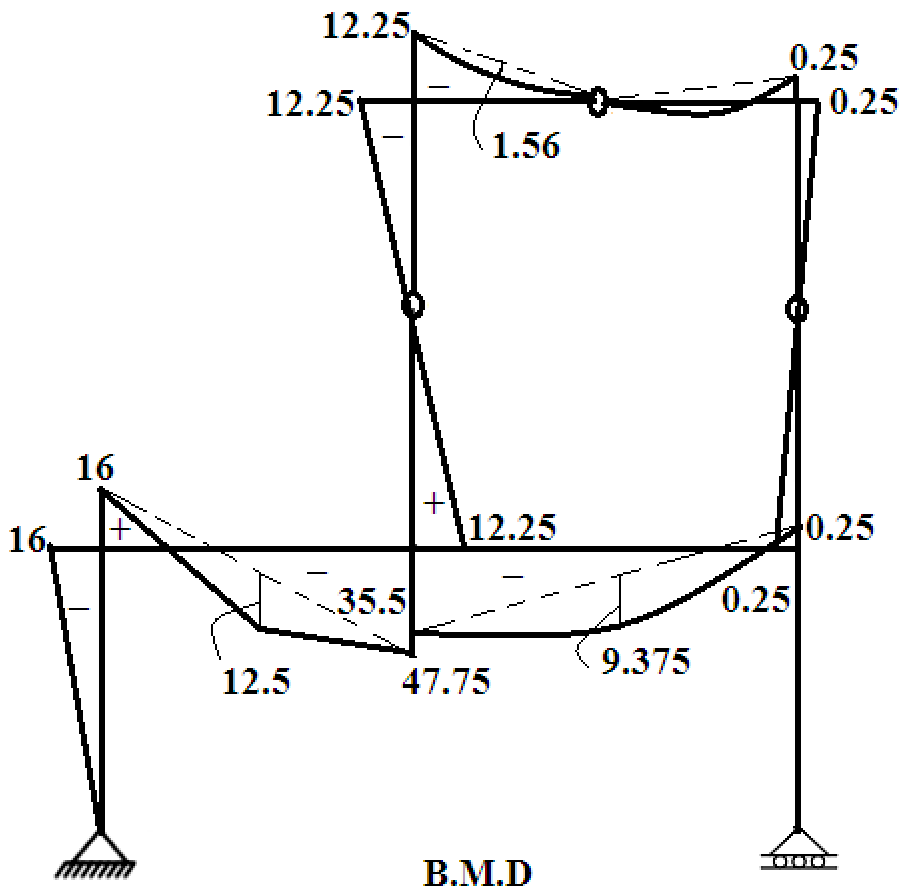


Fig. 4.29

Example (25)

For the given frame in Fig. 4.30.a, draw the normal, shear force and bending moment diagrams.

Solution

- *Finding the reactions*

$$Y_a = 0 \quad Y_b = 0 \quad X_a = 0$$

$\sum M_e$ for the left side $= 10 - 3X_c = 0 \therefore X_c = 3.33t \leftarrow$

$\sum M_e$ for the right side $= 10 - 3X_d = 0 \therefore X_d = 3.33t \leftarrow$

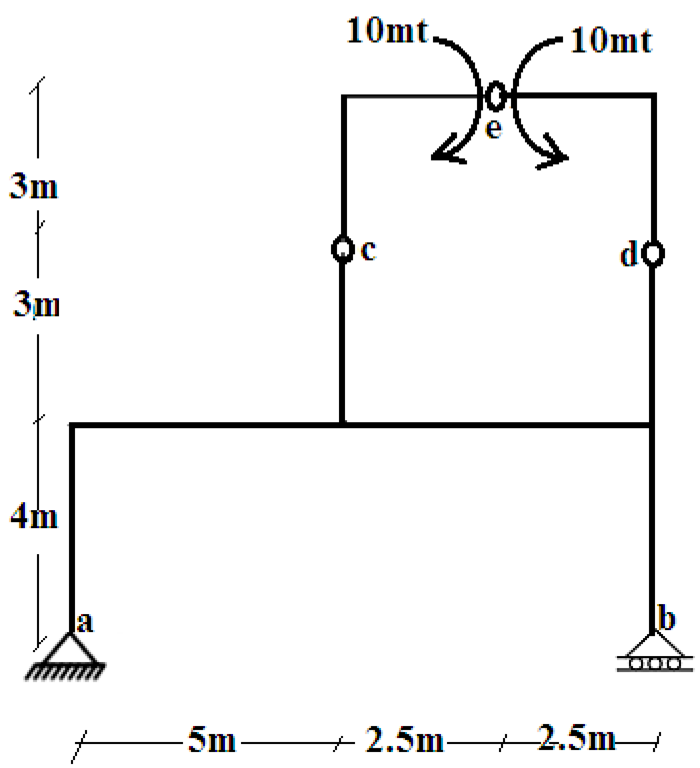
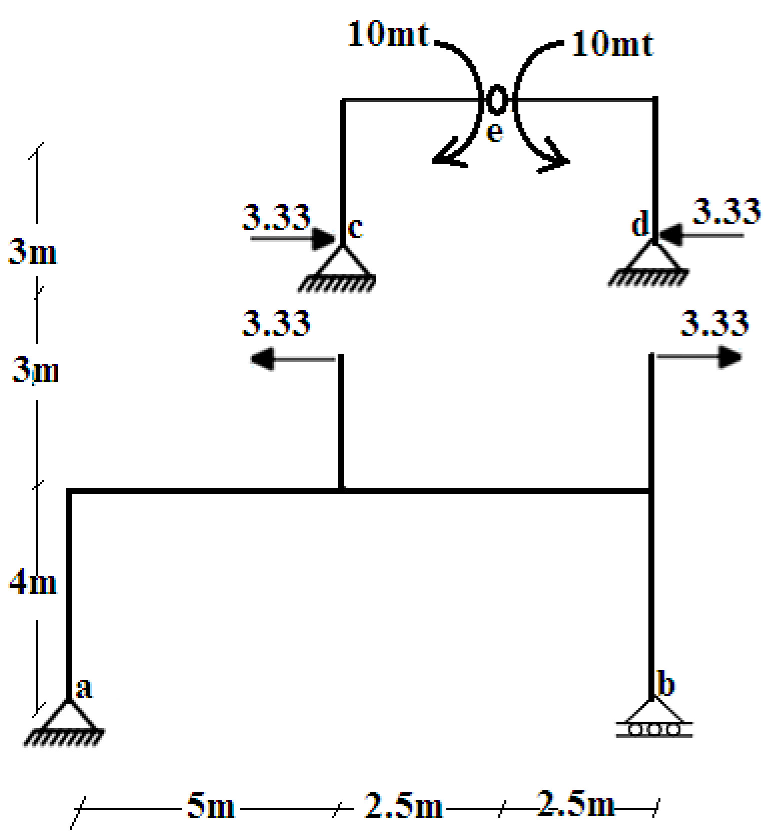

10mt
10mt
e
c
d
3m
3m
4m
a
b
5m
2.5m
2.5m

Fig. 4.30.a


10mt
10mt
e
3.33
c
d
3.33
3.33
3.33
3m
3m
4m
a
b
5m
2.5m
2.5m

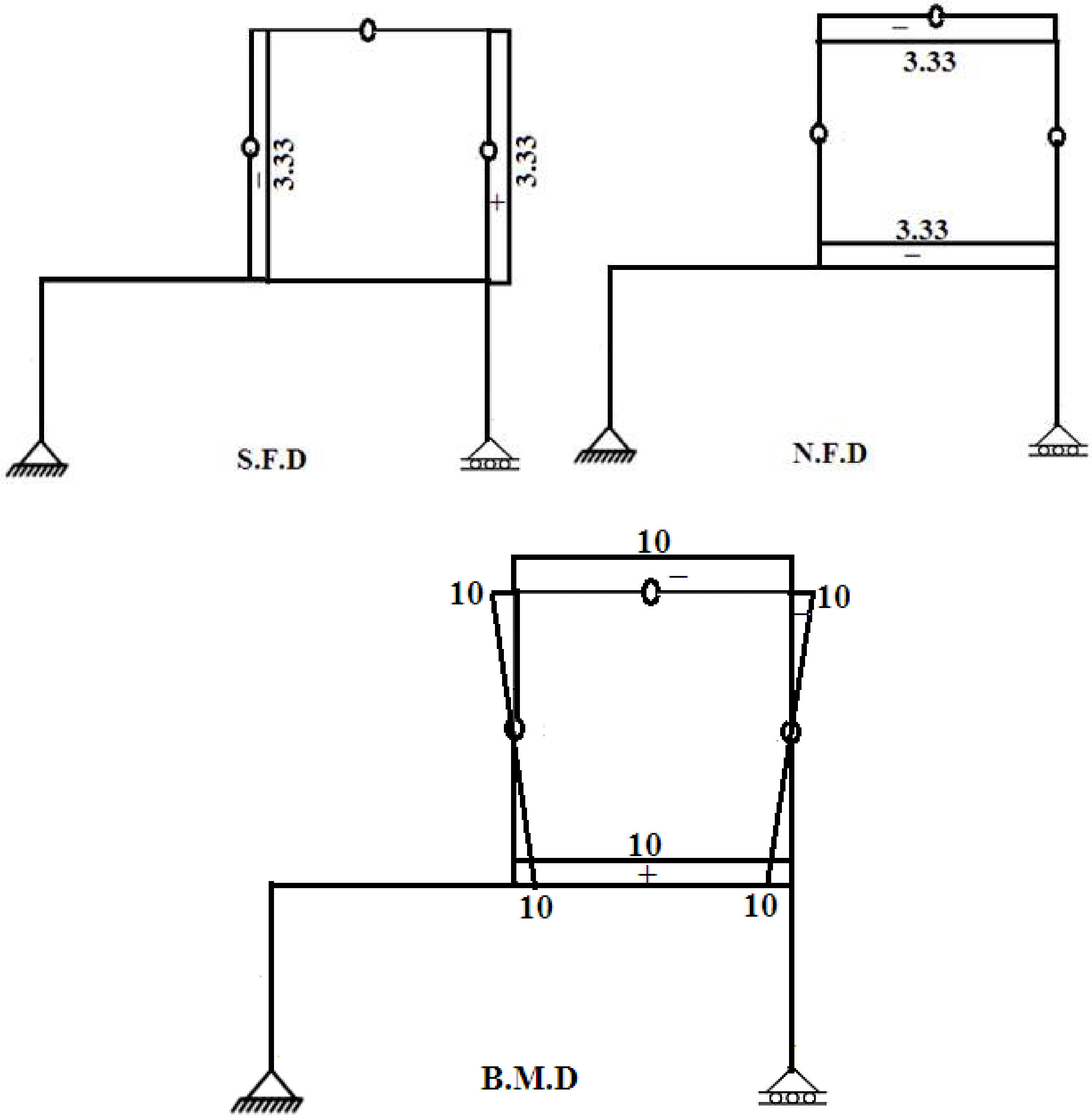


Fig. 4.30.b

Example (26)

For the given structure in Fig. 4.31.a, draw the normal, shear force and bending moment diagrams.

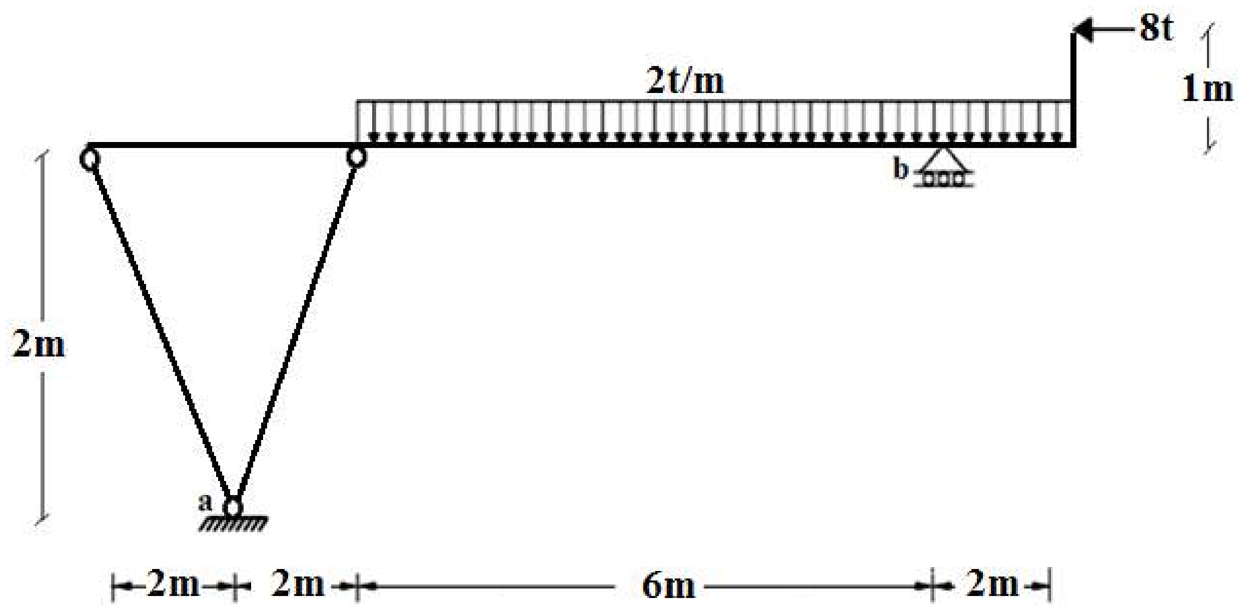


Fig. 4.31.a

Solution

Finding the reactions

$\sum F_x = 0 \ggg X_a = 8t \rightarrow$

$\sum M_a = 12x5 + 4x9 - 8x3 - 8Y_b = 0 \qquad \therefore Y_b = 9t \uparrow$

$\sum M_b = 12x3 + 8x2 - 4x1 - 8Y_a = 0 \qquad \therefore Y_a = 7t \uparrow$

Check $\sum F_y = 12 + 4 - 9 - 7 = 0 \ggg$ ok

- *For joint#1*

$\sum F_x = F_{1x} + 8 - F_{2x} = 0 \ldots\ldots(1)$

$\sum F_y = F_{1y} - 7 + F_{2y} = 0 \ldots\ldots(2)$

$F_{1x} = F_{1y} \ \ldots\ldots(3) \ and \ F_{2x} = F_{2y} \ \ldots\ldots(4)$

By solving the four equations $F_{1x} = F_{1y} = 0.5t \ and \ F_1 = 0.701t \ (tension)$

$F_{2x} = F_{2y} = 7.5t \; and \; F_2 = 10.61t \; (comp)$

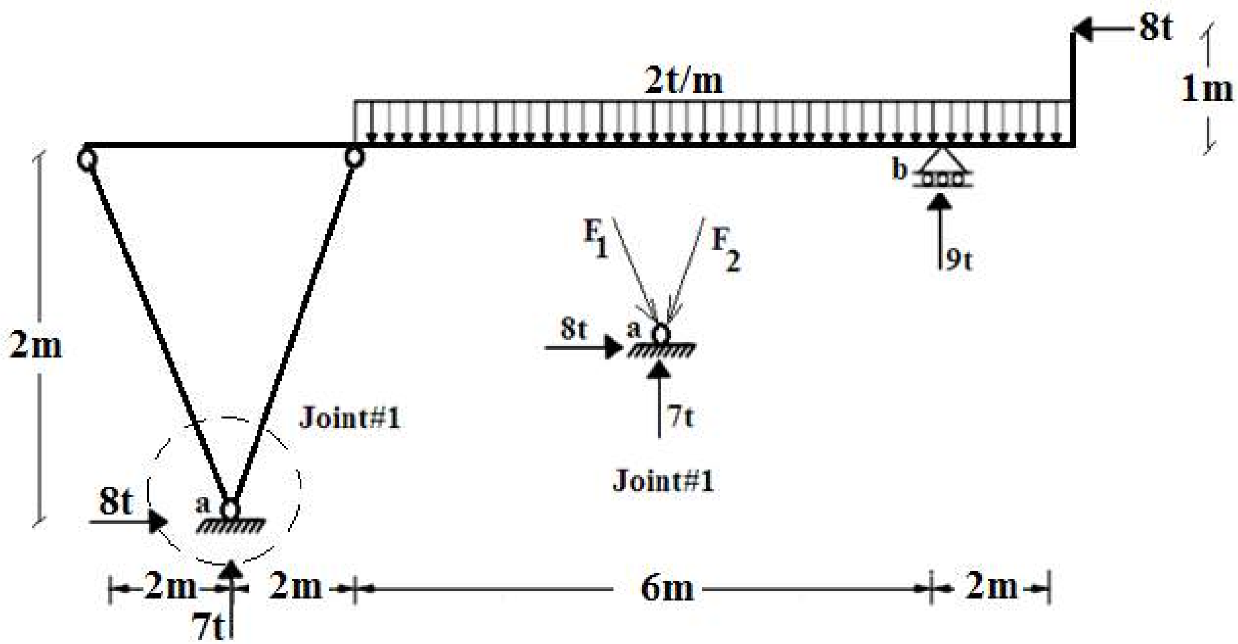


Fig. 4.31.b

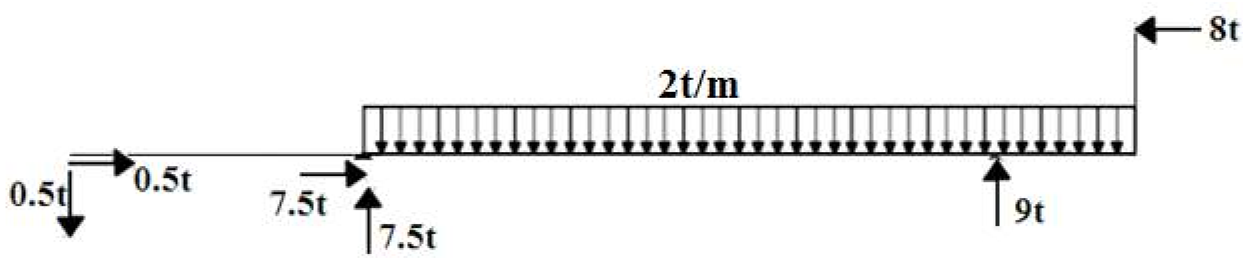
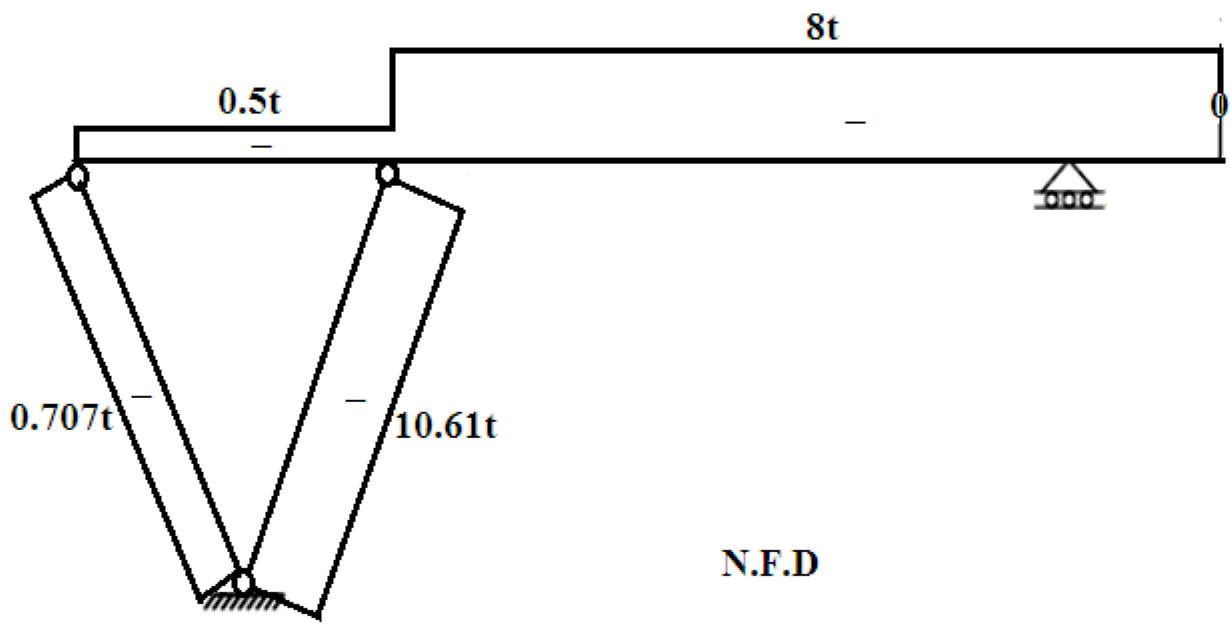
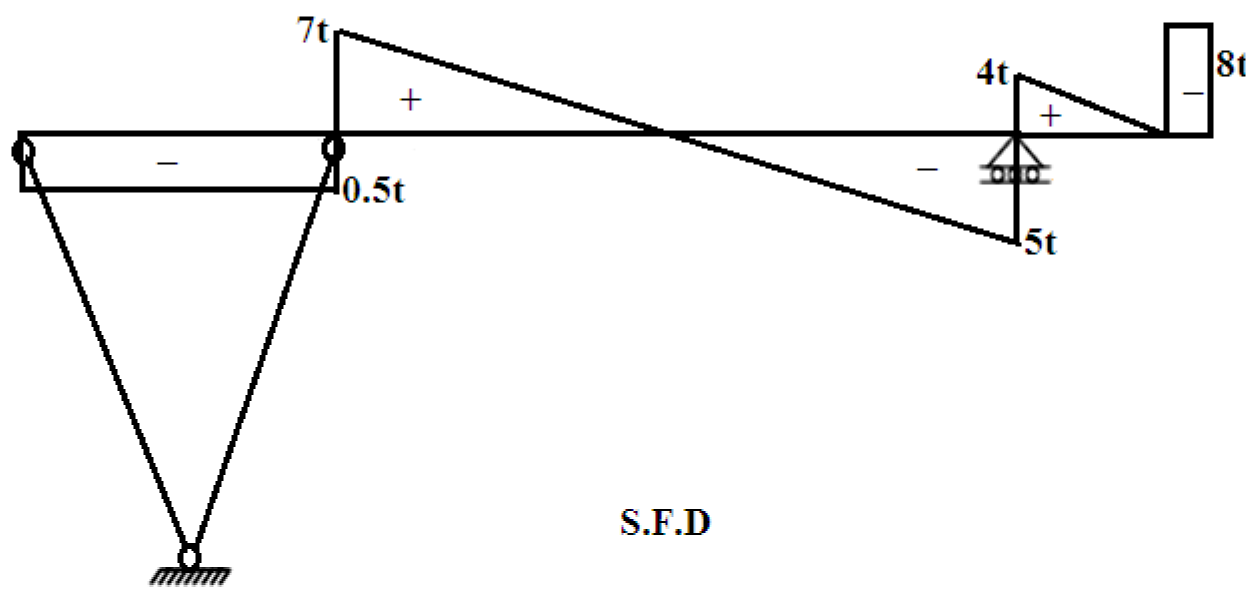


Fig. 4.31.c



N.F.D



S.F.D

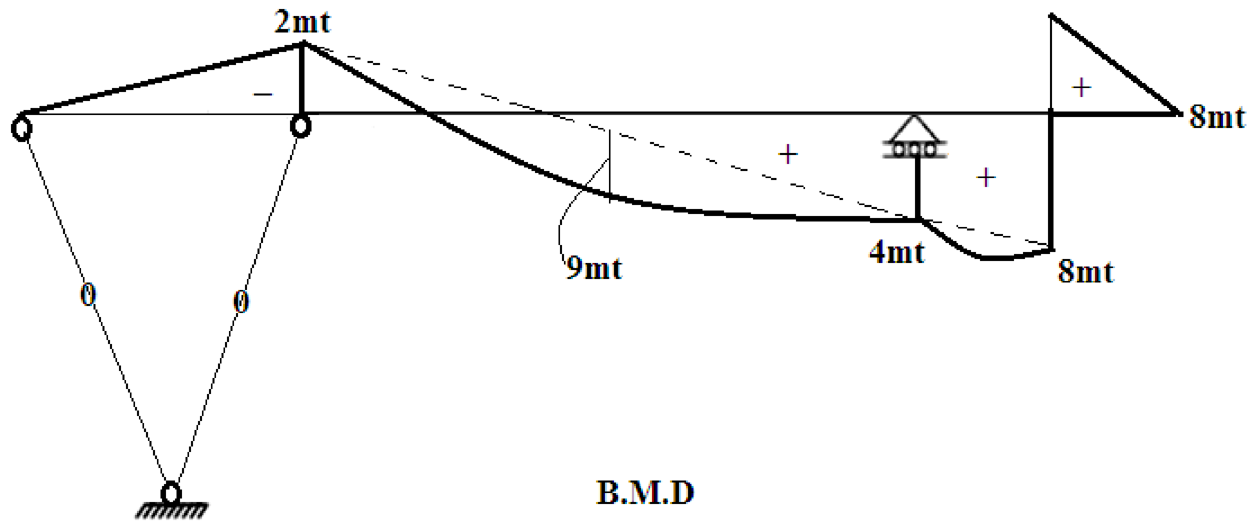


Fig. 4.31.d

Example (27)

For the given structure in Fig. 4.32.a, draw the normal, shear force and bending moment diagrams.

Solution

Finding the reactions

$\sum F_x = 0 \ggg X_b = 0 \rightarrow$

$\sum M_a = 12x3 + 12x9 + 4x13 - 2x2 - 9Y_b = 0 \qquad \therefore Y_b = 21.33t \uparrow$

$\sum M_b = 12x6 + 2x11 - 4x4 - 9Y_a = 0 \qquad \therefore Y_a = 8.67t \uparrow$

Check $\sum F_y = 12 + 12 + 4 + 2 - 21.33 - 8.67 = 0 \ggg$ ok

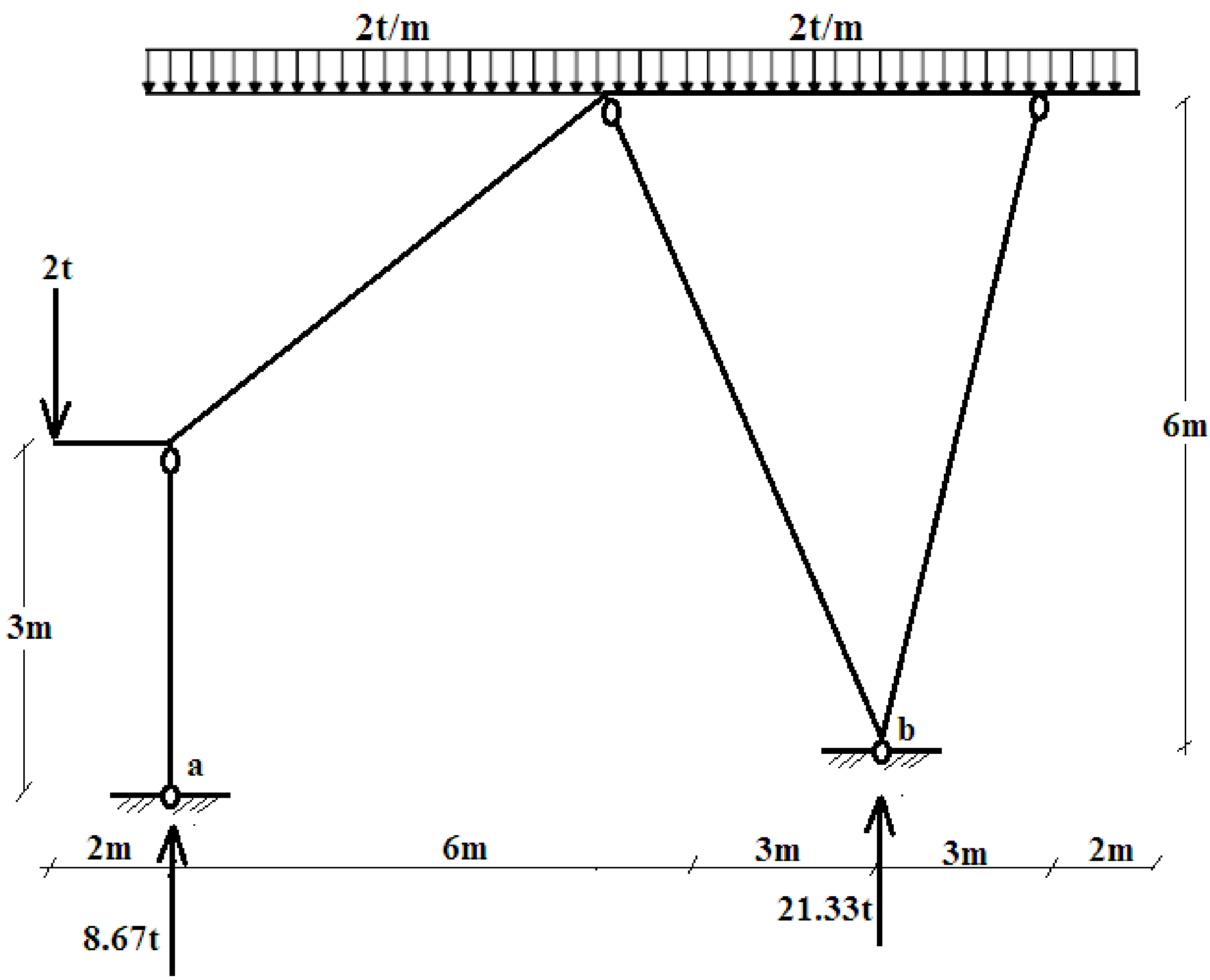


Fig. 4.32.a

- *For joint#b*

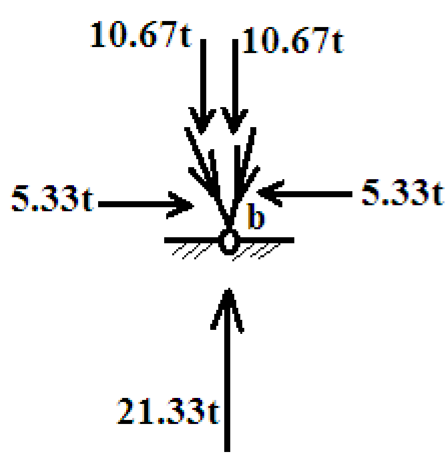
$$\sum F_x = F_{1x} - F_{2x} = 0 \ldots \ldots (1)$$

$$\sum F_y = F_{1y} - 21.33 + F_{2y} = 0 \ldots \ldots (2)$$

$$F_{1x} = 0.5F_{1y} \ \ldots \ldots (3)$$

$$F_{2x} = 0.5F_{2y} \ \ldots \ldots (4)$$

By solving the four equations



Joint (b)

$F_{1x} = 5.33t \ and \ F_{1y} = 10.67t \ and \ F_1 = 11.92t \ (comp)$

$F_{2x} = 5.33t \ and \ F_{2y} = 10.67t \ and \ F_2 = 11.92t \ (comp)$

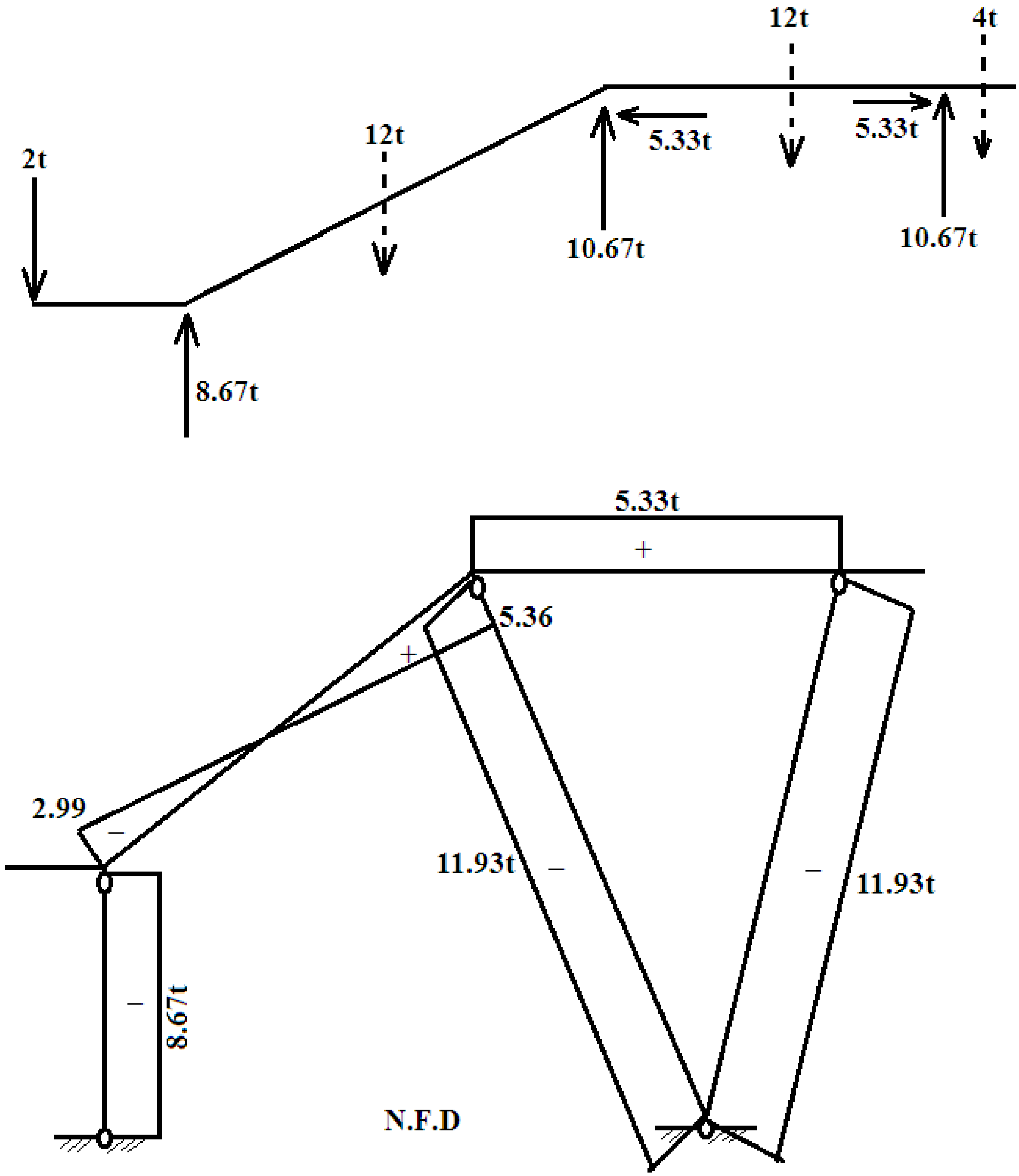
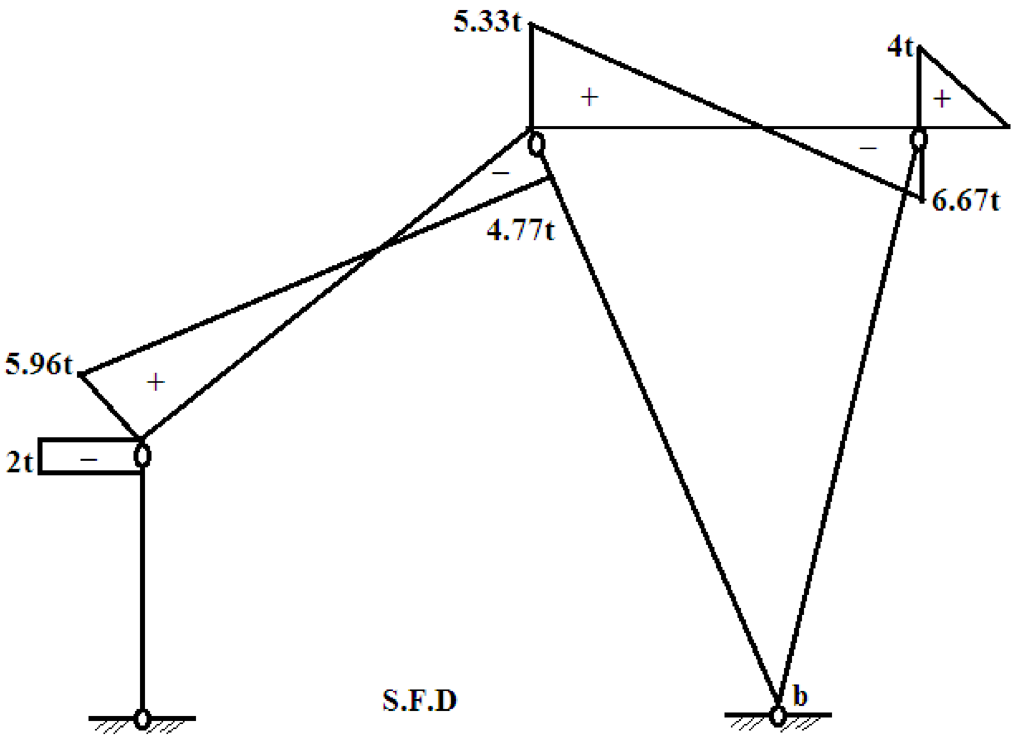
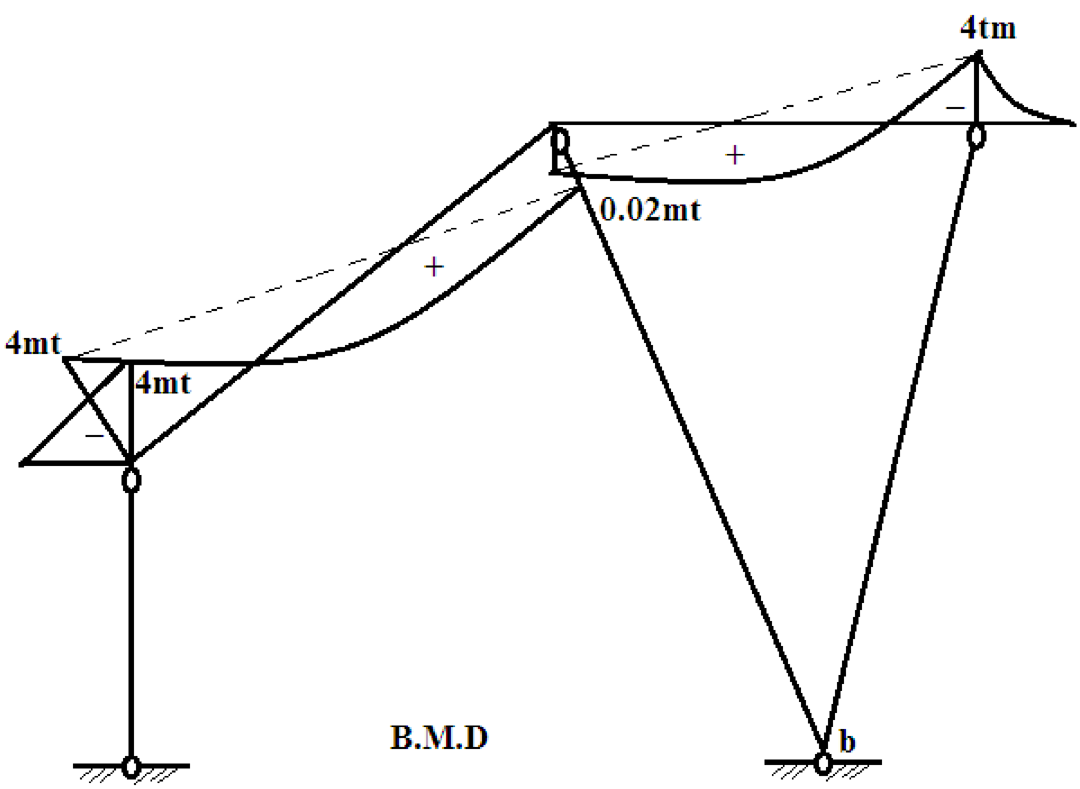


Fig. 4.32.b


5.33t
4t
+
+
–
–
6.67t
4.77t
5.96t
+
2t
–
S.F.D
b


4tm
–
+
0.02mt
+
4mt
4mt
–
B.M.D
b

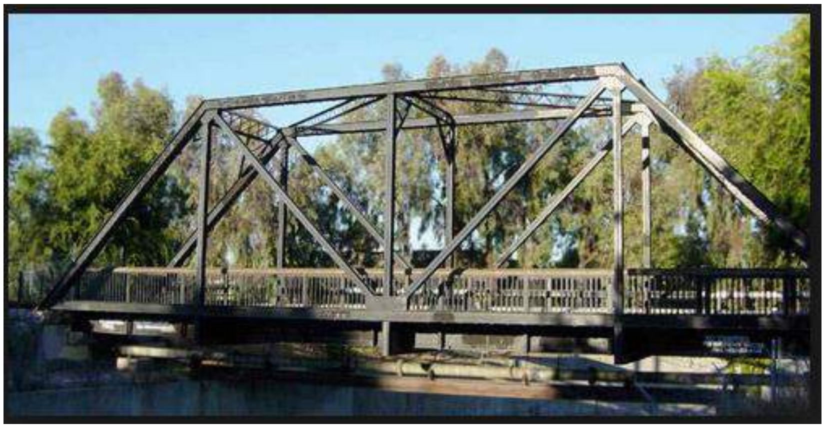
Chapter (5)

TRUSS STRUCTURES

5.1. Introduction

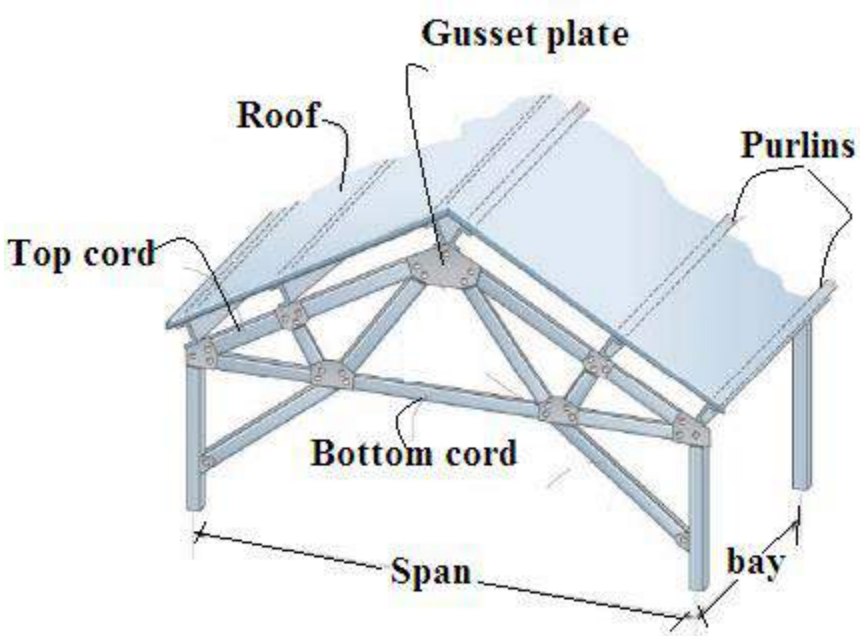
Truss is a type of structures that consist of one or more triangular units. Each unite is constructed with straight slender members whose ends are connected at joints referred to as nodes. This type of structures may be used in building structures or bridges as indicated in Fig. 5.1. In the truss structure, the external forces and reactions to those forces are considered to act only at the nodes and result in forces in the members which are either tensile (pulling the pin) or compressive forces (compress the pin). Moments (torsional forces) are explicitly excluded because all the joints in a truss are treated as pin joints. The components of the truss structure are presented in Fig. 5.2. There are various types of the trusses according to the shape of the trusses as presented in Fig. 5.3.

a) Building



b) Bridge

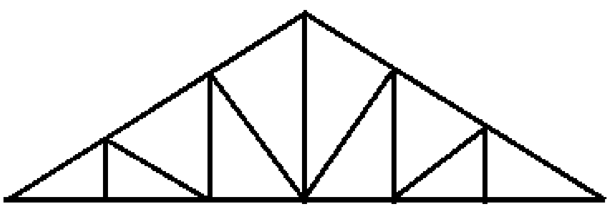
Fig. 5.1: Type of truss structures



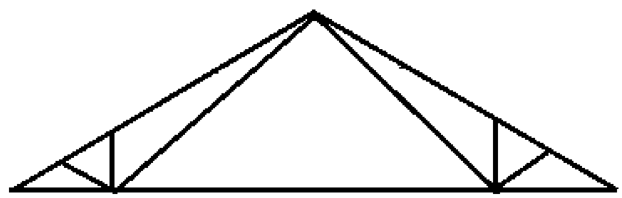
a) Building

b) Bridges

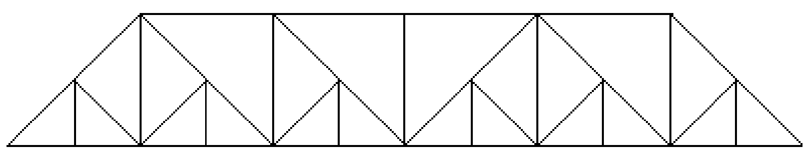
Fig. 5.2: Components of truss structure



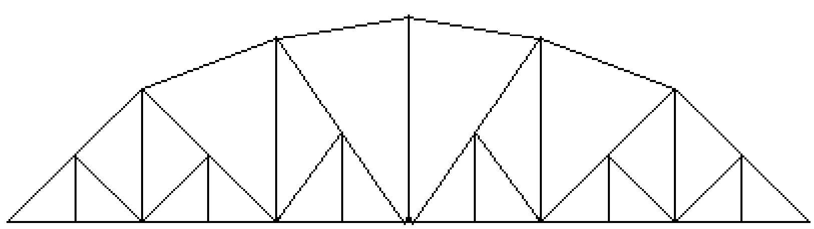
Pratt



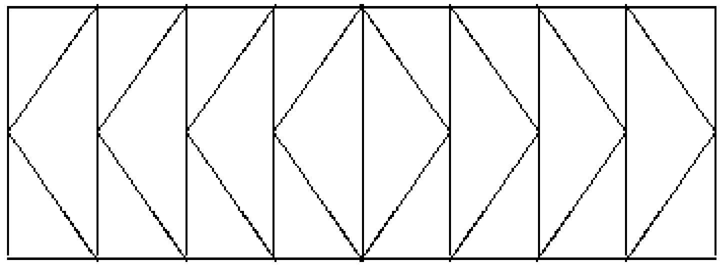
Fan



Baltimoro



Pennsylvania



K Truss

Fig. 5.3: Classification of the trusses

5.2. Determinacy of the Truss

The degree of the determinacy of the truss structures can be calculated from the below equation.

$I = b + r - 2j$……………………………………………………….. (5-1)

Where

b = number of bars

r = number of external support reaction

j = number of joints

If I=0 the trusses is statically determinate

If I > 0, the truss is statically indeterminate I<0 the truss is unstable

The truss may be externally unstable if all of its reactions are concurrent or parallel as shown in Fig. 5.4. Also it may be internally unstable as shown in Fig. 5.5.

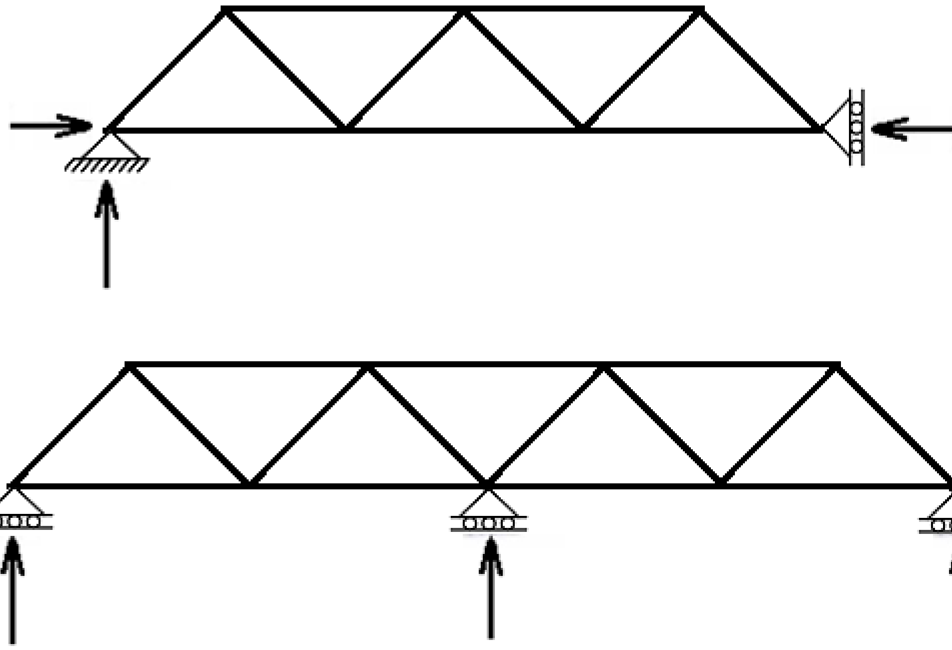


Fig. 5.4

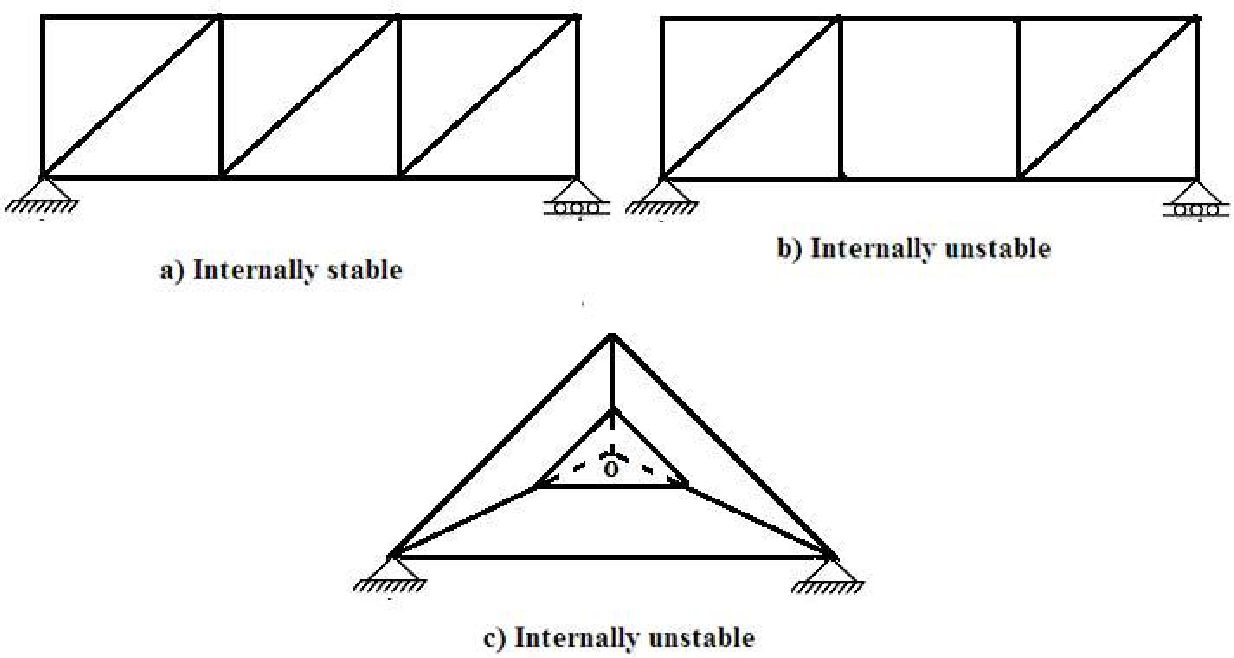

a) Internally stable
b) Internally unstable
o
c) Internally unstable

Fig. 5.5

5.3. Method of Analysis

The analysis method of trusses is based on the assumptions that all members are connected only at their ends by frictionless hinges in plane trusses. Also all loads and support reactions are applied only at the joints.

1. *Method of Joints*

The *method of joints* consists of satisfying the equilibrium equations $\sum F_x = 0\ and\ \sum F_y = 0$ for forces acting on each joint. The below steps are followed to determine the forces in the truss member using this method.

1) Determination all the support reactions using the equations of equilibrium.
2) Selecting a joint with one or two unknowns force.
3) Drawing the free-body diagram of a selected joint.
4) Assuming that all unknown member forces act in tension (pulling the pin) unless the force that can be determined by inspection that the forces are compression loads.
5) Apply the scalar equations of equilibrium, $\Sigma\ FX = 0$ *and* $\Sigma FY = 0$, to determine the unknowns. If the answer is *positive,* then the assumed direction *(tension)* is correct, otherwise it is in the opposite direction (compression).
6) Repeating steps 2 to 5 at each joint in succession until all the required forces are determined.

2. *Method of Section*

This method is used to find analytically the forces in selected members of the truss. In this method, we isolate a portion of truss by an imaginary cut section. This section passes through all or some of selected members. After that we satisfy at the isolation portion of truss the three equilibrium equations

$$\sum F_x = 0 \; and \; \sum F_y = 0 \; and \; \sum M_{at\ any\ point} = 0$$

3. ***Graphic Method***

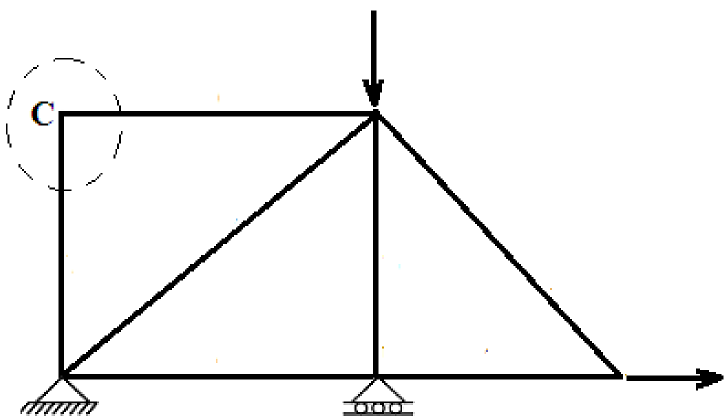
This method depends on the three properties of the force; the force has magnitude, direction and point of applied. In this method, the force can be represented by a straight line or vector. The below steps are followed to determine the forces in the truss member using this method.

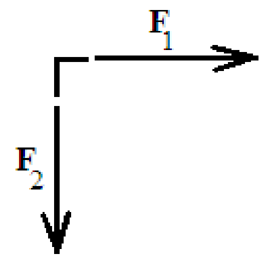
1) Determine the reactions
2) Represent graphically the external forces (reactions and the applied loads) graphically. In this step, the external forces are represent by a diaphragm starts at a point and end at the same point. Each force is represented by its magnitude and direction.
3) Start with a joint having only two unknown forces.
4) Draw vectors parallel the force directions for the joint. These vectors are closed. Measure the length of these straight lines and multiply by the scale factors. These values represent the force magnitude.
5) Go around the joint in the same direction used in drawing the external forces.
6) Repeat the steps 3 to 5.

5.4. Zero members

The force in the truss member equals zero in the below cases

1) If only two non-collinear members are connected to a joint that has no external loads or reactions applied to it, then the force in both members is zero (see Fig. 5.6.a).



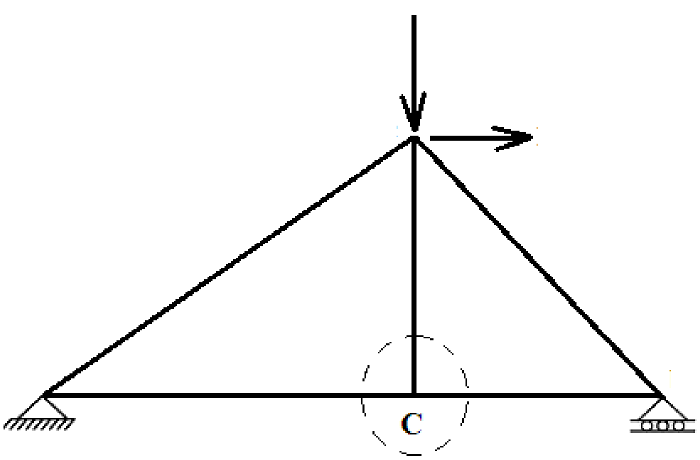


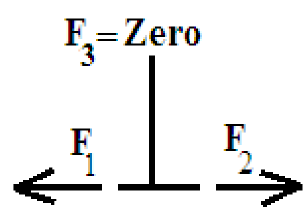
$$\sum F_y = 0 \quad F_1 = 0$$

$$\sum F_x = 0 \quad F_2 = 0$$

Fig. 5.6.a

2) If three members form a truss joint for which two of the members are collinear and there is no external load or reaction at that joint as shown in Fig. 5.6.b, then the third non-collinear member is a zero force.





$$\sum F_y = 0 \quad F_3 = 0$$

$$\sum F_x = 0 \quad F_2 = F_3$$

Fig. 5.6.b

5.4. Solved Examples

Example (1)

Find analytically the forces in all members of given truss in Fig. 5.7. by the method of joint.

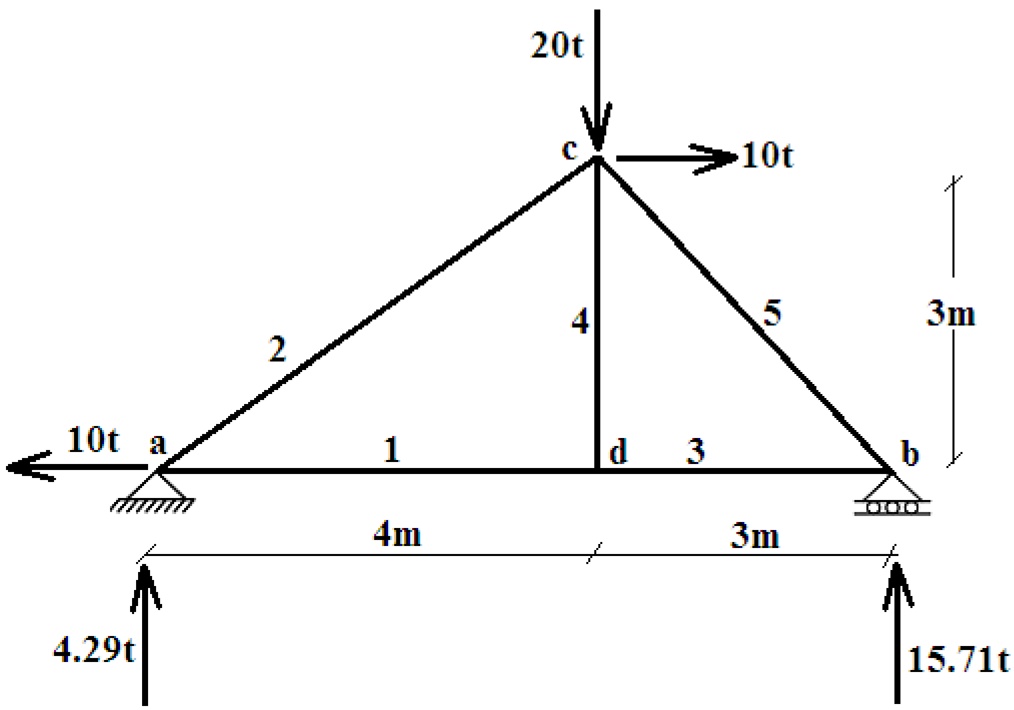


Fig. 5.7.

Solution

- Finding the reactions

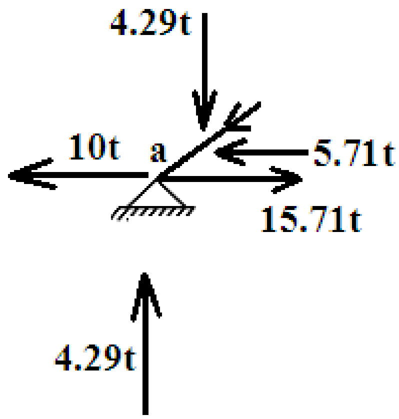
a) $\sum F_x = 0 \ggg\ggg\ggg X_a = 10t \leftarrow$

$b) \sum M_a = 0 \ggg\ggg\ggg 20x4 + 10x3 - Y_b x7 = 0 \therefore Y_b = 15.71t \uparrow$

$c) \sum M_b = 0 \ggg\ggg\ggg 20x3 - 10x3 - Y_a x7 = 0$

$$\therefore Y_a = 4.29 \uparrow$$

d) *Check* $\sum F_y = 20 - 4.29 - 15.71 = 0 \ggg\ggg \text{ ok}$

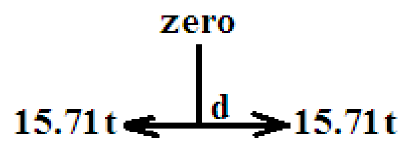


$\sum F_y = 0 \quad F_{y2} = 4.29\text{t} \downarrow$

$$F_{x2} = \frac{4}{3} 4.29 = 5.71t$$

$\sum F_x = 0 \quad F_{x1} = 15.71\text{t} \rightarrow$

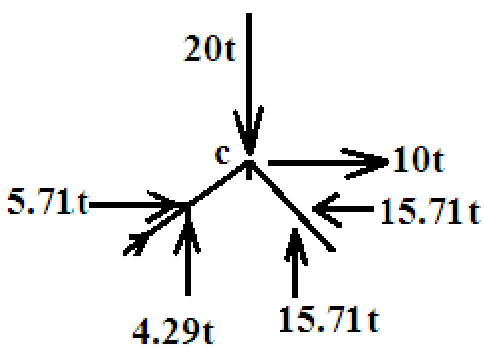
Joint (a)



$\sum F_y = 0 \quad F_4 = 0$

$\sum F_x = 0 \quad F_3 = 15.71\text{t} \rightarrow$

Joint (d)

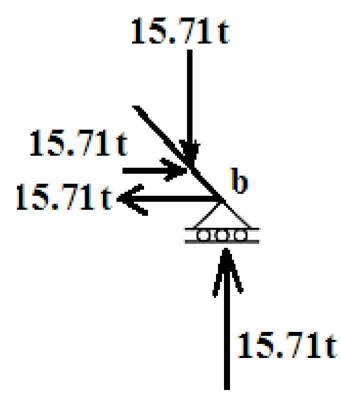


$\sum F_y = 0 \quad F_{y5} = 15.71\text{t} \uparrow$

$$F_{x2} = \frac{3}{3} 15.71 = 15.71t \leftarrow$$

$\sum F_x = 0 \quad \text{ok (check)}$

Joint (d)



$\sum F_y = 0 \quad ok$

$\sum F_x = 0 \quad \text{ok (check)}$

Joint (b)

Member	1	2	3	4	5
Force	15.71	7.14	15.71	0	22.22
Type	T	C	T	…..	C

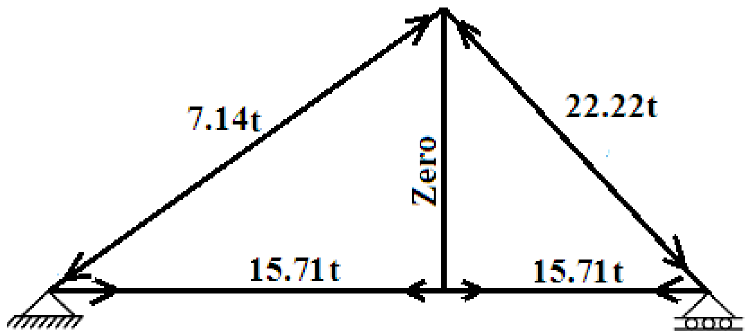


Fig. 5.7. b

Example (2)

Find analytically the forces in all members of given truss in Fig. 5.8. by the method of joint.

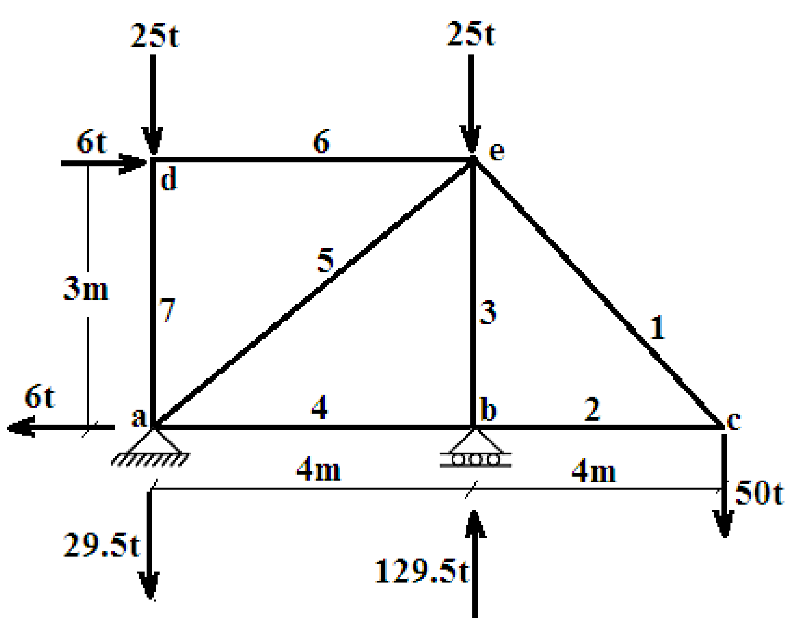


Fig. 5.8.a

Solution

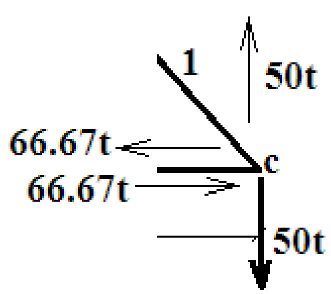
- Finding the reactions

e) $\sum F_x = 0 \ggg\ggg X_a = 6t \leftarrow$

f) $\sum M_a = 0 \ggg\ggg 25x4 + 50x8 + 6x3 - Y_b x4 = 0 \therefore Y_b = 129.5t \uparrow$

g) $\sum M_b = 0 \ggg\ggg 50x4 + 6x3 - 25x4 - Y_a x4 = 0 \therefore Y_b = 29.5t \downarrow$

h) $\underline{Check} \sum F_y = 25 + 50 + 25 + 29.5 - 129.5 = 0 \ggg\ggg$ ok

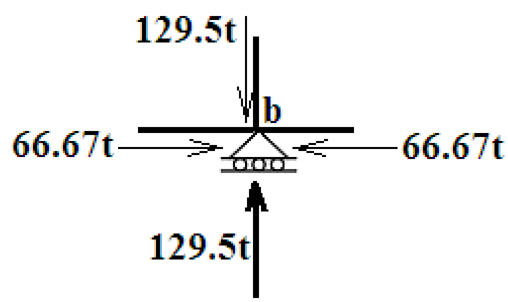


$\sum F_y = 0 \;\; F_{y1} = 50t \uparrow$

$$F_{x2} = \frac{4}{3} 50 = 66.67t \leftarrow$$

$\sum F_x = 0 \;\; F_2 = 66.67t \rightarrow$

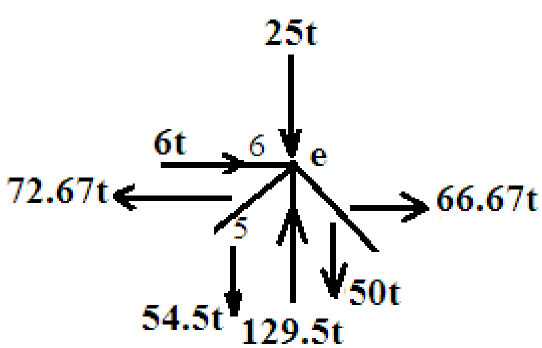
Joint (c)



$\sum F_y = 0 \;\; F_3 = 129.5t \downarrow$

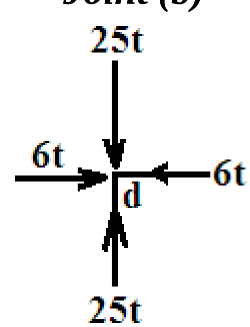
$\sum F_x = 0 \;\; F_4 = 66.67t \rightarrow$

Joint (b)



$\sum F_y = 0 \;\; F_{y5} = 54.5t \downarrow$

$$F_{x5} = \frac{4}{3} 54.5 = 72.67t \leftarrow$$

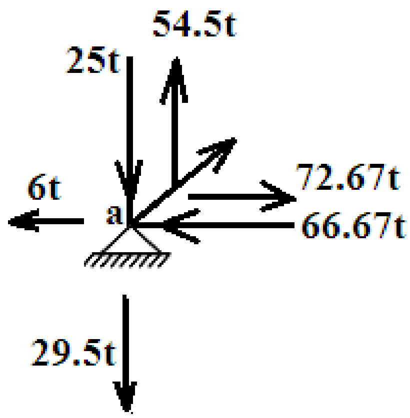


$\sum F_y = 0 \;\; F_7 = 25t \uparrow$

$\sum F_x = 0$ ok

$$\sum F_x = 0 \;\; F_6 = 6t \rightarrow$$

Joint (e) ***Joint (d)***



$\sum F_y = 0 \;\; ok$

$\sum F_x = 0$ ok (check)

Joint (a)

Member	1	2	3	4	5	6	7
Force	83.33	66.67	129.5	66.67	90.84	6	25
Type	T	C	C	C	T	C	C

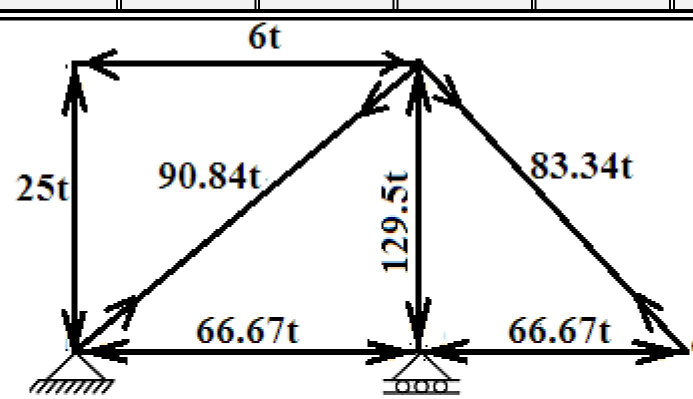


Fig. 5.8.b

Example (3)

Find analytically the forces in all members of given truss in Fig. 5.9.a by the method of joint.

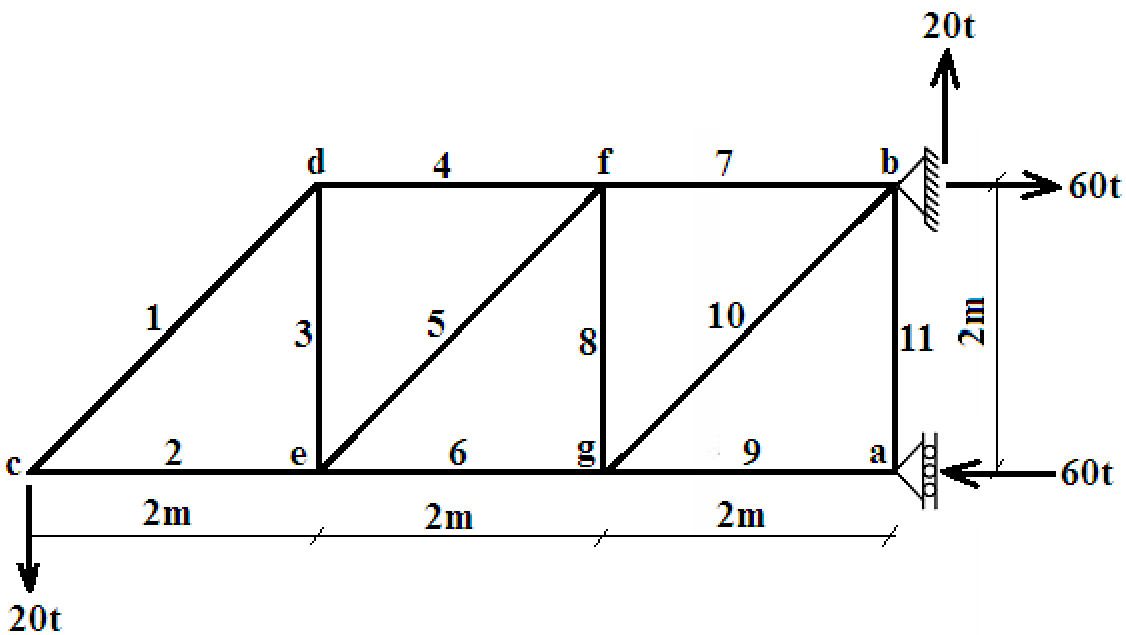


Fig. 5.9.a

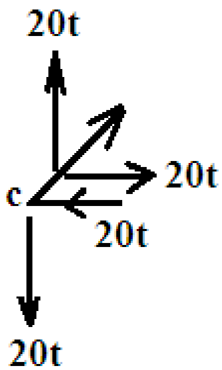
Solution

Finding the reactions

$\sum M_a = 20x6 - 2X_b = 0 \qquad \therefore X_b = 60t \rightarrow$

$\sum M_b = 20x6 - 2X_a = 0 \qquad \therefore X_a = 60t \leftarrow$

Check $\sum F_y = Y_b = 20t \uparrow$

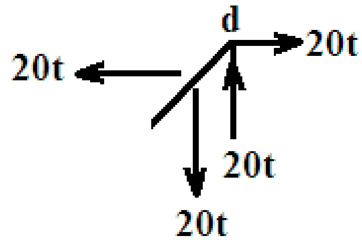


$\sum F_y = 0 \quad F_{y1} = 20t \uparrow$

$F_{x1} = F_{y1} = 20t \rightarrow$

$\sum F_x = 0 \quad F_2 = 20t \leftarrow$

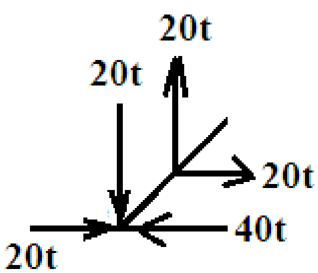
Joint#c



$\sum F_y = 0 \quad F_3 = 20t \uparrow$

$\sum F_x = 0 \quad F_4 = 20t \rightarrow$

Joint#b

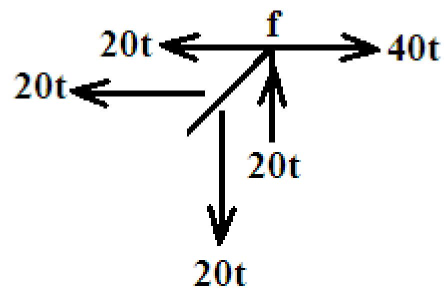


$\sum F_y = 0 \quad F_{y5} = 20t \uparrow$

$F_{x5} = F_{y5} = 20t \rightarrow$

$$\sum F_x = 0 \quad F_6 = 40t \leftarrow$$

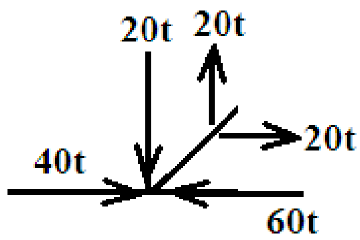
Joint#e



$\sum F_y = 0 \quad F_7 = 20t \uparrow$

$\sum F_x = 0 \quad F_8 = 40t \rightarrow$

Joint#f

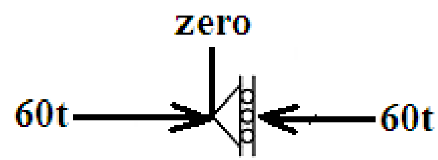


$\sum F_y = 0 \;\; F_{y9} = 20t \uparrow$

$F_{x5} = F_{y9} = 20t \rightarrow$

$\sum F_x = 0 \;\; F_{10} = 60t \leftarrow$

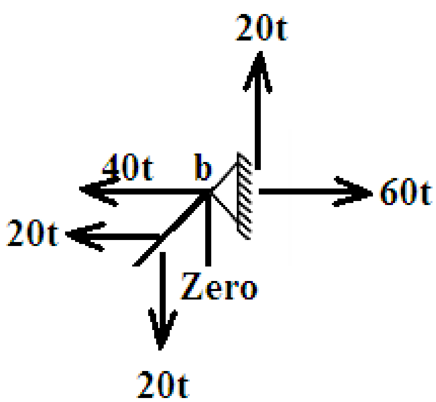
Joint#g



$\sum F_y = 0 \;\; F_{11} = \text{zero}$

$\sum F_x = 0 \;\; 0k$

Joint#a



$\sum F_y = 0 \;\; \text{ok}$

$\sum F_x = 0 \;\; 0k$

Joint#b

Member	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
Force	21	28.28	20	20	28.28	40	40	20	60
Type	C	T	C	T	T	C	T	C	C

Member	10	11
Force	28.28	0
Type	T	…

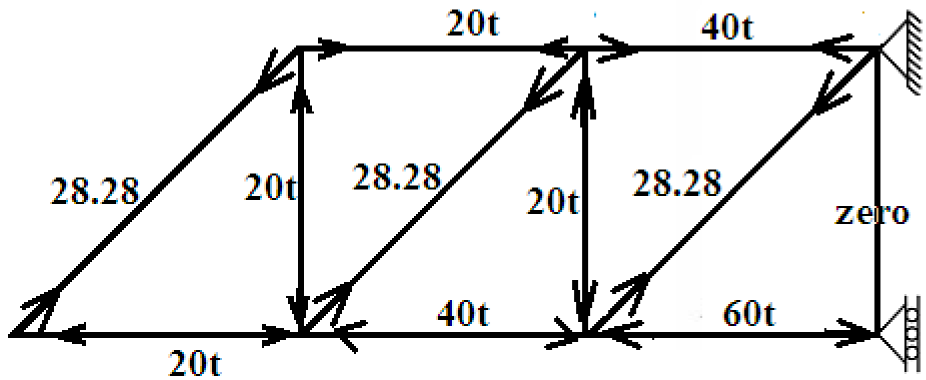


Fig. 5.9.b

Example (4)

Find analytically the forces in all members of given truss in Fig. 5.10.a by the method of joint.

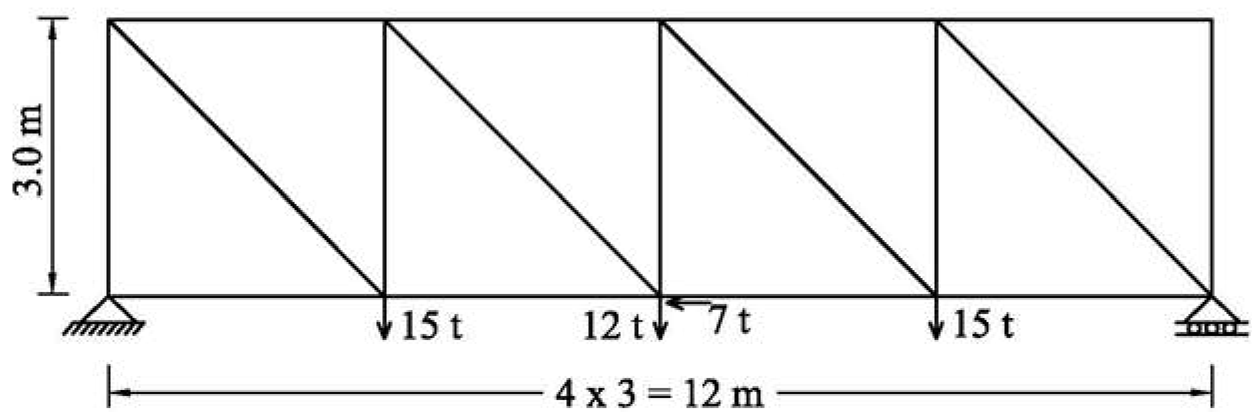


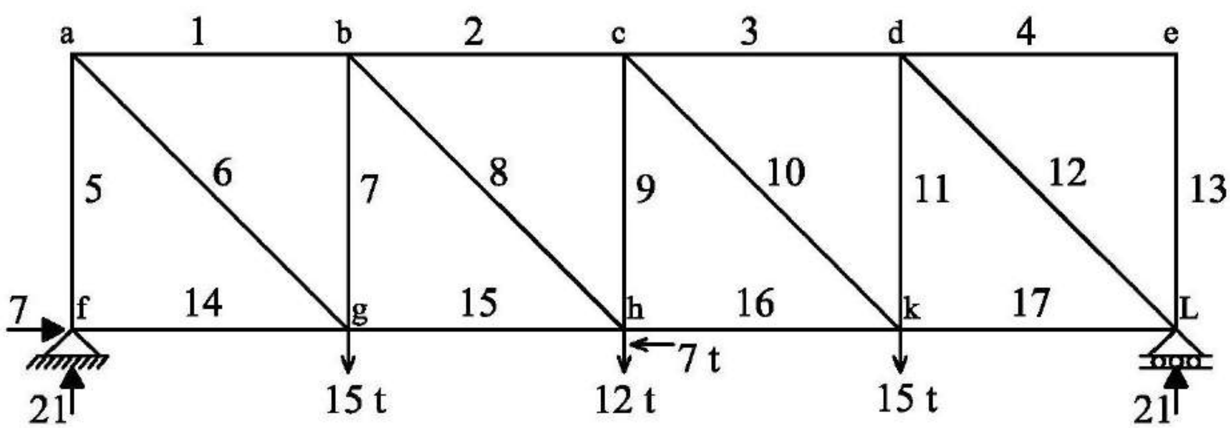
Fig. 5.10.a

Solution

Finding the reactions

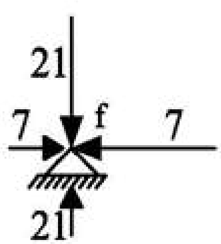
$\sum M_a = 15x3 + 12x6 + 15x9 - 12Y_b = 0 \qquad \therefore Y_b = 21t \uparrow$

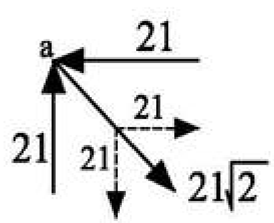
$\sum M_b = 15x3 + 12x6 + 15x9 - 12Y_a = 0 \qquad \therefore Y_a = 21t \uparrow$



Check $\sum F_y = 0$ ok

Fig. 5.10.b





$\sum F_y = 0 \;\; F_5 = 21t \downarrow$

$\sum F_x = 0 \;\; F_{14} = 7t \leftarrow$

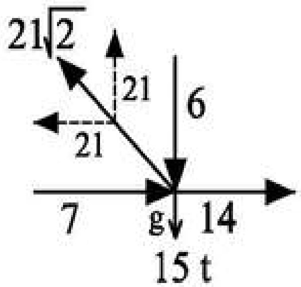
Joint#f

$\sum F_y = 0 \;\; F_{y6} = 21t \downarrow$

$F_{x6} = F_{y6} = 21t \rightarrow$

$\sum F_x = 0 \;\; F_1 = 21t \leftarrow$

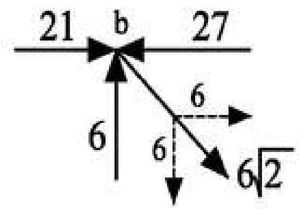
Joint#a



$\sum F_y = 0 \;\; F_7 = 6t \downarrow$

$\sum F_x = 0 \;\; F_{15} = 14t \rightarrow$

Joint#g

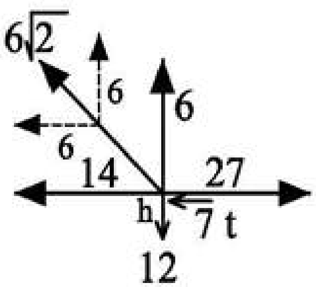


$\sum F_y = 0 \;\; F_8 = 6t \downarrow$

$F_{x8} = F_{y8} = 6t \rightarrow$

$\sum F_x = 0 \;\; F_2 = 27t \leftarrow$

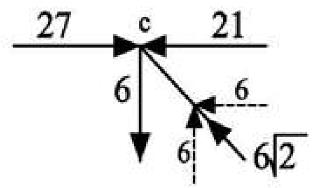
Joint#b



$\sum F_y = 0 \;\; F_9 = 6t \uparrow$

$\sum F_x = 0 \;\; F_{16} = 27t \rightarrow$

Joint#h

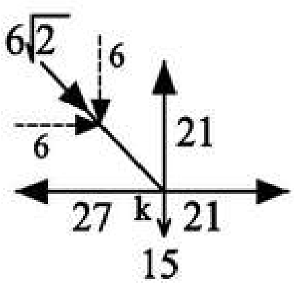


$\sum F_y = 0 \;\; F_{y10} = 6t \uparrow$

$F_{x10} = F_{y10} = 6t \leftarrow$

$\sum F_x = 0 \;\; F_3 = 21t \leftarrow$

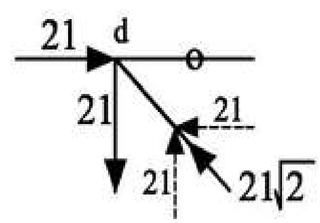
Joint#c



$\sum F_y = 0 \;\; F_{11} = 21t \uparrow$

$\sum F_x = 0 \;\; F_{17} = 21t \rightarrow$

Joint#k

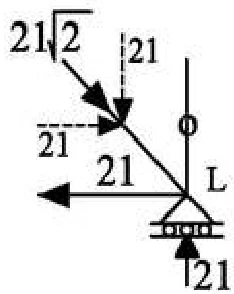


$\sum F_y = 0 \;\; F_{y12} = 21t \uparrow$

$F_{x12} = F_{y12} = 21t \leftarrow$

$\sum F_x = 0 \;\; F_4 = 0$

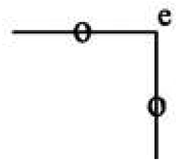
Joint#d



$\sum F_y = 0 \;\; F_{13} = 0$

$\sum F_x = 0 \;\; \text{ok}$

Joint#L



$\sum F_y = 0 \;\; \text{ok}$

ok

Joint#e

Member	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
Force	21	27	21	0	21	$21\sqrt{2}$	6	$6\sqrt{2}$	6
Type	C	C	C	…..	C	T	C	T	T

Member	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**
Force	$6\sqrt{2}$	21	$21\sqrt{2}$	0	7	14	27	21
Type	C	T	C	….	C	T	T	T

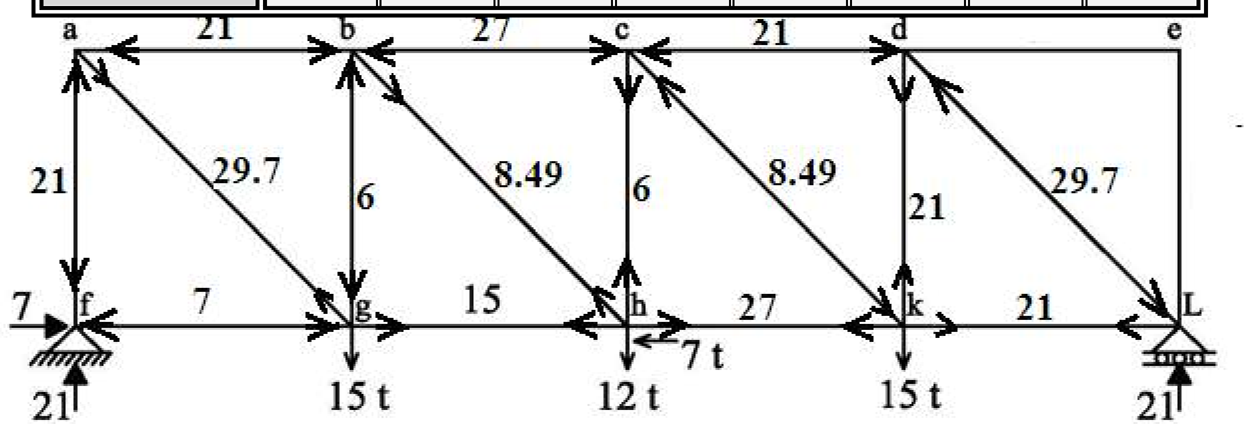


Fig. 5.10.c

Example (5)

Find analytically the forces in all members of given truss in Fig. 5.11.a by the method of joint.

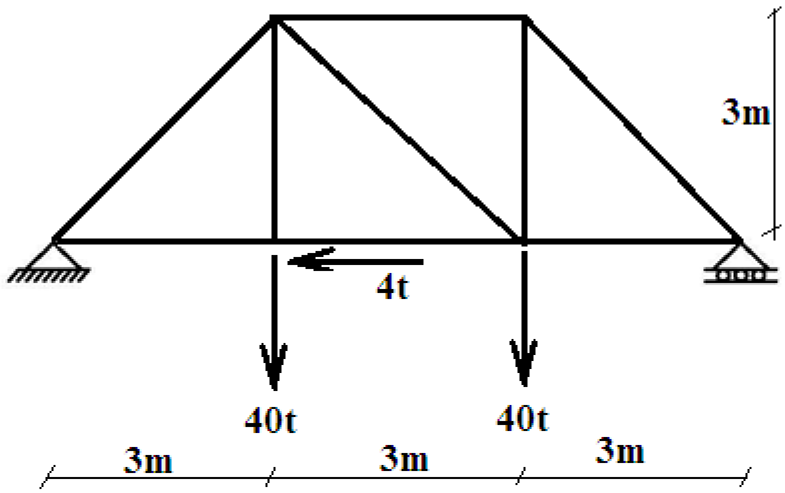


Fig. 5.11.a

Solution

Finding the reactions

$\sum F_x = 0 \qquad X_a = 4t \rightarrow \text{ok}$

$\sum M_a = 40x3 + 40x6 - 9Y_b = 0 \qquad \therefore Y_b = 40t \uparrow$

$\sum M_b = 40x3 + 40x6 - 9Y_a = 0 \qquad \therefore Y_a = 40t \uparrow$

Check $\sum F_y = 0$ ok

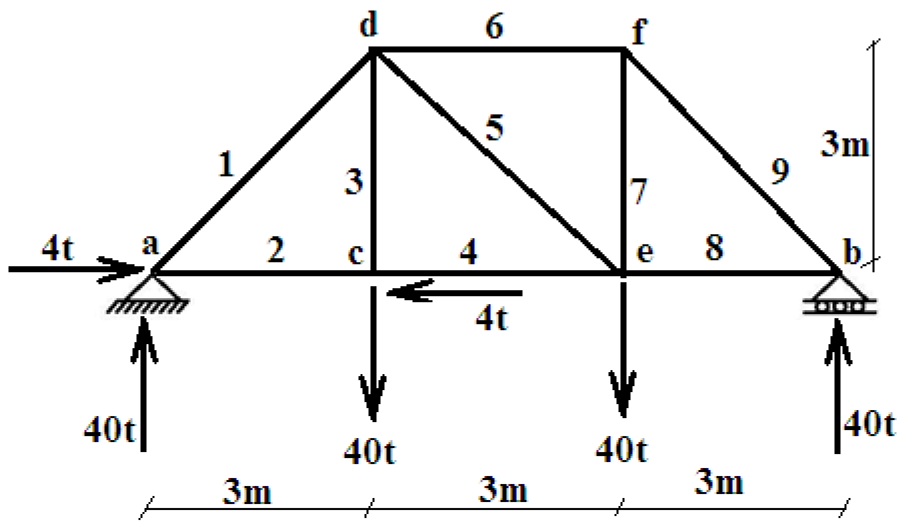
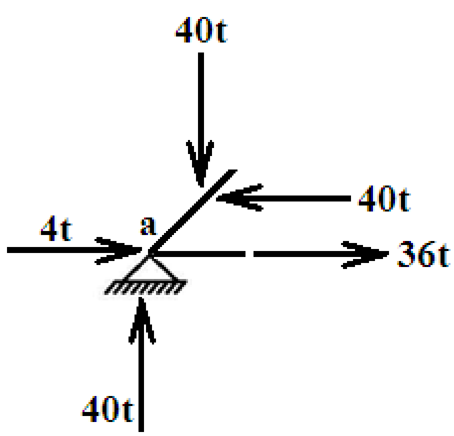


Fig. 5.11.b

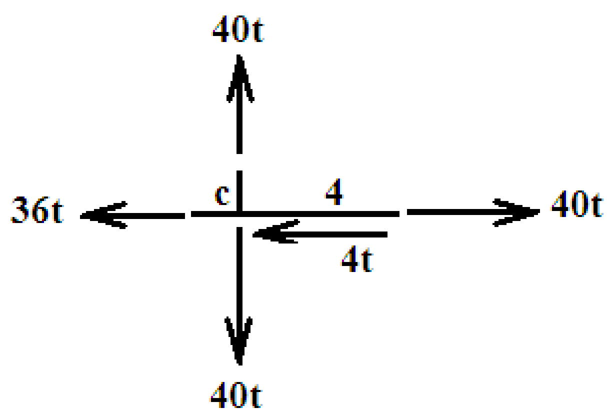


$\sum F_y = 0 \quad F_{y1} = 40t \uparrow$

$F_{x1} = F_{y1} = 40t \leftarrow$

$\sum F_x = 0 \quad F_2 = 36t \rightarrow$

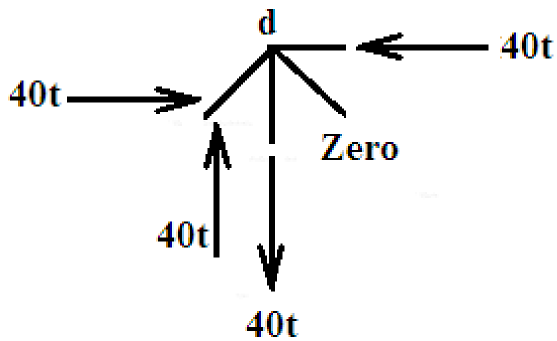
Joint (a)

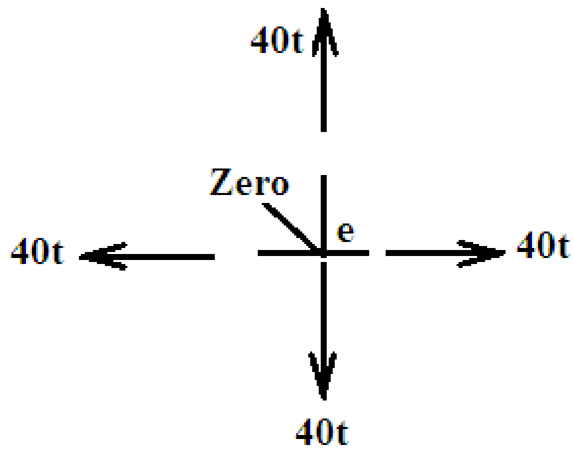


$\sum F_y = 0 \quad F_3 = 40t \uparrow$

$\sum F_x = 0 \quad F_4 = 40t \rightarrow$

Joint (c)





$\sum F_y = 0 \;\; F_{y5} = 0$

$F_{x5} = F_{y5} = 0$

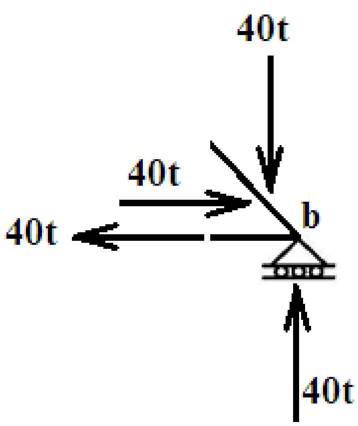
$\sum F_x = 0 \;\; F_6 = 40t \leftarrow$

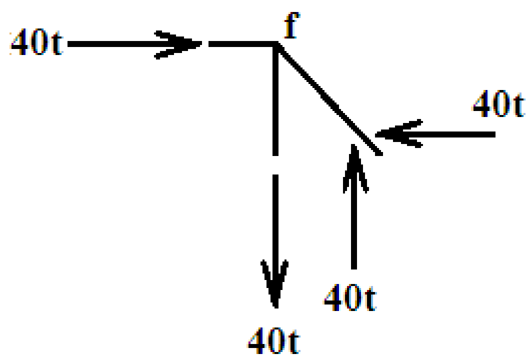
Joint (d)

$\sum F_y = 0 \;\; F_7 = 40t \uparrow$

$\sum F_x = 0 \;\; F_8 = 40t \rightarrow$

Joint (e)





$\sum F_y = 0 \;\; F_{y9} = 40t \downarrow$

$F_{x5} = F_{y5} = 40t \rightarrow$

$\sum F_x = 0$ ok

Joint (b)

$\sum F_y = 0$ ok

$\sum F_x = 0$ ok

Joint (f)

Member	1	2	3	4	5	6	7	8	9
Force	56.57	36	40	40	0	40	40	40	56.57
Type	C	T	T	T	-----	C	T	T	C

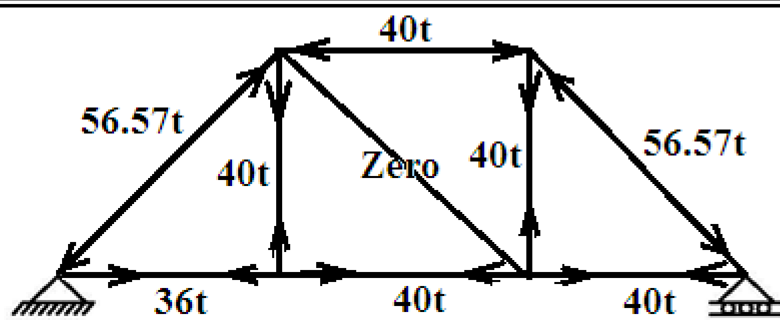


Fig. 5.11.c

Example (6)

Find analytically the forces in all members of given truss in Fig. 5.12.a. by the method of joint.

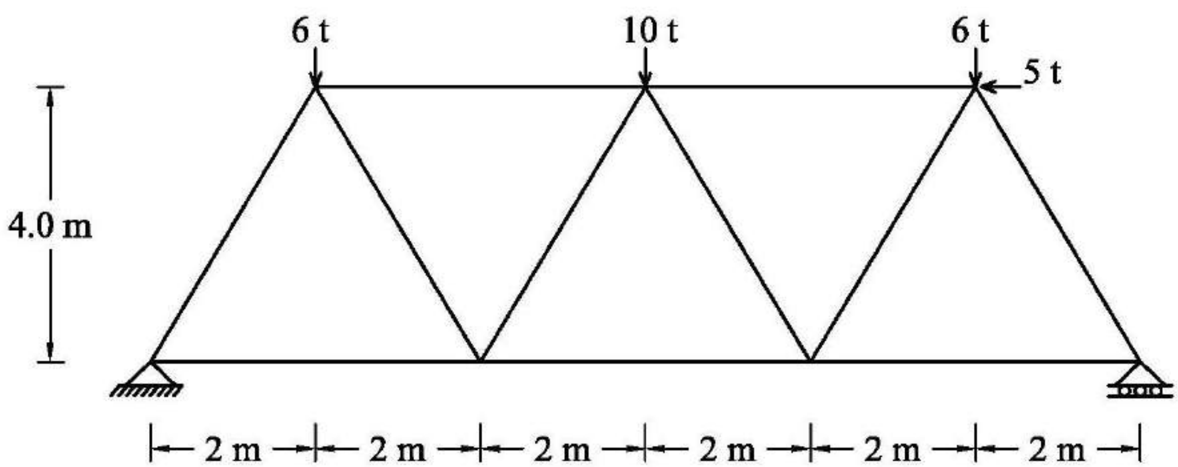


Fig. 5.12.a

Solution

Finding the reactions

$$\sum F_x = 0 \quad F_{xd} = 5t \rightarrow$$

$$\sum M_d = 6x2 + 10x6 + 6x10 - 5x4 - 12Y_g = 0 \qquad \therefore Y_g = 9.33t \uparrow$$

$$\sum M_g = 6x2 + 10x6 + 6x10 + 5x4 - 12Y_d = 0 \qquad \therefore Y_d = 12.67t \uparrow$$

Check $\sum F_y = 0$ ok

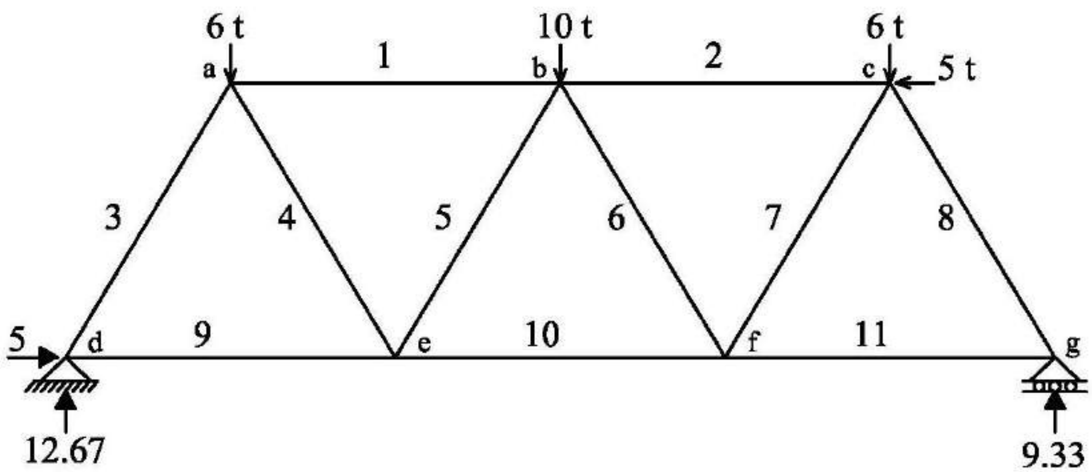
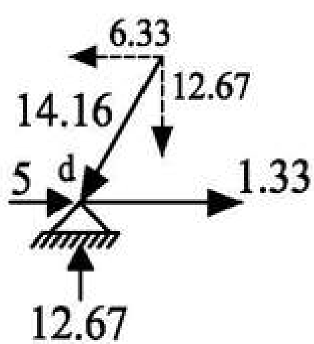


Fig. 5.12. b

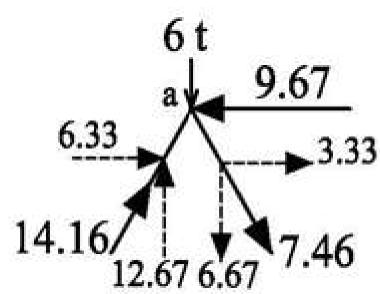


$$\sum F_y = 0 \quad F_{y3} = 12.67t \downarrow$$

$$F_{x3} = 0.5F_{y3} = 6.33t \leftarrow$$

$$\sum F_x = 0 \quad F_9 = 1.33t \rightarrow$$

Joint#d

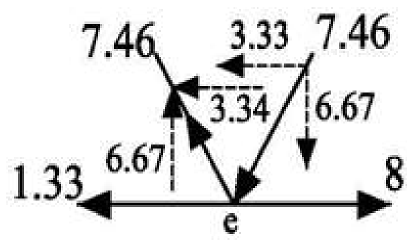


$$\sum F_y = 0 \quad F_{y4} = 6.67t \downarrow$$

$$F_{x3} = 0.5F_{y4} = 3.33t \rightarrow$$

$$\sum F_x = 0 \quad F_1 = 9.67t \leftarrow$$

Joint#a

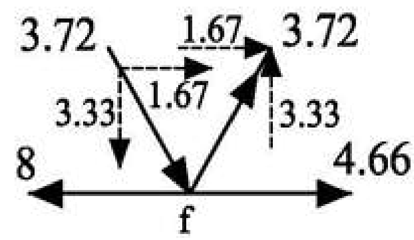


$\sum F_y = 0 \;\; F_{y5} = 6.67t \downarrow$

$F_{x3} = 0.5F_{y5} = 3.33t \leftarrow$

$\sum F_x = 0 \;\; F_{10} = 8t \rightarrow$

Joint#e

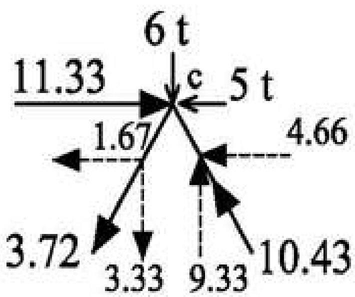


$\sum F_y = 0 \;\; F_{y7} = 3.33t \uparrow$

$F_{x3} = 0.5F_{y7} = 1.67t \rightarrow$

$\sum F_x = 0 \;\; F_{11} = 4.66t \rightarrow$

Joint#f

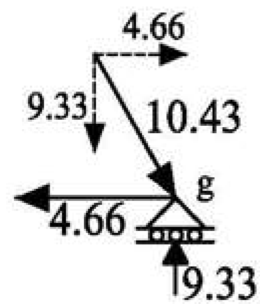


$\sum F_y = 0 \;\; F_{y8} = 9.33t \uparrow$

$F_{x3} = 0.5F_{y8} = 4.66t \leftarrow$

$\sum F_x = 0 \;\; \text{ok}$

Joint#c



$\sum F_y = 0 \;\; ok \;\; \sum F_x = 0 \;\; \text{ok}$

Joint#g

Member	**1**	**2**	**3**	**4**	**5**	**6**
Force	9.67	11.33	14.16	7.46	7.46	3.72
Type	C	C	C	T	C	C

Member	**7**	**8**	**9**	**10**	**11**
Force	3.72	10.43	1.33	8	4.66
Type	T	C	T	T	T

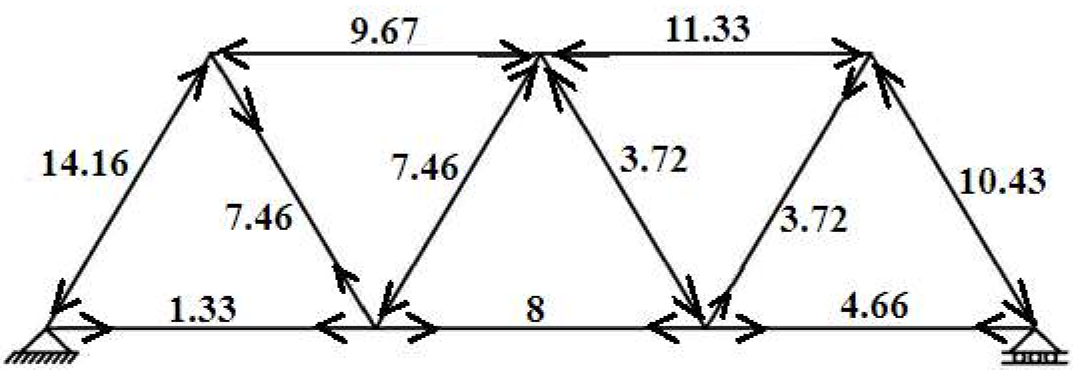


Fig. 5.12.c

Example (6)

Find analytically the forces in all members of given truss in Fig. 5.13.a. by the method of joint.

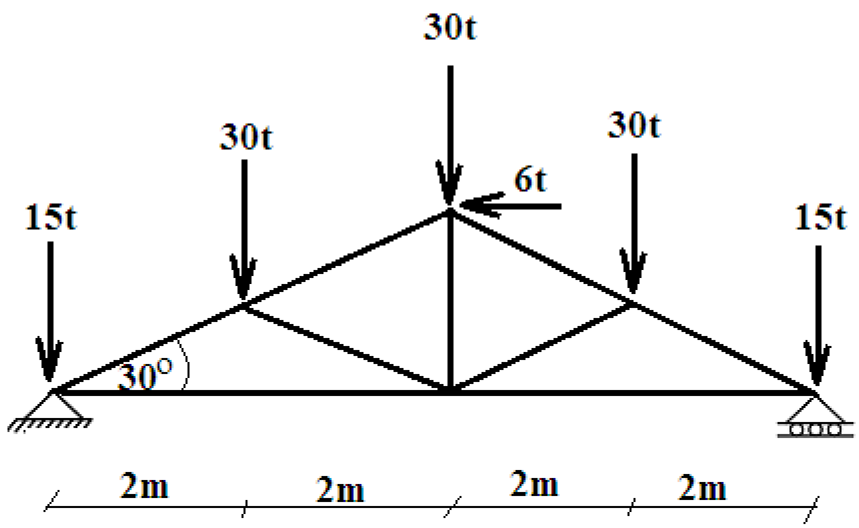


Fig. 5.13.a

Solution

Finding the reactions

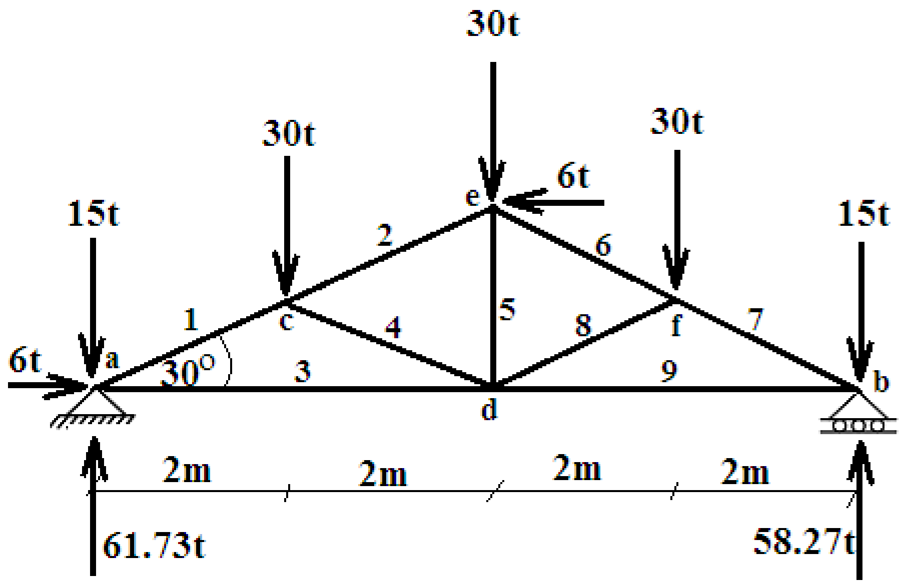
$$\sum F_x = 0 \; X_a = 6t \rightarrow$$

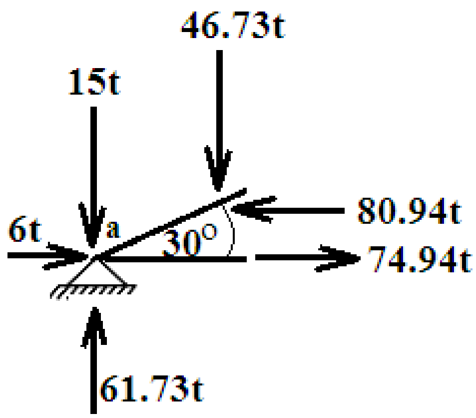
$$\sum M_a = 30x2 + 30x4 + 30x6 + 15x8 - 6x2.31 - 8Y_b = 0$$

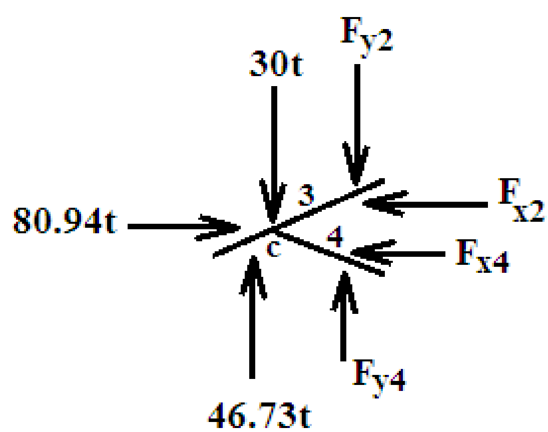
$$\therefore Y_b = 58.27 \uparrow$$

$\sum M_b = 30x2 + 30x4 + 30x6 + 15x8 + 6x2.31 - 8Y_a = 0 \qquad Y_a = 61.73 \uparrow$

<u>*Check*</u> $\sum F_y = 0$ ok







$\sum F_y = 0 \;\; F_{y1} = 46.73t \downarrow$

$$F_{x1} = \frac{F_{y1}}{tan30} = 80.94 \leftarrow$$

$\sum F_x = 0 \;\; F_3 = 74.94t \rightarrow$

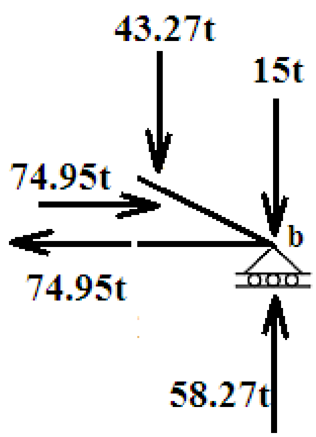
$\sum F_y = 0 \;\; F_{y2} - F_{y4} = 16.73t \downarrow$

$$F_{x2} = \frac{F_{y2}}{tan30}$$

$\sum F_x = 0 \;\; F_{x2} + F_{x4} = 80.94t$

$$F_{x4} = \frac{F_{y4}}{tan30}$$

Joint#a



$$\sum \mathrm{F_y} = 0 \quad F_{y7} = 43.27\mathrm{t} \downarrow$$

$$F_{x7} = \frac{F_{y7}}{tan30} = 74.95 \rightarrow$$

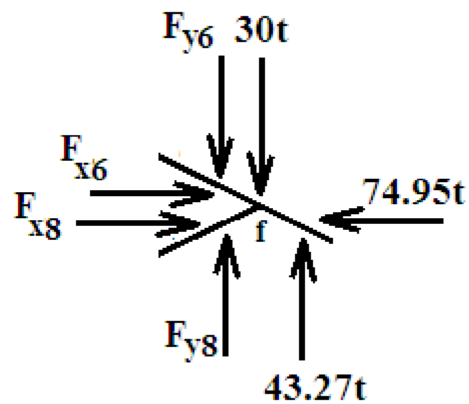
$$\sum \mathrm{F_x} = 0 \quad F_9 = 74.95\mathrm{t} \leftarrow$$

Joint#b

$$F_{y2} = 31.73t \downarrow \quad F_{y4} = 15.00t \uparrow$$

$$F_{x2} = 54.95t \leftarrow \quad F_{x4} = 25.98t \leftarrow$$

Joint#c



$$\sum \mathrm{F_y} = 0 \quad F_{y6} - F_{y8} = 13.27\mathrm{t} \downarrow$$

$$F_{x6} = \frac{F_{y6}}{tan30}$$

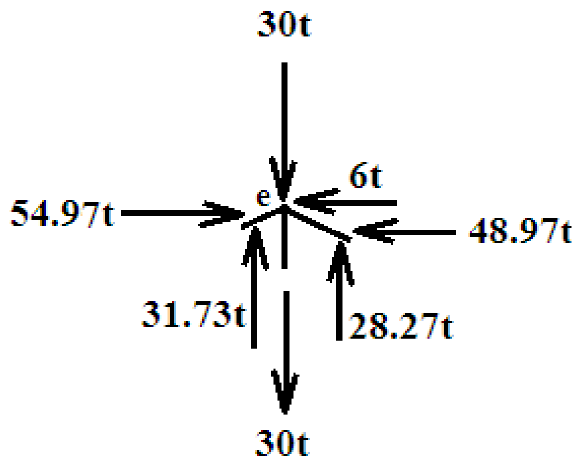
$$\sum \mathrm{F_x} = 0 \quad F_{x6} + F_{x8} = 74.95\mathrm{t}$$

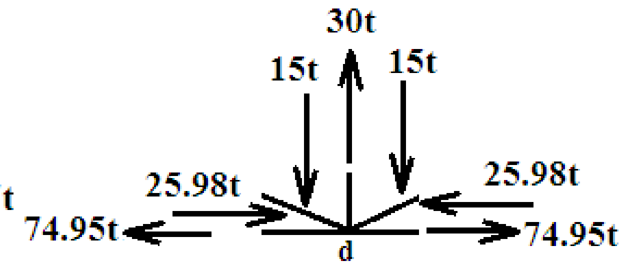
$$F_{x8} = \frac{F_{y8}}{tan30}$$

$$F_{y6} = 28.27t \downarrow \quad F_{y8} = 15.00t \uparrow$$

$$F_{x6} = 48.97t \rightarrow \quad F_{x8} = 25.98t \rightarrow$$

Joint#f





$\sum F_y = 0 \;\; F_5 = 30t \downarrow$

$\sum F_x = 0 \;\; ok$

Joint#e

$\sum F_y = 0 \;\; ok$

$\sum F_x = 0 \;\; ok$

Joint#d

Member	**1**	**2**	**3**	**4**	**5**	**6**
Force	93.46	63.45	74.94	30	30	56.54
Type	C	C	T	C	T	C

Member	**7**	**8**	**9**
Force	86.54	30	74.95
Type	C	C	T

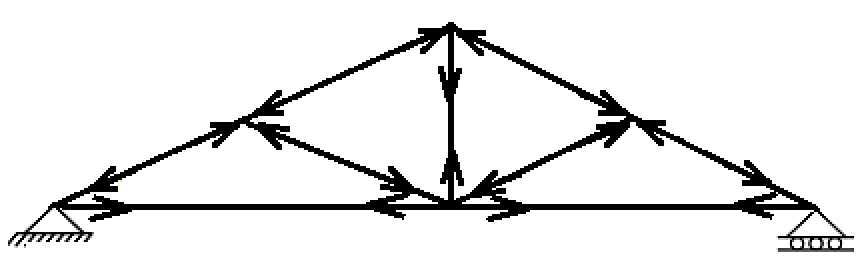


Fig. 5.13.b

Example (7)

Find analytically the forces in all members of given truss in Fig. 5.14.a by the method of joint.

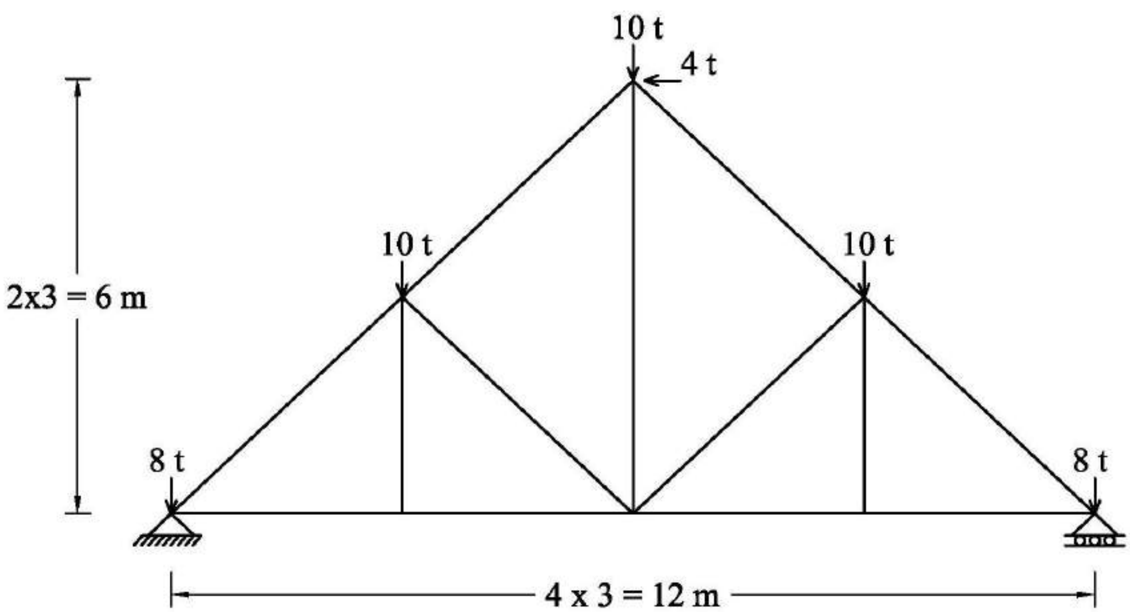


Fig. 5.14. a

Solution

<u>Finding the reactions</u>

$$\sum F_x = 0 \; X_a = 6t \rightarrow$$

$$\sum M_a = 10x3 + 10x6 + 10x9 + 8x12 - 4x6 - 12Y_h = 0$$

$$\therefore Y_h = 21 \uparrow$$

$$\sum M_h = 10x3 + 10x6 + 10x9 + 8x12 + 4x6 - 12Y_a = 0 \quad \therefore Y_a = 25t \uparrow$$

<u>*Check*</u> $\sum F_y = 0$ ok

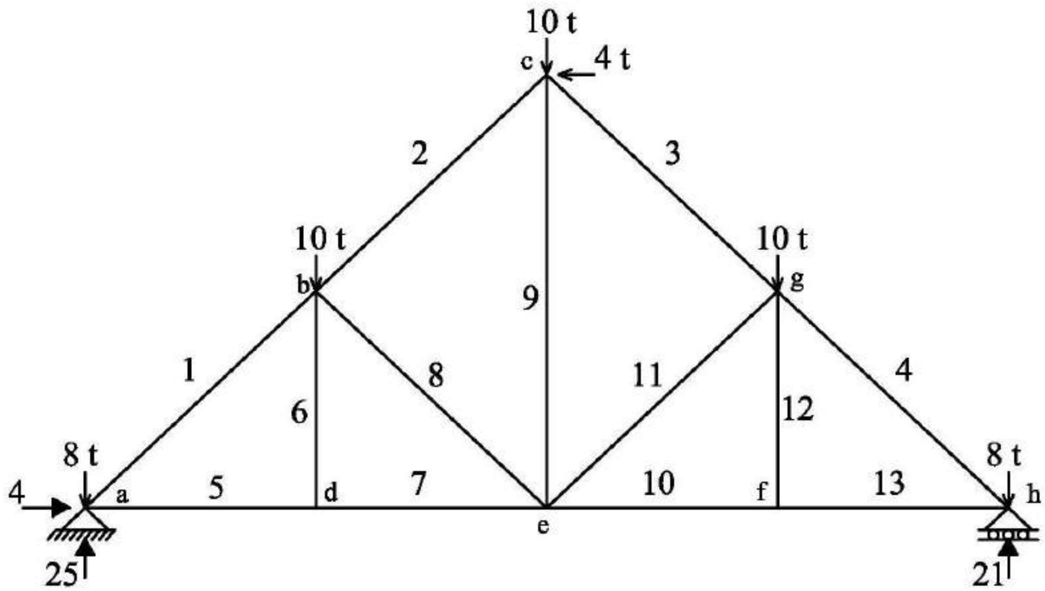
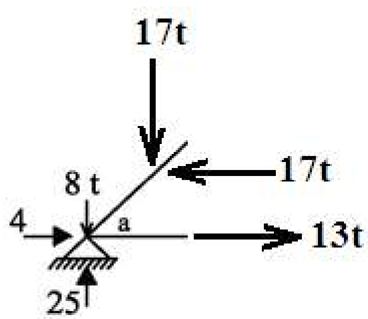


Fig. 5. 14. b

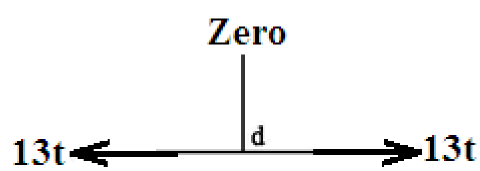


$\sum F_y = 0 \;\; F_{y1} = 17t \downarrow$

$F_{x5} = F_{y1} = 17t \leftarrow$

$\sum F_x = 0 \;\; F_5 = 13t \rightarrow$

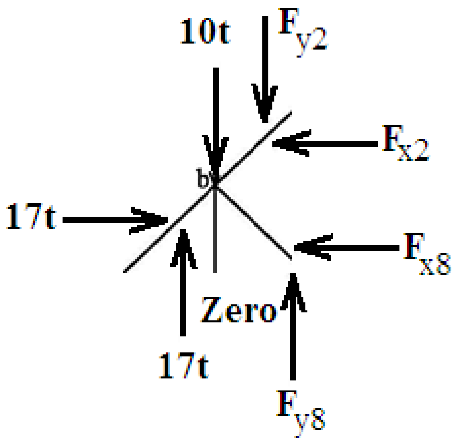
Joint#a



$\sum F_y = 0 \;\; F_6 = \text{zero}$

$\sum F_x = 0 \;\; F_7 = 13t \rightarrow$

Joint#d



$\sum F_y = 0 \;\; F_{y2} - F_{y8} = 7$

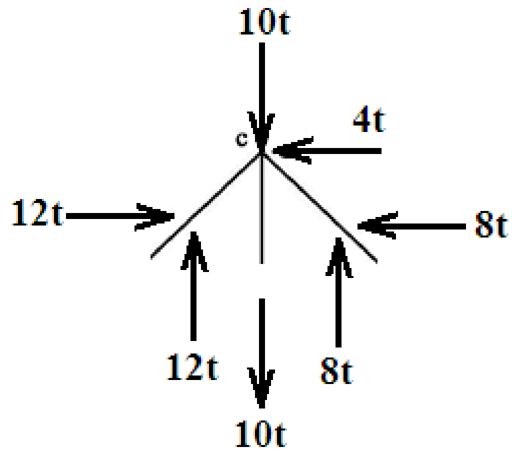
$F_{x2} = F_{y2} \; and \; F_{x8} = F_{y8}$

$\sum F_x = 0 \;\; F_{x2} + F_{x8} = 17$

$F_{x2} = 12t \leftarrow F_{y2} = 12t \downarrow$

$F_{x8} = 5 \leftarrow \;\; F_{y8} = 5t \uparrow$

Joint#b

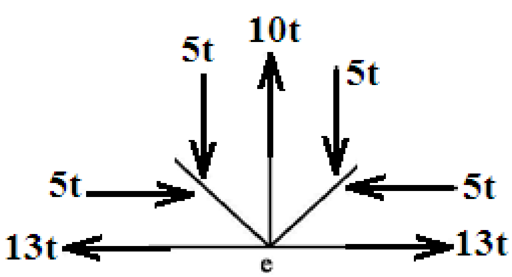


$\sum F_x = 0 \;\; F_{x3} = 8t \uparrow$

$F_{x3} = F_{y3} = 8t \leftarrow$

$\sum F_y = 0 \;\; F_9 = 10t \downarrow$

Joint#c

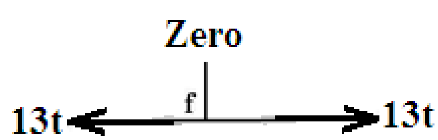


$\sum F_y = 0 \;\; F_{y11} = 5t \downarrow$

$F_{x11} = 5t \leftarrow$

$\sum F_x = 0 \;\; F_{10} = 13t \rightarrow$

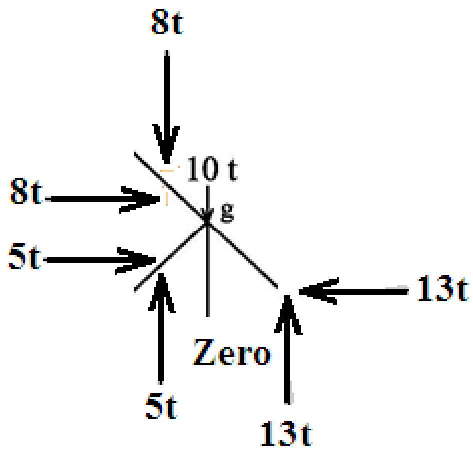
Joint#e

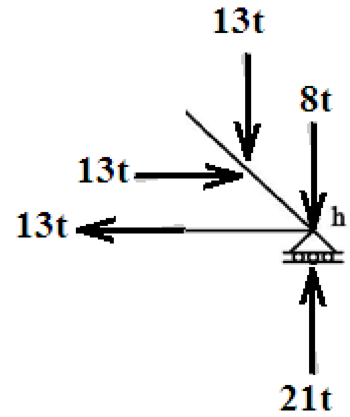


$\sum F_y = 0 \;\; F_{y12} = zero$

$\sum F_x = 0 \;\; F_{13} = 13t \rightarrow$

Joint#f





$\sum F_y = 0 \;\; F_{y4} = 13\text{t} \uparrow$

$F_{x4} = F_{y4} = 13t \leftarrow$

$\sum F_x = 0$ ok

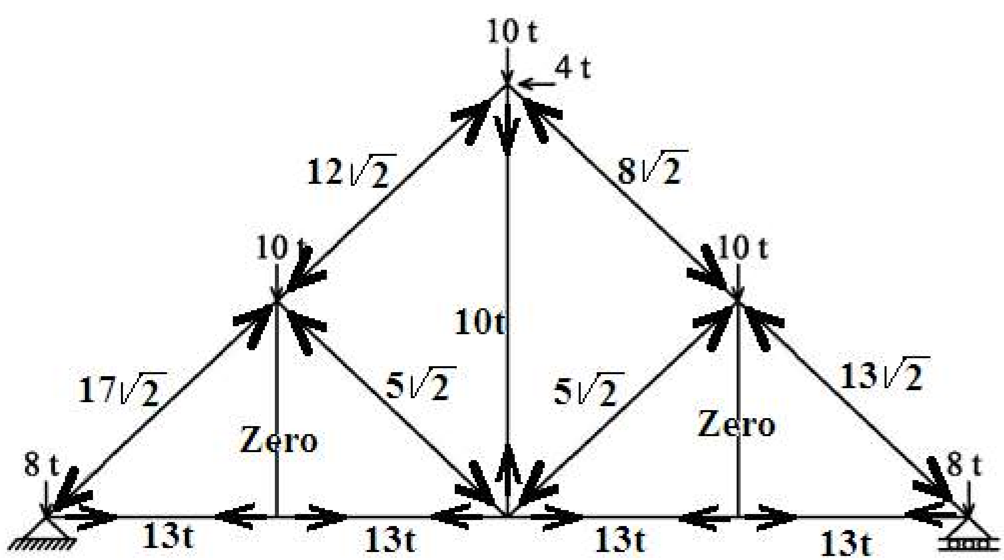
Joint#g

$\sum F_y = 0$ ok

$\sum F_x = 0$ ok

Joint#h

Member	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**
Force	$17\sqrt{2}$	$12\sqrt{2}$	$8\sqrt{2}$	$13\sqrt{2}$	13	0	13	$5\sqrt{2}$	10	13	$5\sqrt{2}$	0	13
Type	C	C	C	C	T	….	T	C	T	T	C	….	T



Example (8)

Find analytically the forces in all members of given truss in Fig. 5.15. a by the method of joint.

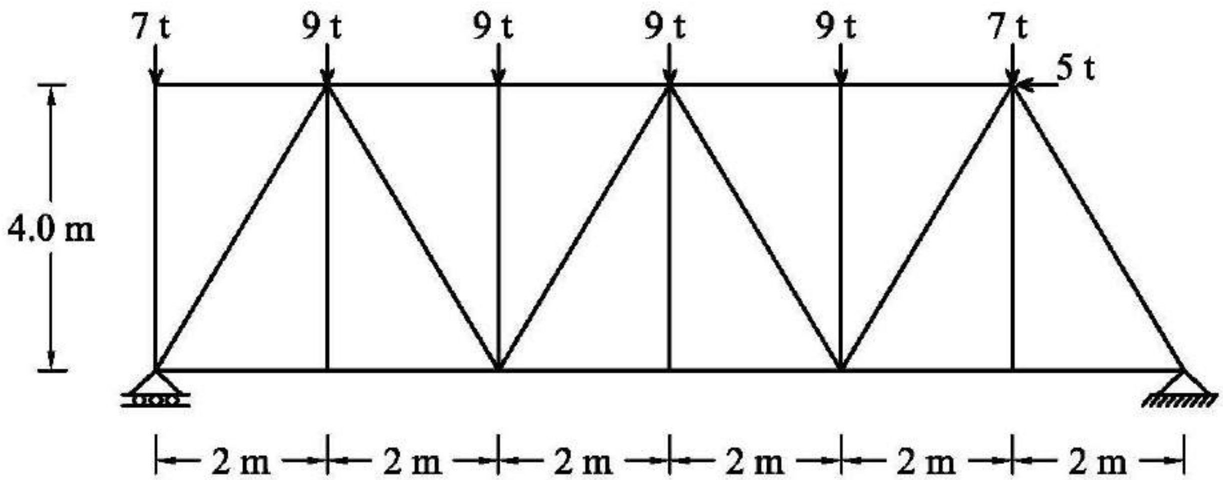


Fig. 5.15.a

Solution

<u>*Finding the reactions*</u>

$$\sum F_x = 0 \quad X_o = 5t \rightarrow$$

$$\sum M_g = 9x2 + 9x4 + 9x6 + 9x8 + 7x10 - 5x4 - 12Y_o = 0$$

$$\therefore Y_o = 19.17 \uparrow$$

$$\sum M_o = 7x2 + 9x4 + 9x6 + 9x8 + 9x10 + 7x12 + 5x4 - 12Y_g = 0$$

$$\therefore Y_g = 30.83 \uparrow$$

<u>*Check*</u> $\sum F_y = 0$ ok

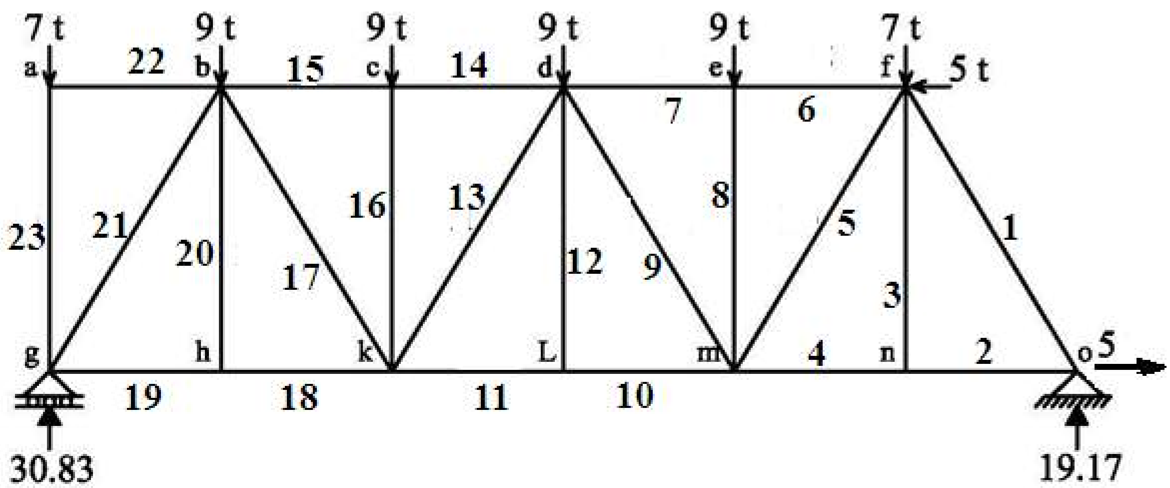
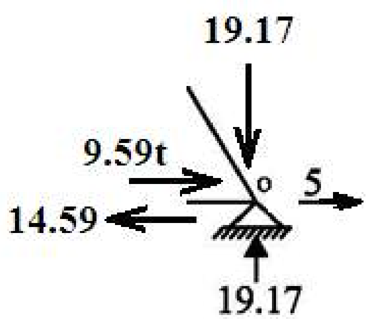


Fig. 5.15.b

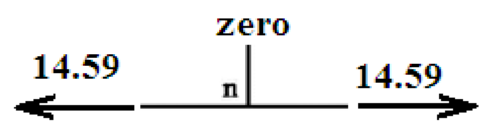


$\sum F_y = 0 \quad F_{y1} = 19.17t \downarrow$

$F_{x1} = 0.5F_{y1} = 9.59t \rightarrow$

$\sum F_x = 0 \quad F_2 = 14.59t \leftarrow$

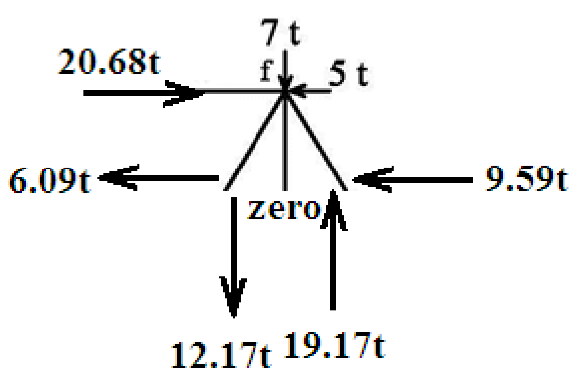
Joint#o



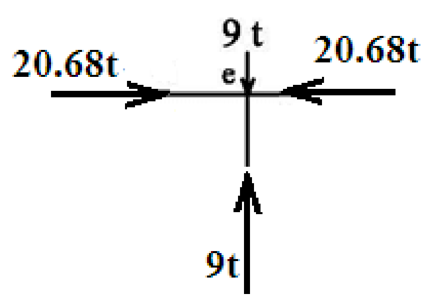
$\sum F_y = 0 \quad F_{y3} = 0$

$\sum F_x = 0 \quad F_4 = 14.59t \leftarrow$

Joint#n



$\sum F_y = 0 \quad F_{y5} = 12.17t \downarrow$

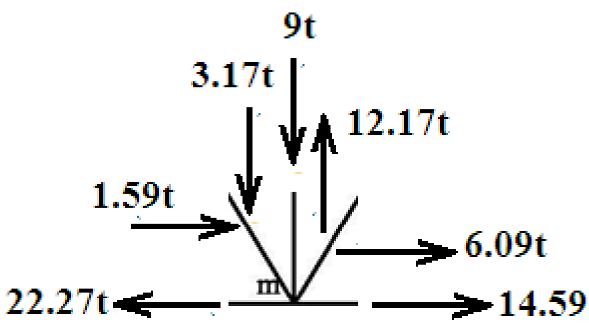


$\sum F_y = 0 \quad F_8 = 9t \uparrow$

$F_{x5} = 0.5F_{y5} = 6.09t \downarrow$

$\sum F_x = 0 \;\; F_6 = 20.68 \rightarrow$

Joint#f

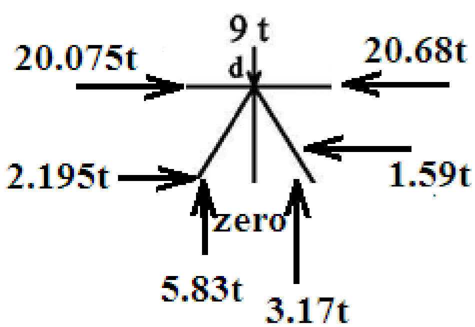


$\sum F_y = 0 \;\; F_{y9} = 3.17t \downarrow$

$F_{x5} = 0.5F_{y9} = 1.59t \rightarrow$

$\sum F_x = 0 \;\; F_{10} = 22.27t \leftarrow$

Joint#m

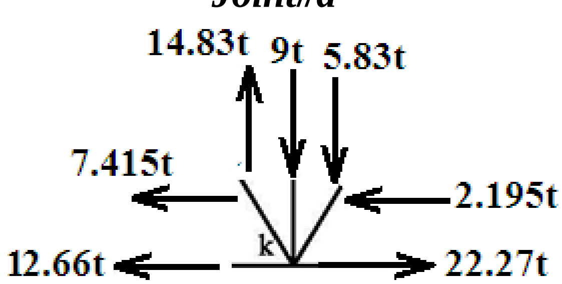


$\sum F_y = 0 \;\; F_{y13} = 5.83t \uparrow$

$F_{x5} = 0.5F_{y13} = 2.195t \rightarrow$

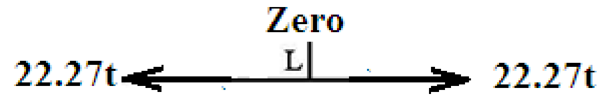
$\sum F_x = 0 \;\; F_{14} = 20.075t \rightarrow$

Joint#d



$\sum F_x = 0 \;\; F_7 = 20.68t \leftarrow$

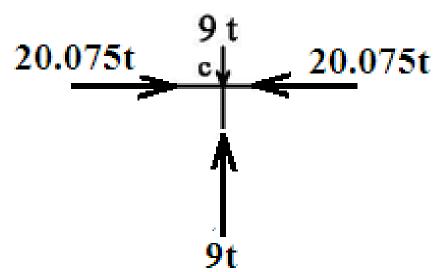
Joint#e



$\sum F_y = 0 \;\; F_{12} = 0$

$\sum F_x = 0 \;\; F_{11} = 22.27t \leftarrow$

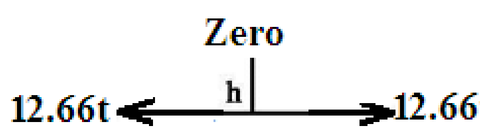
Joint#L



$\sum F_y = 0 \;\; F_{16} = 9t \uparrow$

$\sum F_x = 0 \;\; F_{15} = 20.075t \rightarrow$

Joint#c

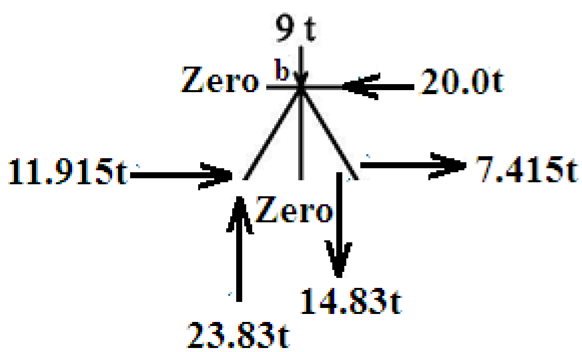


$\sum F_y = 0 \;\; F_{y17} = 14.83t \uparrow$

$F_{x5} = 0.5F_{y17} = 7.415t \leftarrow$

$\sum F_x = 0 \;\; F_{18} = 12.66t \leftarrow$

Joint#k



$\sum F_y = 0 \;\; F_{y21} = 23.83t \uparrow$

$F_{x5} = 0.5F_{y21} = 11.915t \rightarrow$

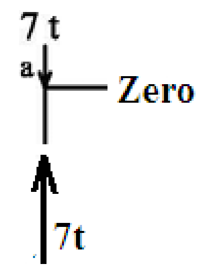
$\sum F_x = 0 \;\; F_{22} = zero$

Joint#b

$\sum F_y = 0 \;\; F_{20} = zero$

$\sum F_x = 0 \;\; F_{19} = 12.66t \leftarrow$

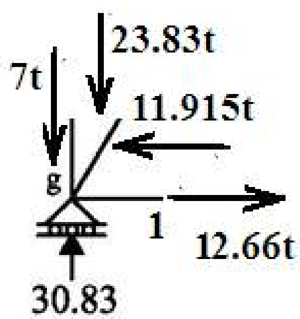
Joint#h



$\sum F_y = 0 \;\; F_{23} = 7t \uparrow$

$\sum F_x = 0 \;\; ok$

Joint#a



$\sum F_y = 0 \;\; ok$

$\sum F_x = 0 \;\; ok$

Joint#g

Member	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Force	21.43	14.59	zero	14.59	13.61	20.68	20.68	9	3.55	22.27
Type	C	T	---	T	T	C	T	C	C	T

Member	11	12	13	14	15	16	17	18	19	20
Force	22.27	0	6.23	20.07	20.075	9	16.58	12.66	12.66	zero
Type	T	----	C	C	C	C	T	T	T	----
Member	21	22	23							
Force	26.64	zero	7							
Type	C	---	C							

<u>Example (9)</u>

Find analytically the forces in all members of given truss in Fig. 5.16. a by the method of joint.

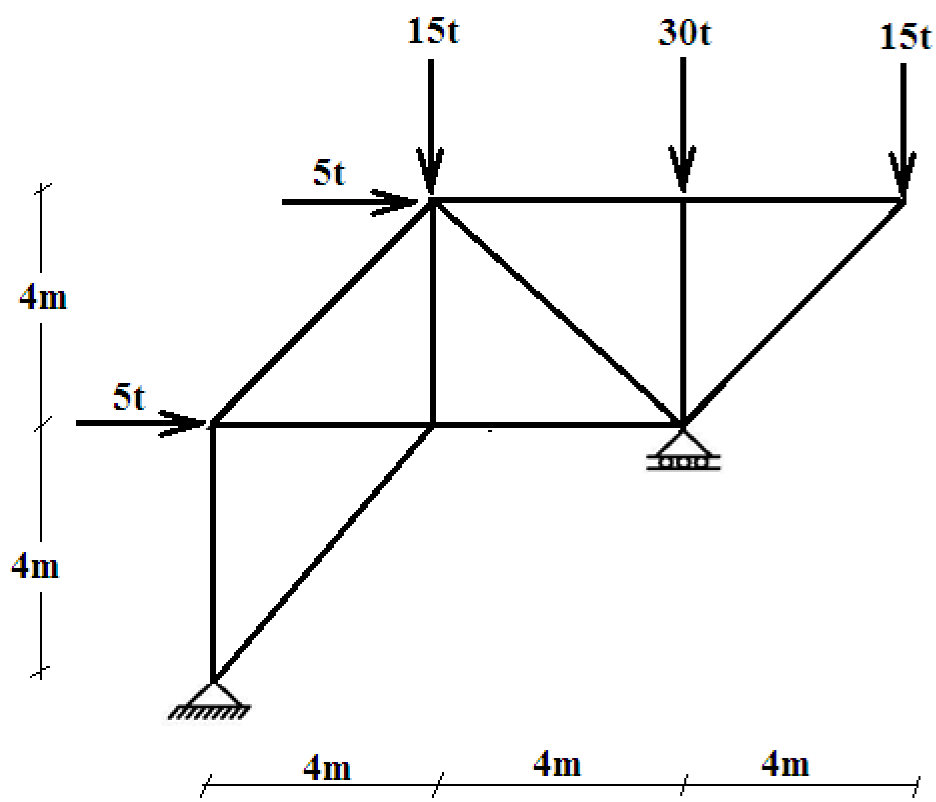


Fig. 5.16.a

Solution

Finding the reactions

$\sum F_x = 0 \; X_a = 10t \leftarrow$

$\sum M_a = 5x4 + 5x8 + 15x4 + 30x8 + 15x12 - 8Y_b = 0$

$$\therefore Y_b = 67.5 \uparrow$$

$\sum M_b = -15x4 + 15x4 - 5x4 - 10x4 - 8Y_a = 0$

$$\therefore Y_a = 7.5 \downarrow$$

Check $\sum F_y = 0$ ok

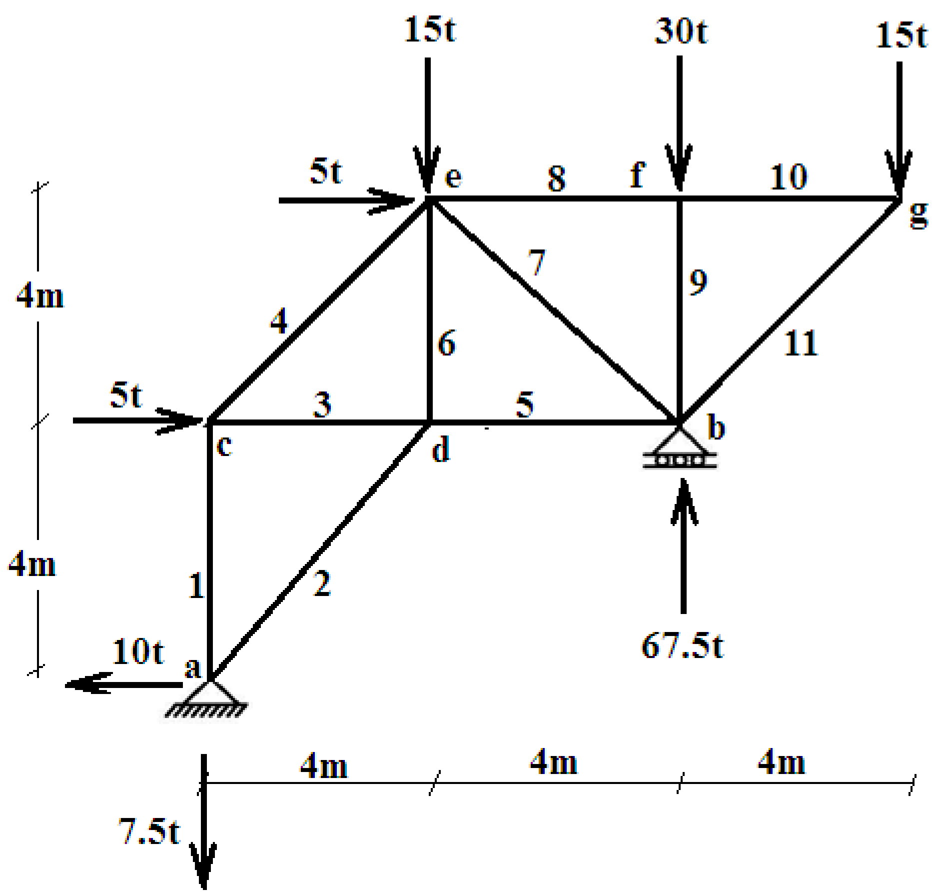
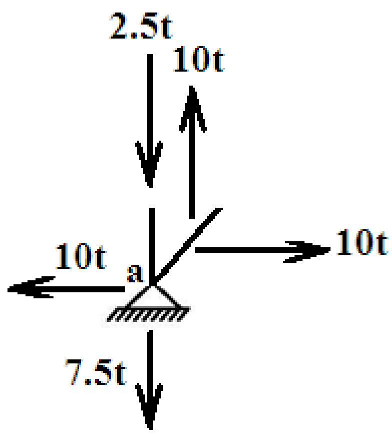


Fig. 5. 16.b

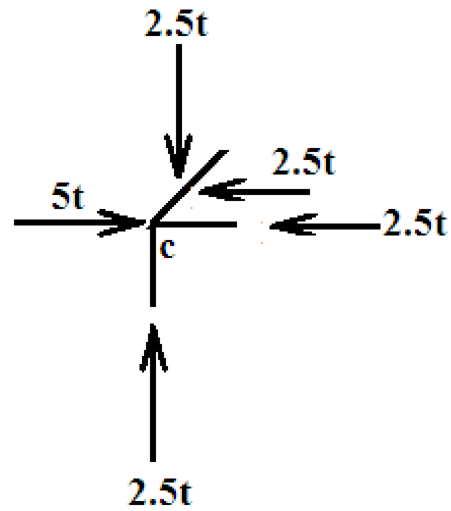


$\sum F_x = 0 \;\; F_{x2} = 10 \rightarrow$

$F_{x2} = F_{y2} = 10t \uparrow$

$\sum F_y = 0 \;\; F_1 = 2.5t \downarrow$

Joint (a)

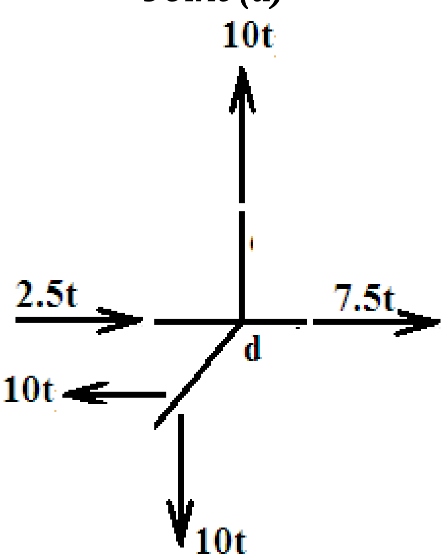


$\sum F_y = 0 \;\; F_{y4} = 2.5t \downarrow$

$F_{x4} = F_{x4} = 2.5t \leftarrow$

$\sum F_x = 0 \;\; F_3 = 2.5t \leftarrow$

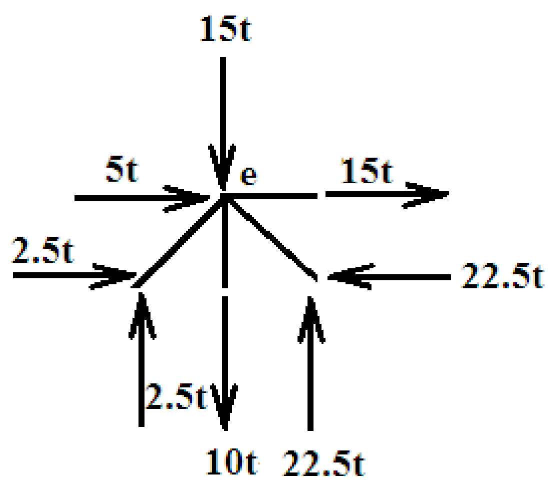
Joint (c)



$\sum F_y = 0 \;\; F_6 = 10t \uparrow$

$\sum F_x = 0 \;\; F_5 = 7.5t \rightarrow$

Joint (d)

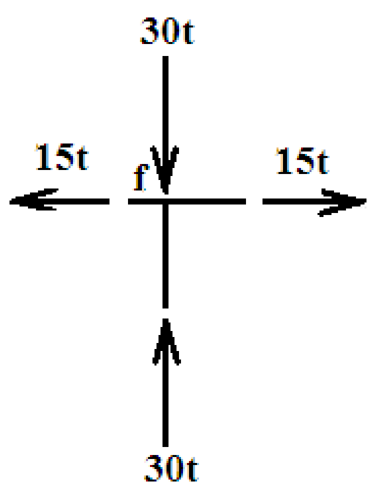


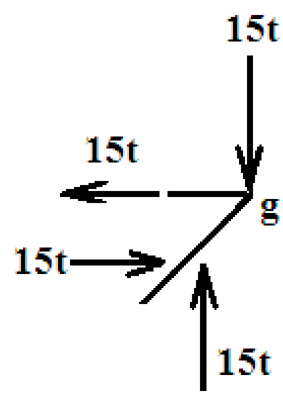
$\sum F_y = 0 \;\; F_{y7} = 22.5t \uparrow$

$F_{x7} = F_{x7} = 22.5t \leftarrow$

$\sum F_x = 0 \;\; F_8 = 15t \rightarrow$

Joint (e)





$\sum F_y = 0 \;\; F_9 = 30t \uparrow$

$\sum F_x = 0 \;\; F_{10} = 15t \rightarrow$

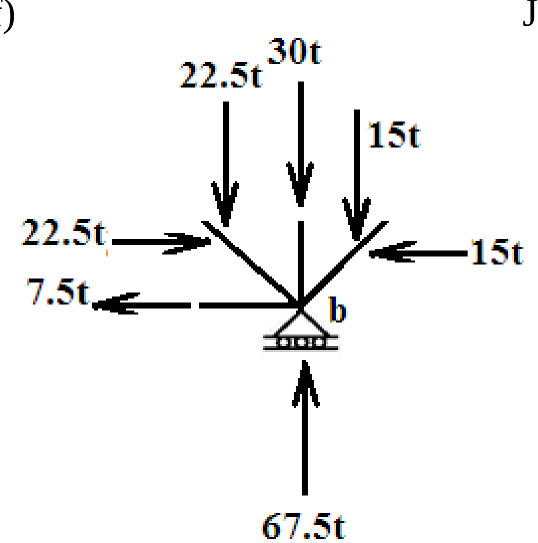
Joint (f)

$\sum F_y = 0 \;\; F_{y11} = 15t \uparrow$

$F_{x11} = F_{y11} = 15t \rightarrow$

$\sum F_x = 0 \;\;$ ok

Joint (g)



$\sum F_y = 0 \;\;$ ok

$\sum F_x = 0 \;\;$ ok

Joint (b)

Member	1	2	3	4	5	6	7	8	9
Force	2.5	14.14	2.5	3.536	7.5	10	31.82	15	30
Type	C	T	C	C	T	T	C	T	C

Member	10	11
Force	15	21.21
Type	T	C

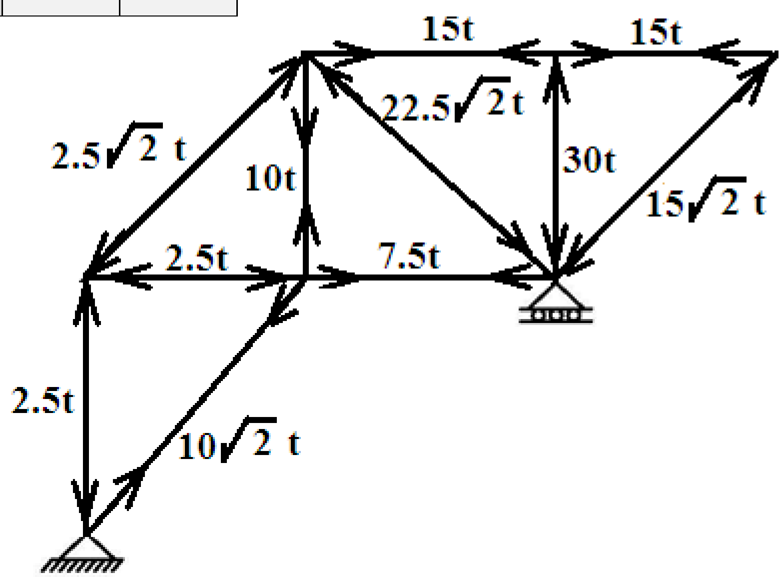


Fig. 5.16.a

Example (10)

Find analytically the forces in all members of given truss in Fig. 5.17.a by the method of joint.

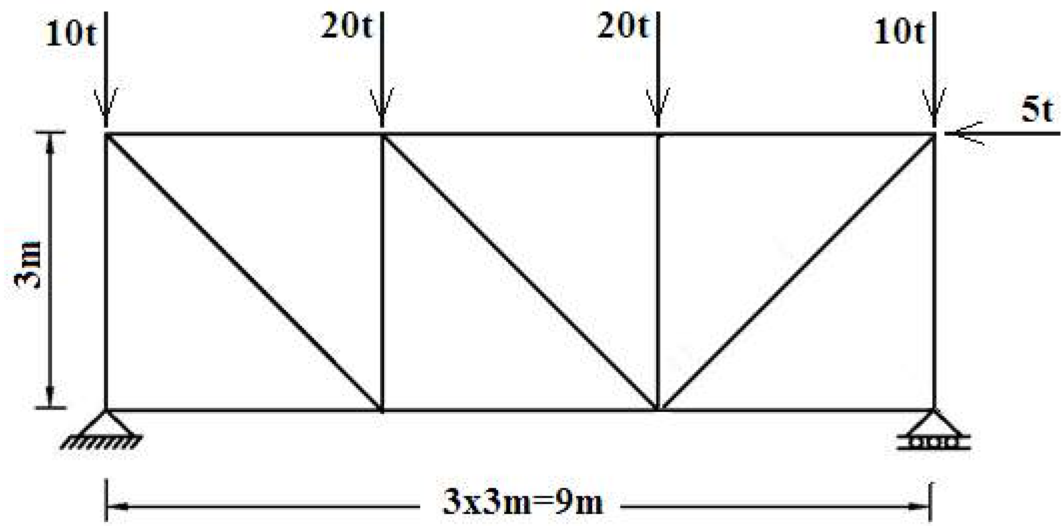


Fig. 5. 17.a

Solution

<u>*Finding the reactions*</u>

$\sum F_x = 0 \; X_a = 5t \rightarrow$

$\sum M_a = 20x3 + 20x6 + 10x9 - 5x3 - 9Y_b = 0$

$\therefore Y_b = 28.33 \uparrow$

$\sum M_b = 20x3 + 20x6 + 10x9 + 5x3 - 9Y_a = 0$

$\therefore Y_a = 31.67 \uparrow$

<u>*Check*</u> $\sum F_y = 0$ ok

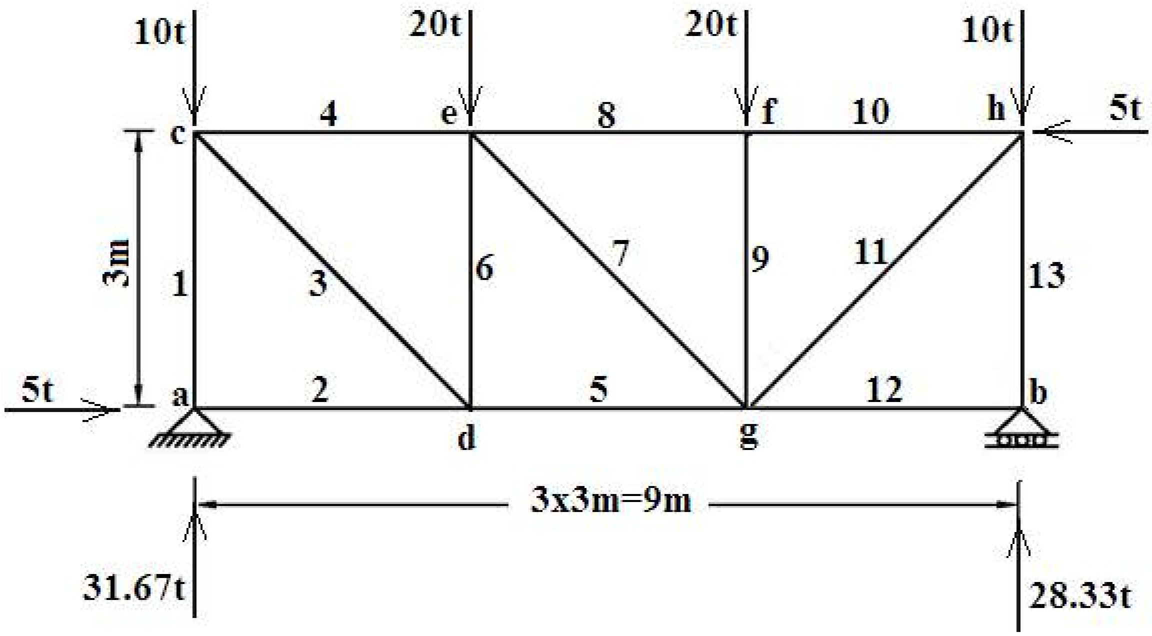
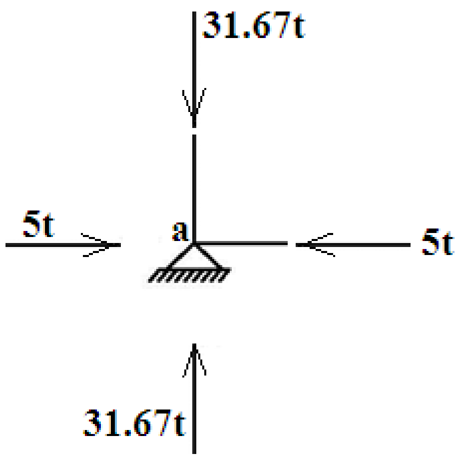
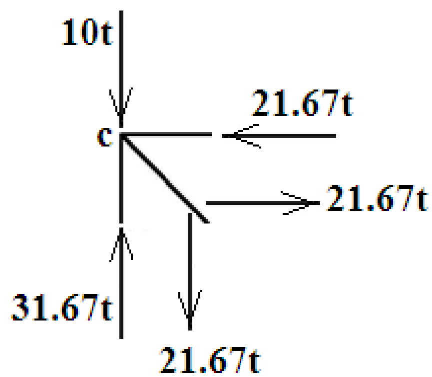


Fig. 5. 17.b





$\sum F_y = 0 \;\; F_1 = 31.67\text{t} \downarrow$

$\sum F_x = 0 \;\; F_2 = 5\text{t} \leftarrow$

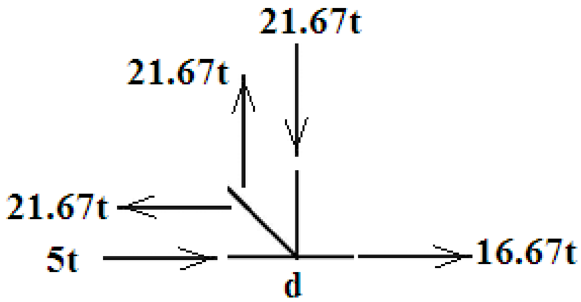
Joint (a)

$\sum F_y = 0 \;\; F_{y3} = 21.67\text{t} \downarrow$

$F_{x3} = F_{x3} = 21.67t \rightarrow$

$\sum F_x = 0 \;\; F_4 = 21.67\text{t} \leftarrow$

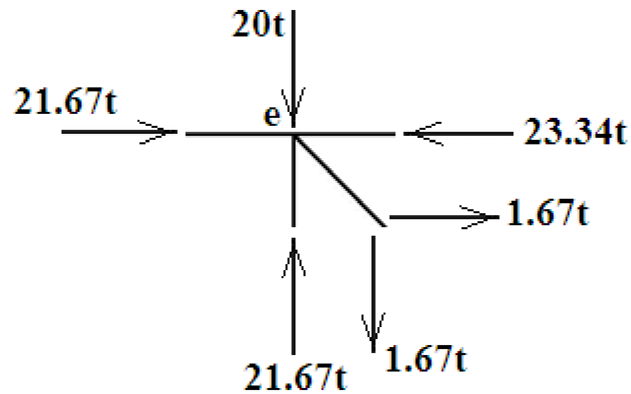
Joint (c)



$\sum F_y = 0 \;\; F_6 = 21.67t \downarrow$

$\sum F_x = 0 \;\; F_5 = 16.67t \rightarrow$

Joint (d)

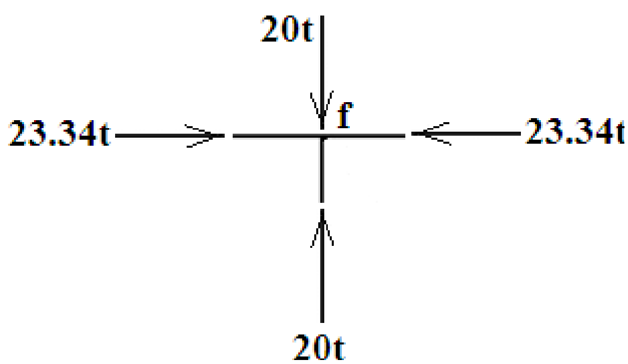


$\sum F_y = 0 \;\; F_{y7} = 1.67t \downarrow$

$F_{x7} = F_{x7} = 1.67t \rightarrow$

$\sum F_x = 0 \;\; F_8 = 23.34t \leftarrow$

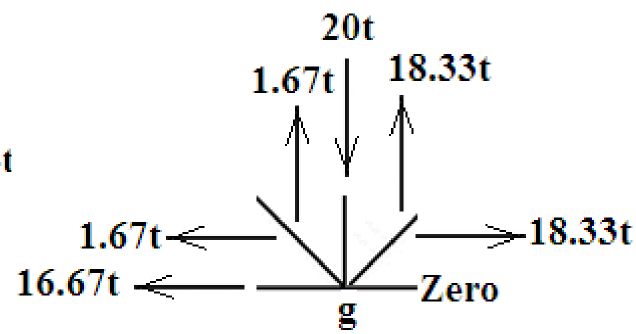
Joint (e)



$\sum F_y = 0 \;\; F_9 = 20t \uparrow$

$\sum F_x = 0 \;\; F_{10} = 23.34t \leftarrow$

Joint (f)

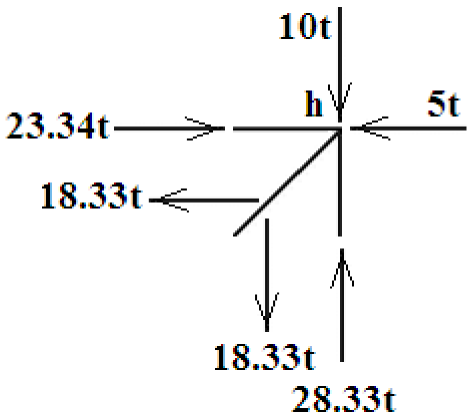


$\sum F_y = 0 \;\; F_{y11} = 18.33t \uparrow$

$F_{x11} = F_{x11} = 18.33t \rightarrow$

$\sum F_x = 0 \;\; F_{12} = \text{zero}$

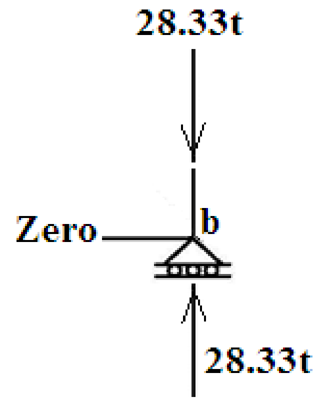
Joint (g)



$\sum F_y = 0 \;\; F_{13} = 28.33t \uparrow$

$Check \sum F_x = 0$ ok

Joint (h)



$check \sum F_y = 0$ ok

$Check \sum F_x = 0$ ok

Joint (b)

Member	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Force	31.67	5	30.65	21.67	16.67	21.67	2.36	23.34	20	23.34
Type	C	C	T	C	T	C	T	C	C	C
Member	11	12	13							
Force	25.92	0	28.33							
Type	T	----	C							

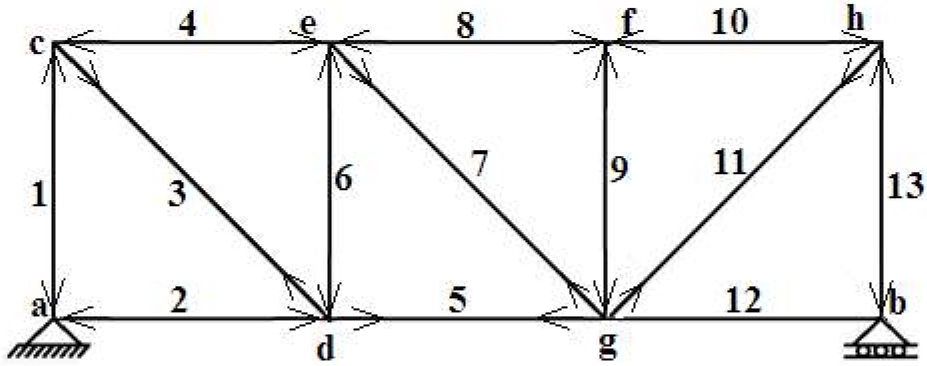


Fig. 5. 17.c

Example (11)

Find analytically the forces in all members of given truss in Fig. 5.18.a by the method of joint.

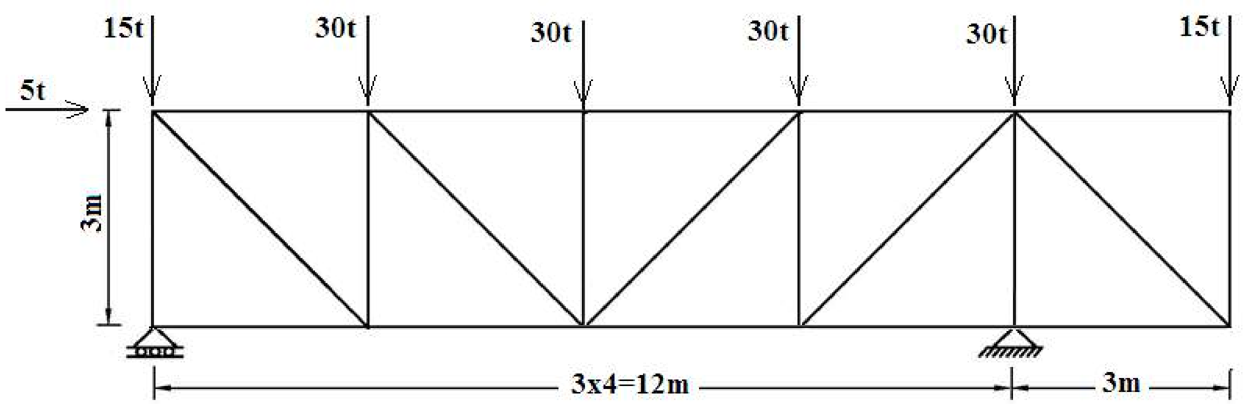


Fig. 5.18.a

Solution

Finding the reactions

$\sum F_x = 0 \; X_b = 5t \leftarrow$

$\sum M_a = 30x3 + 30x6 + 30x12 + 15x15 + 5x3 - 12Y_b = 0$

$$\therefore Y_b = 95t \uparrow$$

$\sum M_b = -15x3 + 30x3 + 30x6 + 30x9 + 15x12 - 5x3 - 12Y_a = 0$

$$\therefore Y_a = 55t \uparrow$$

Check $\sum F_y = 0$ ok

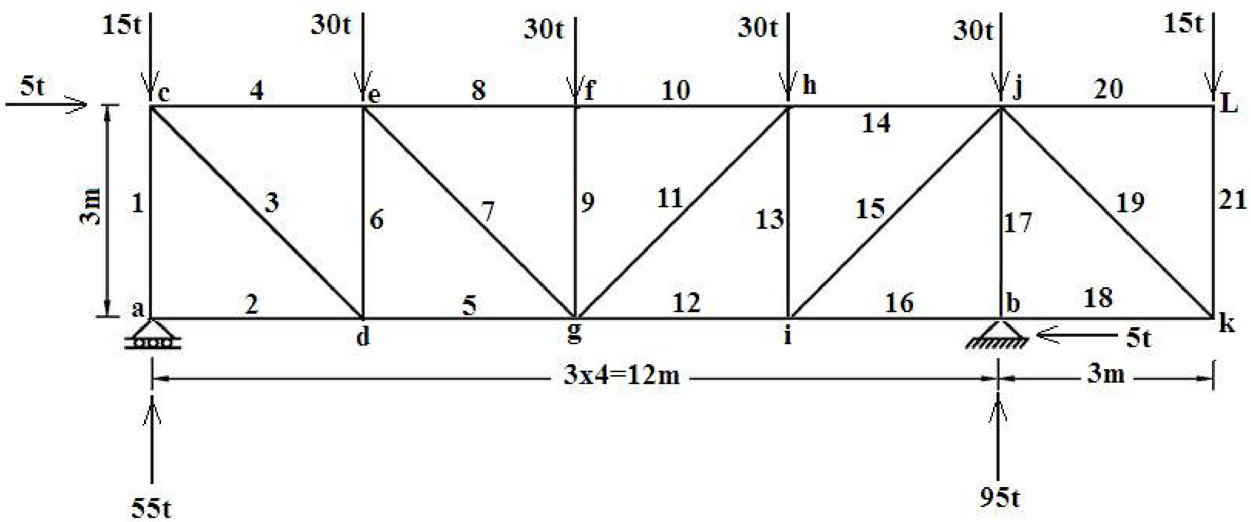
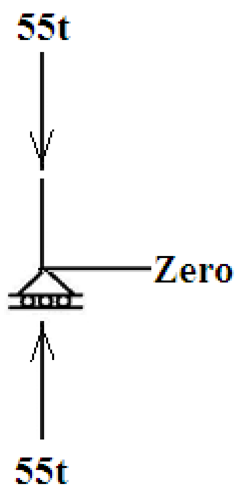


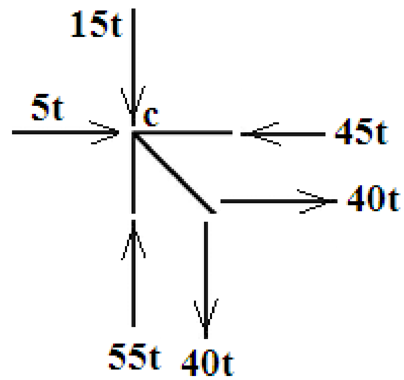
Fig. 5.18.b



$\sum F_y = 0 \;\; F_1 = 55t \uparrow$

$\sum F_x = 0 \;\; F_2 = \text{zero}$

Joint (a)

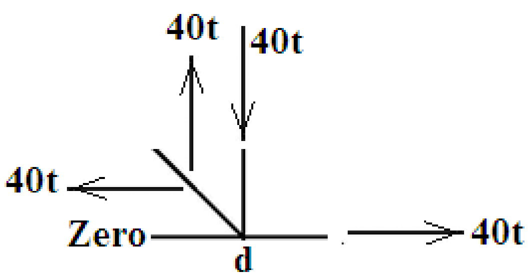


$\sum F_y = 0 \;\; F_{y3} = 40t \downarrow$

$F_{x3} = F_{x3} = 40t \rightarrow$

$\sum F_x = 0 \;\; F_4 = 45t \leftarrow$

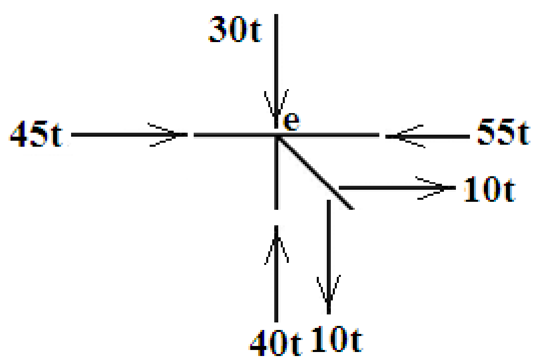
Joint (c)



$\sum F_y = 0 \;\; F_5 = 40t \downarrow$

$\sum F_x = 0 \;\; F_6 = 40t \rightarrow$

Joint (d)

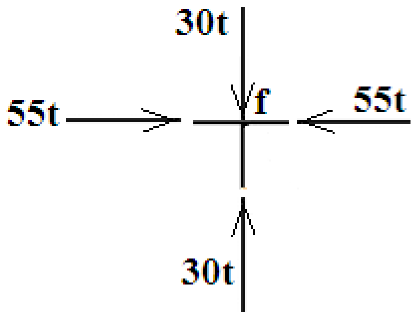


$\sum F_y = 0 \;\; F_{y7} = 10t \downarrow$

$F_{x7} = F_{x7} = 10t \rightarrow$

$\sum F_x = 0 \;\; F_8 = 55t \leftarrow$

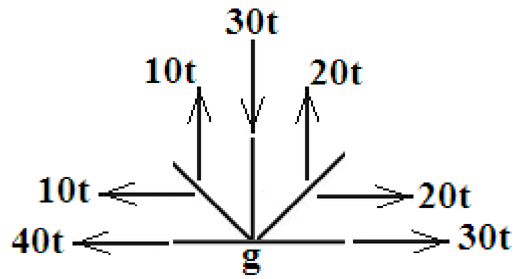
Joint (e)



$\sum F_y = 0 \quad F_9 = 30t \uparrow$

$\sum F_x = 0 \quad F_{10} = 55t \leftarrow$

Joint (f)

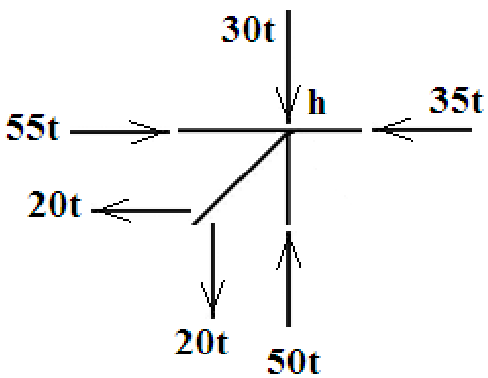


$\sum F_y = 0 \quad F_{y11} = 20t \uparrow$

$F_{x11} = F_{x11} = 20t \rightarrow$

$\sum F_x = 0 \quad F_{12} = 30t \rightarrow$

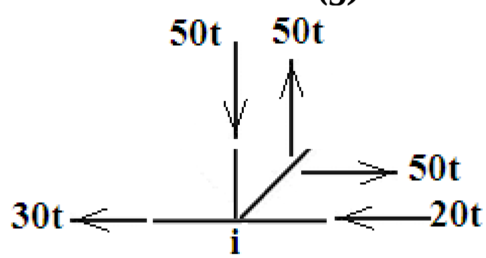
Joint (g)



$\sum F_y = 0 \quad F_{13} = 50t \uparrow$

$\sum F_x = 0 \quad F_{14} = 35t \leftarrow$

Joint (h)

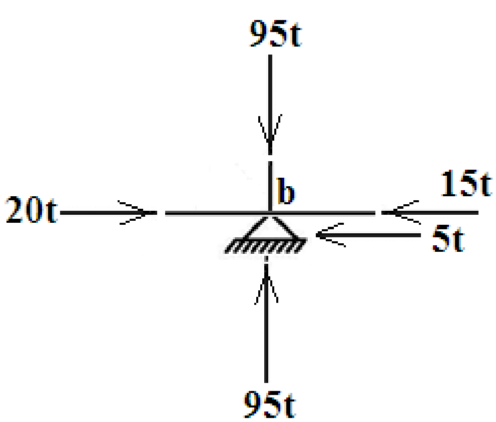


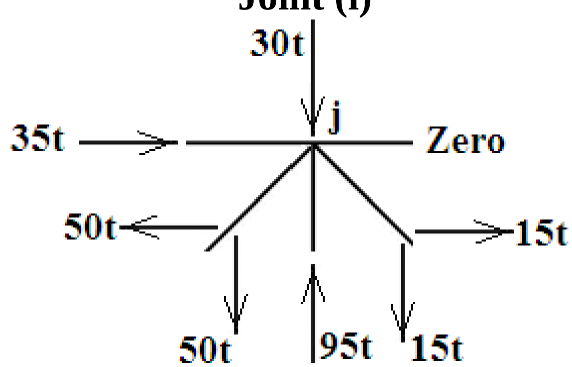
$\sum F_y = 0 \quad F_{y15} = 50t \uparrow$

$F_{x15} = F_{x15} = 50t \rightarrow$

$\sum F_x = 0 \quad F_{16} = 20t \leftarrow$

Joint (i)

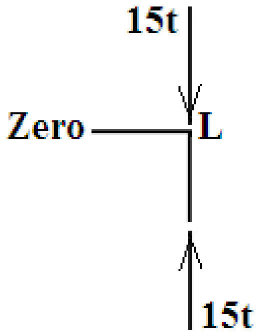




$\sum F_y = 0 \;\; F_{17} = 95t \downarrow$

$\sum F_x = 0 \;\; F_{18} = 15t \leftarrow$

Joint (b)



$\sum F_y = 0 \;\; F_{21} = 15t \uparrow$

$\text{Check} \sum F_x = 0 \;\; \text{ok}$

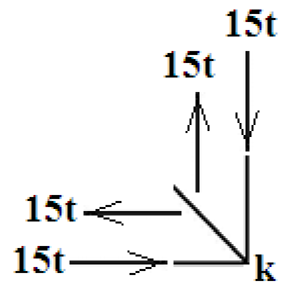
Joint (L)

$\sum F_y = 0 \;\; F_{y19} = 15t \downarrow$

$F_{x19} = F_{x19} = 15t \rightarrow$

$\sum F_x = 0 \;\; F_{20} = 0$

Joint (j)



$Check \sum F_y = 0 \;\; \text{ok}$

$\text{Check} \sum F_x = 0 \;\; \text{ok}$

Joint (k)

Member	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Force	55	Zero	56.56	45	40	40	14.14	55	30	55
Type	C	---	T	C	T	C	T	C	C	C
Member	11	12	13	14	15	16	17	18	19	20
Force	28.28	30	50	35	70.71	20	95	15	21.21	zero
Type	T	C	C	C	T	C	C	C	T	----
Member	21									
Force	15									
Type	C									

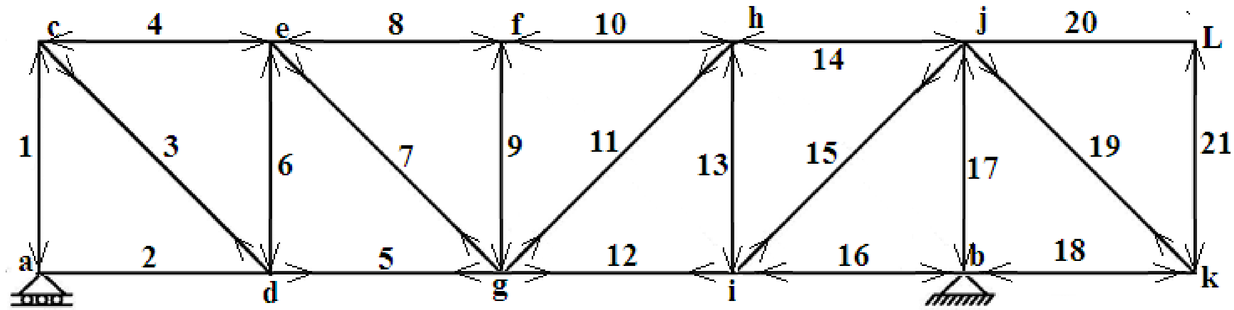


Fig. 5.18.c

Example (12)

Find analytically the forces in selected members of given truss in Fig. 5.19.a by the method of section.

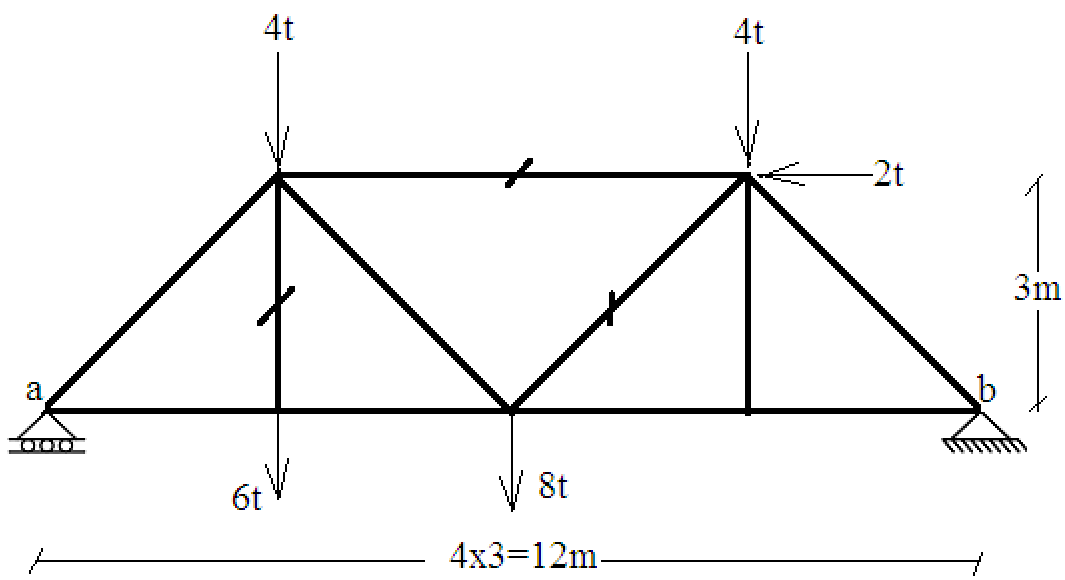


Fig. 5. 19.a

Solution

Finding the reactions

$\sum F_x = 0 \; X_b = 2t \leftarrow$

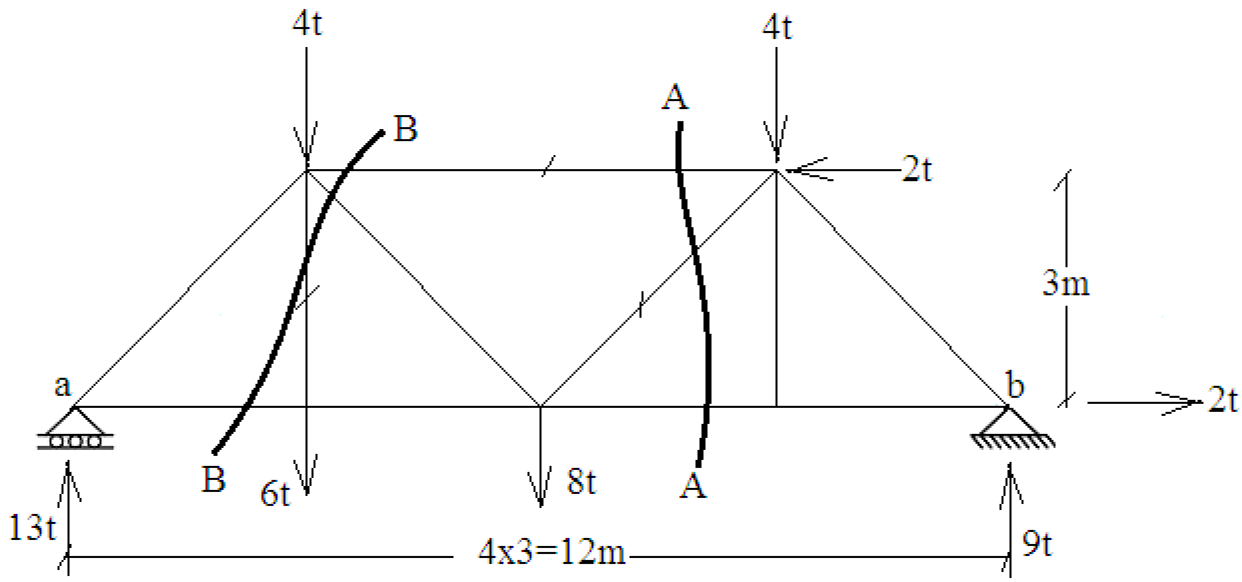
$\sum M_a = 6x3 + 4x3 + 8x6 + 4x9 - 2x3 - 12Y_b = 0$

$$\therefore Y_b = 9t \uparrow$$

$$\sum M_b = 4x3 + 8x6 + 4x9 + 6x9 + 2x3 - 12Y_a = 0$$

$$\therefore Y_a = 13t \uparrow$$

<u>*Check*</u> $\sum F_y = 0$ ok



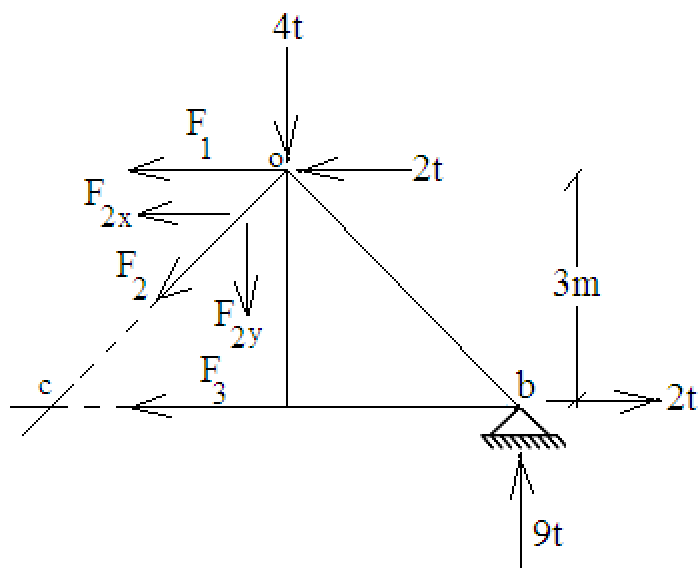
- For Sec. A-A

$\sum F_y = 0 \quad F_{2y} = 9 - 4 = 5t \downarrow$

$F_{2y} = F_{2x} \quad F_{2x} = 5t$

$F_2 = 5\sqrt{2}\ (tension)$

$\sum M_o = 9x3 + 2x3 - F_3x3 = 0$



$$F_3 = 11t \leftarrow$$

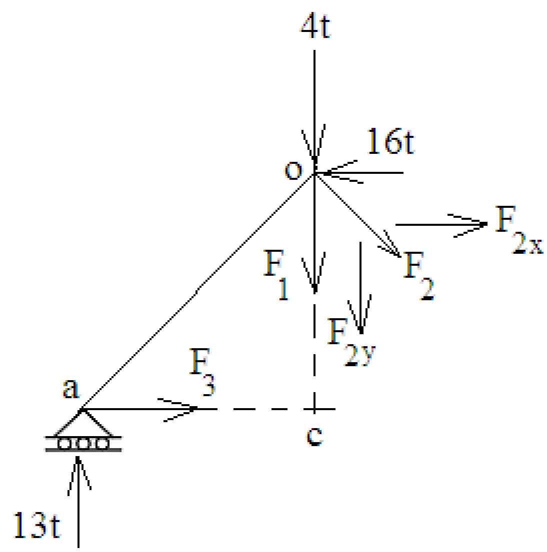
$\sum M_c = 9x6 - 4x3 + 2x3 + F_1x3 = 0$

$$F_1 = 16t \rightarrow (compresion)$$

$$\sum F_x = 0 \quad 2+5+11-2-16=0$$

- For Sec. B-B

$$\sum M_o = 13x3 - F_3x3 = 0$$



$$F_3 = 13t \leftarrow$$

$$\sum M_c = 16x3 - 13x3 - F_{2x}x3 = 0$$

$$F_{2x} = 3t \rightarrow$$

$$F_{2y} = F_{2x} \;\; F_{2y} = 3t \;\downarrow$$

$$F_2 = 3\sqrt{2}\,t$$

$$Check \sum F_x = 0 \quad 3+13-16=0$$

$$\sum F_y = 0 \qquad F_1 = 13-4-3 = 6t \uparrow (tension)$$

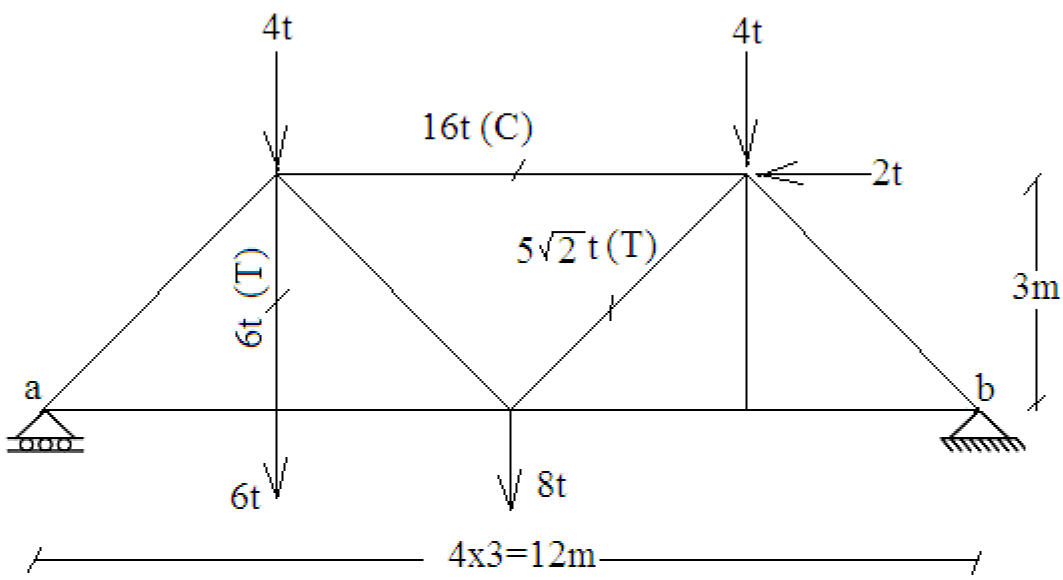


Fig. 5.19.b

Example (13)

Find analytically the forces in selected members of given truss in Fig. 5.20.a by the method of section.

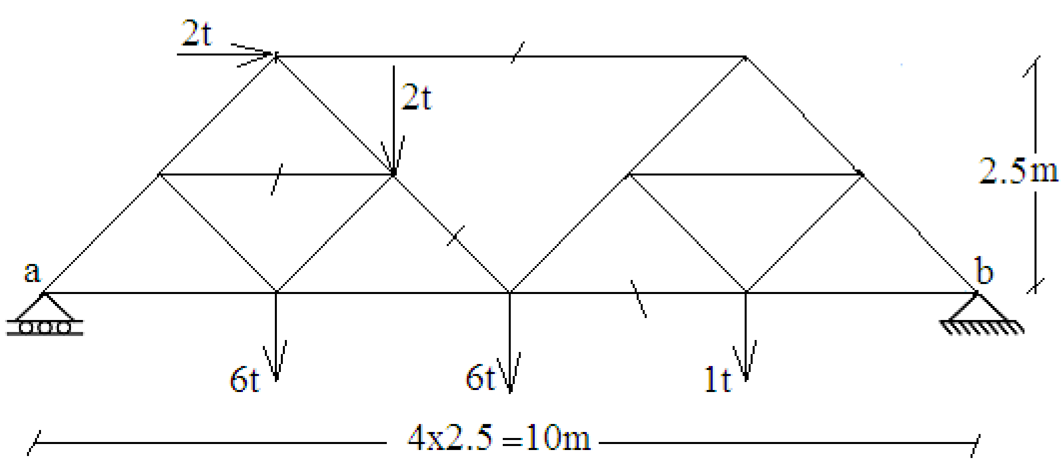


Fig. 5.20.a

Solution

Finding the reactions

$\sum F_x = 0 \; X_b = 2t \leftarrow$

$$\sum M_a = 6x2.5 + 6x5 + 1x7.5 + 2x3.75 + 2x2.5 - 10Y_b = 0$$

$$\therefore Y_b = 6.5t \uparrow$$

$$\sum M_b = 6x7.5 + 6x5 + 1x2.5 + 2x6.25 - 2x2.5 - 10Y_a = 0$$

$$\therefore Y_a = 8.5t \uparrow$$

<u>*Check*</u> $\sum F_y = 0$ ok

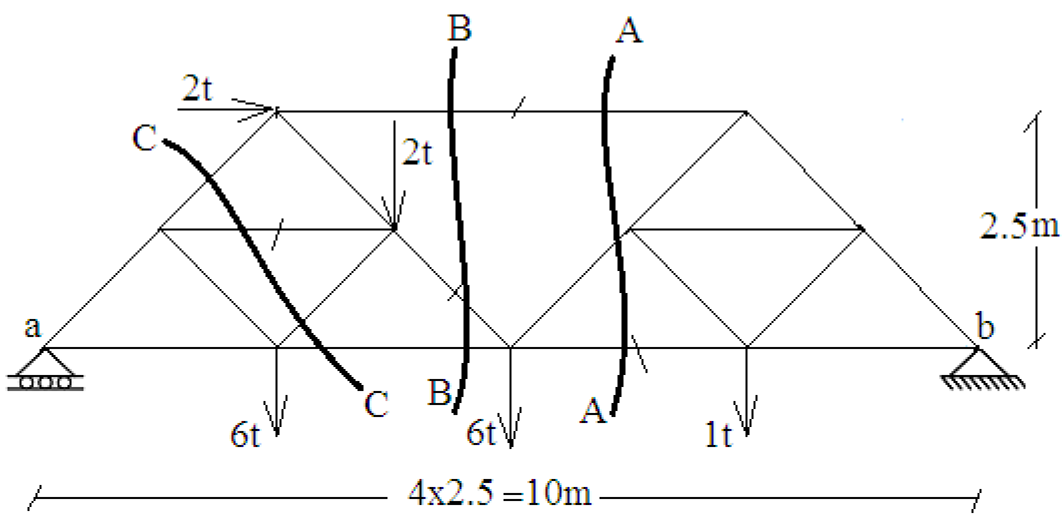
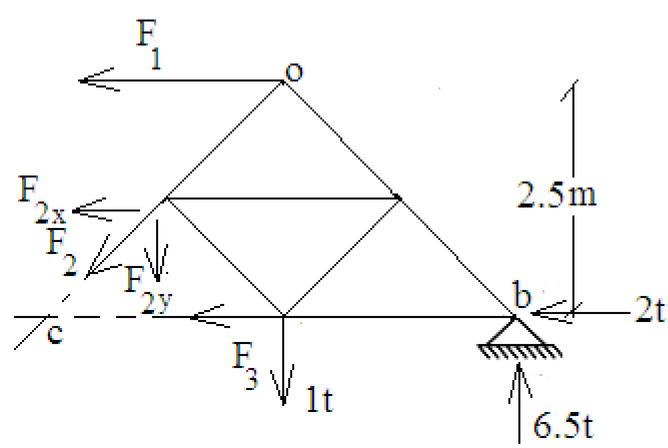


Fig. 5.20.b

➢ For Sec. A-A

$\sum F_y = 0 \quad F_{2y} = 6.5 - 1 = 5.5t \downarrow$



$F_{2y} = F_{2x} \;\; F_{2x} = 5.5t$

$F_2 = 5.5\sqrt{2}$

$\sum M_c = 1x2.5 - 6.5x5 - F_1x2.5 = 0$

$$F_1 = 12t \rightarrow (compresion)$$

$\sum M_o = 6.5x2.5 - 2x2.5 - F_3x2.5 = 0$

$$F_3 = 4.5t \leftarrow (tension)$$

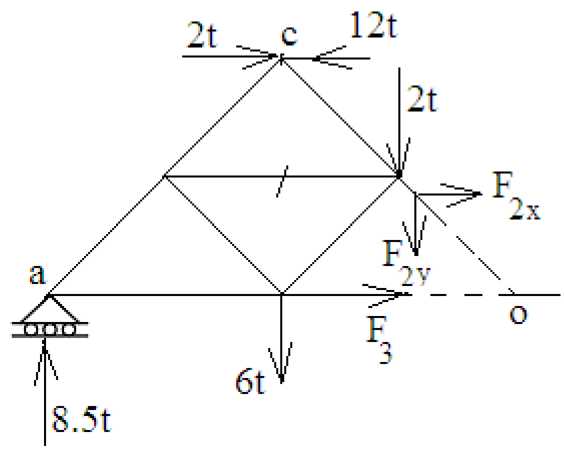
$$check \sum F_x = 0 \quad 2 + 4.5 + 5.5 - 12 = 0$$

- <u>For Sec. B-B</u>

$\sum M_c = 8.5x2.5 + 2x1.25 - F_3x2.5 = 0$

$F_3 = 9.5t \rightarrow (tension)$

$check \sum M_0 = 10x2.5 - 8.5x5 + 2x1.25 + 6x2.5 = 0 \; ok$



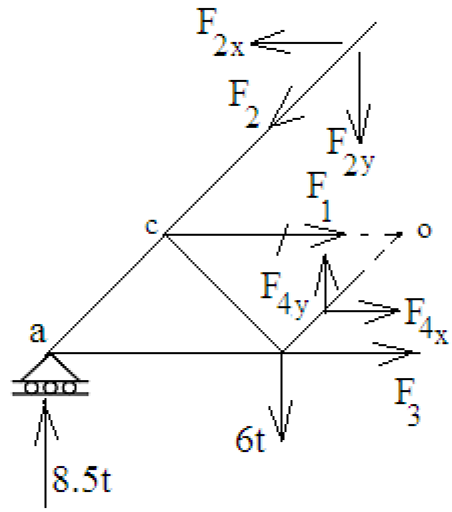
$\sum F_y = 0 \qquad F_{2y} = 8.5 - 2 - 6 = o.5t \downarrow$

$F_{2y} = F_{2x} \;\; F_{2x} = 0.5t \;\rightarrow$

$F_2 = 0.5\sqrt{2} \;\; (tension)$

$check \sum F_x = 0 \quad 1\,2 - 2 - 9.5 - 0.5 = 0 \;\; ok$

- <u>For Sec. C-C</u>



$\sum M_c = 8.5x1.25 + 6x1.25 - 9.5x1.25 - F_4x1.7677 = 0$

$F_4 = 5/\sqrt{2}\,(tension)$

$\sum M_0 = 8.5x3.75 - 6x1.25 - 9.5x1.25 - F_2x1.25\sqrt{2} = 0$

$F_2 \;= 5\sqrt{2}(comp)$

$check \sum F_y \;= 8.5 - 6 + 2.5 - 5 = 0 \; ok$

$\sum F_x = 0 \quad 9.5 + 2.5 - 5 + F_1 = 0$

$F_1 = 7t \leftarrow (compresion)$

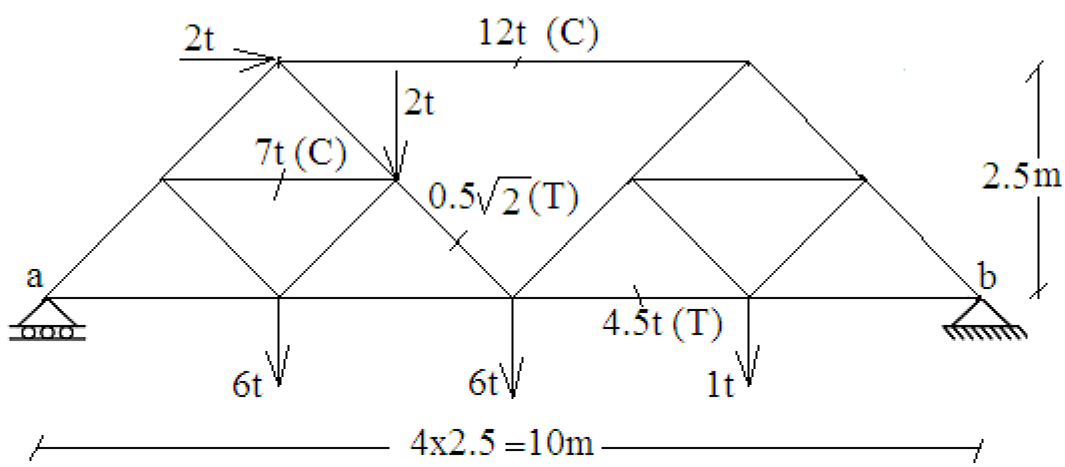


Fig. 5.20.b

Example (13)

Find analytically the forces in selected members of given truss in Fig. 5.21.a by the method of section

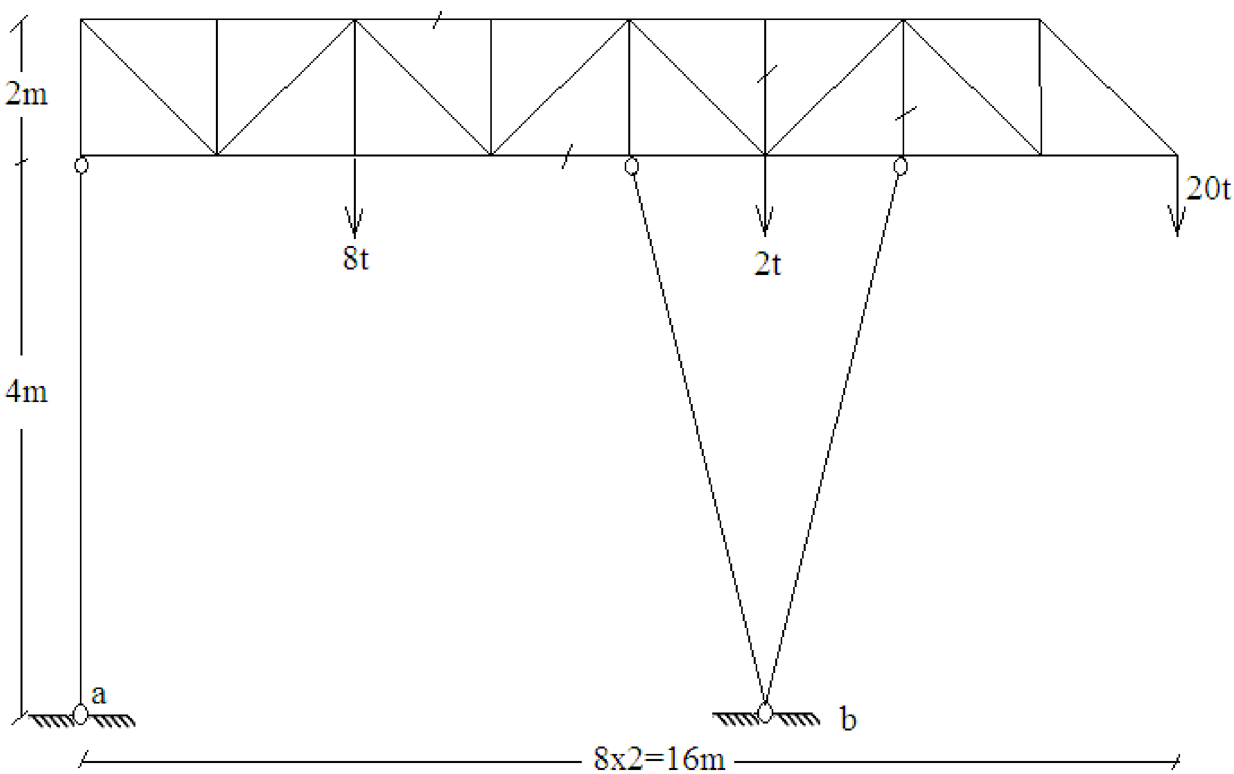


Fig. 5.21.a

Solution

Finding the reactions

$$\sum F_x = 0 \quad X_b = 0$$

$$\sum M_a = 8x4 + 2x10 + 20x16 - 10Y_b = 0$$

$$\therefore Y_b = 37.2t \uparrow$$

$$\sum M_b = 8x6 - 20x6 - 10Y_a = 0$$

$$\therefore Y_a = 7.2t \downarrow$$

Check $\sum F_y = 0$ ok

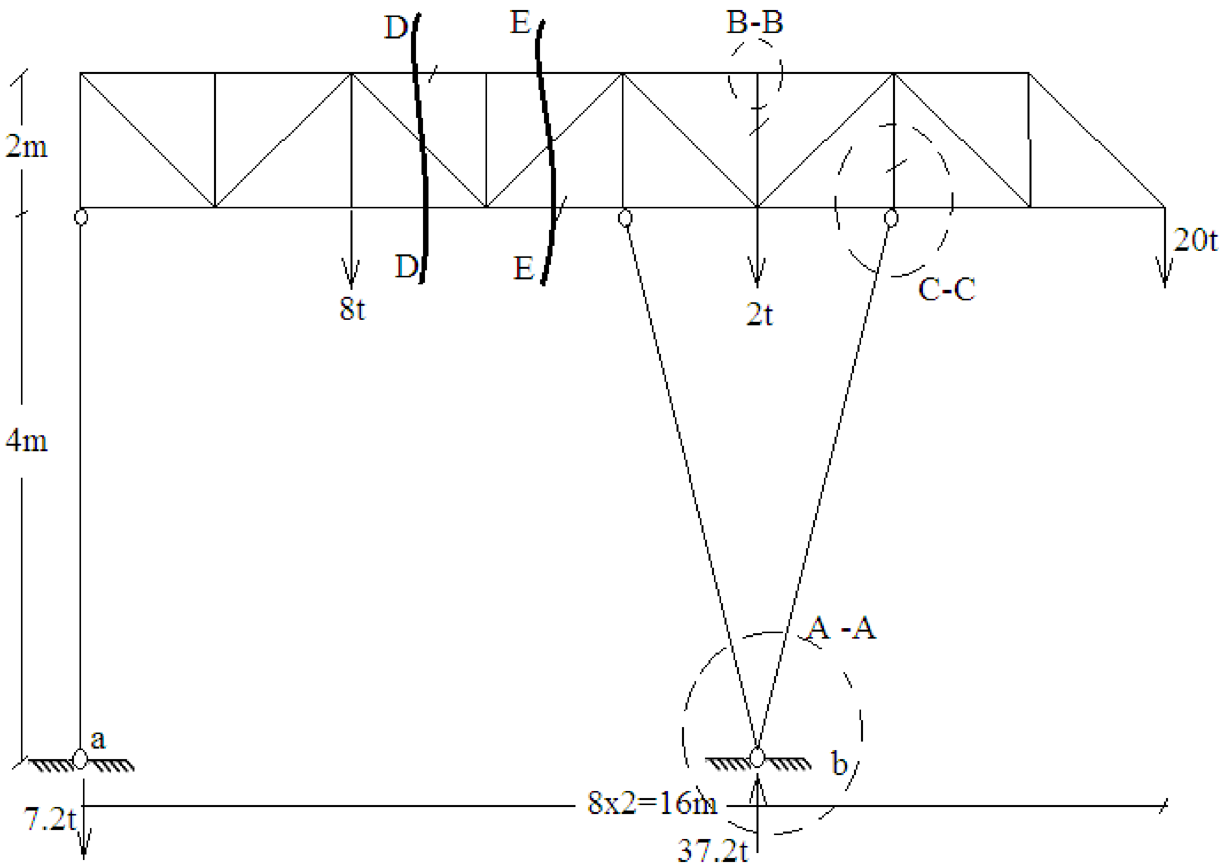


Fig. 5.21.b

- *For Sec. A-A*

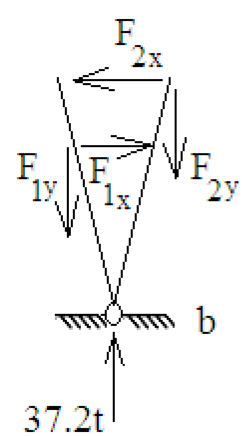
$\sum F_y = 0 \qquad F_{2y} + F_{1y} = 37.2t$

$\sum F_x = 0 \quad F_{2x} - F_{1x} = 0 \;\therefore F_{2y} = F_{1y}$

$F_{2y} = F_{1y} = 18.6t \downarrow$

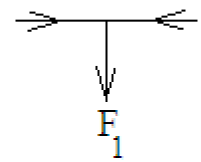
$F_{2y} = 2F_{2x} \;\; F_{2x} = 9.3t \leftarrow$

$F_{1y} = 2F_{1x} \;\; F_{1x} = 9.3t \rightarrow$



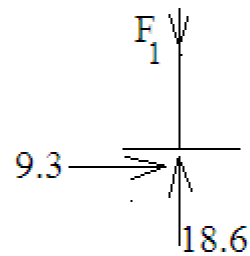
- *For Sec. B-B*

$\sum F_y = 0 \quad F_1 = zero$



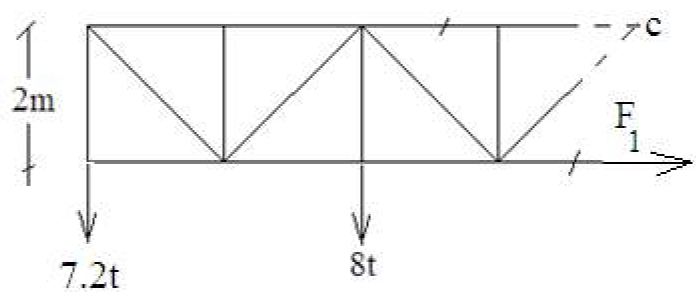
- *For Sec. c-c*

$\sum F_y = 0 \quad F_1 = 18.6t\ (compresion)$



- *For Sec. E-E*

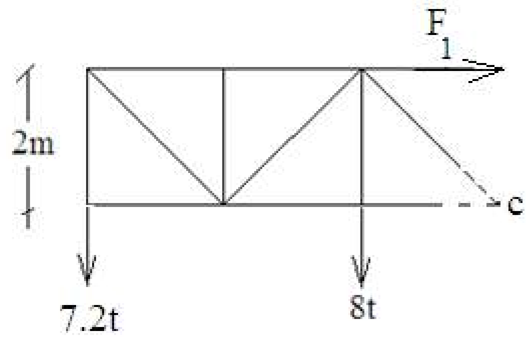
$\sum M_c = 8x4 + 7.2x8 + F_1 x2 = 0$



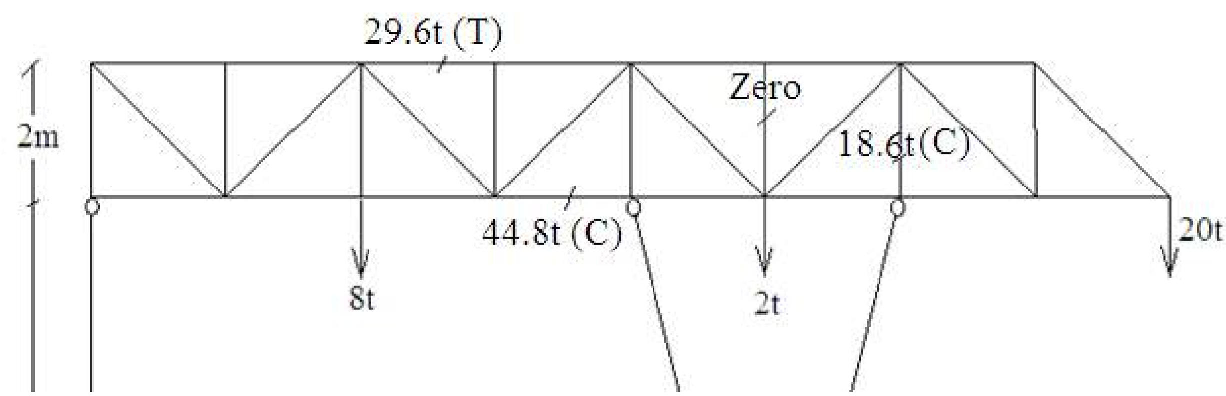
$F_2 = 44.8\ (compression)$

- *For Sec. D-D*

$\sum M_c = 8x2 + 7.2x6 - F_1x2 = 0$



$F_2 = 29.6\ (tension)$



Example (14)

Find analytically the forces in selected members of given truss in Fig. 5.22.a by the method of section

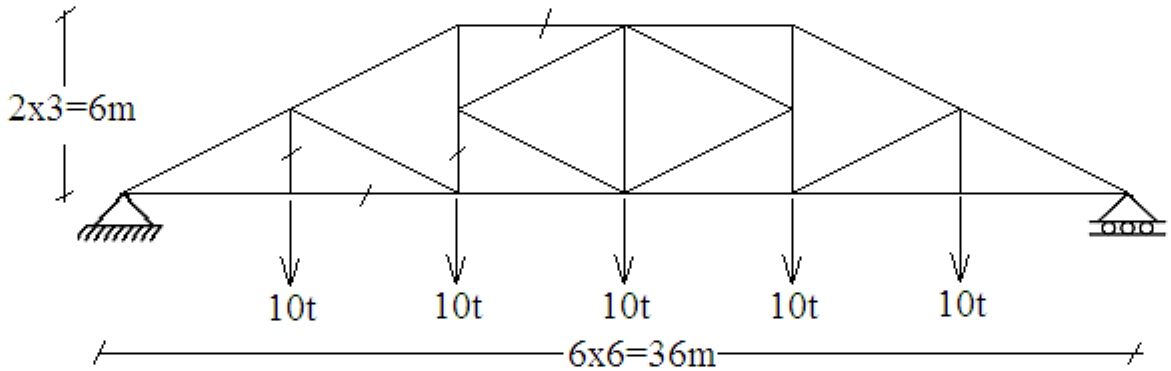


Fig. 5.22.a

Solution

Finding the reactions

$\sum F_x = 0 \; X_a = 0$

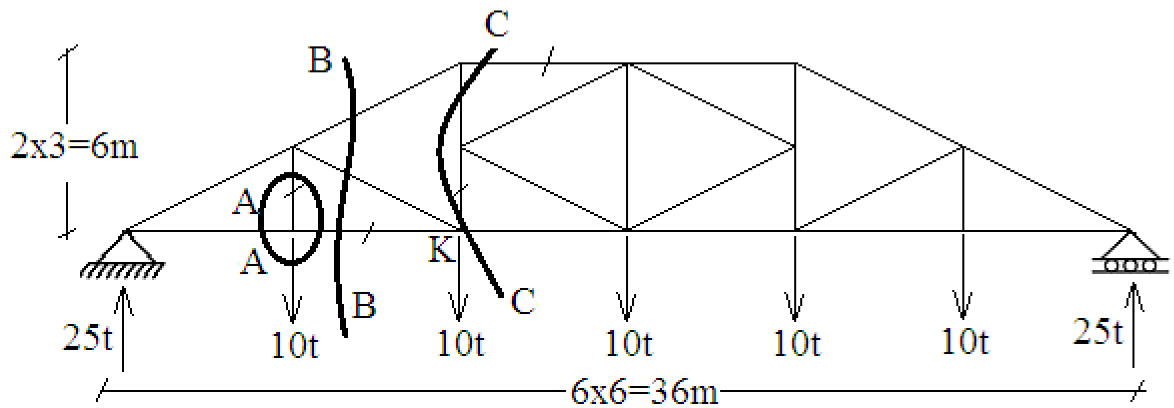
$\sum M_a = 10x6 + 10x12 + 10x18 + 10x24 + 10x30 - 36Y_b = 0$

$$\therefore Y_b = 25t \uparrow$$

$\sum M_b = 10x6 + 10x12 + 10x18 + 10x24 + 10x30 - 36Y_a = 0$

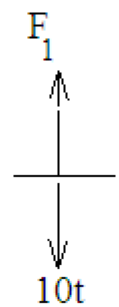
$$\therefore Y_a = 25t \uparrow$$

Check $\sum F_y = 0$ ok

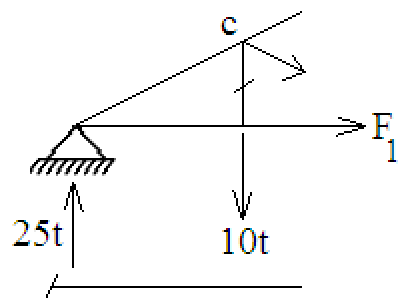


- <u>*For Sec. A-A*</u>

$\sum F_y = 0 \qquad F_1 = 10t\ (tension)$



- <u>*For Sec. B-B*</u>

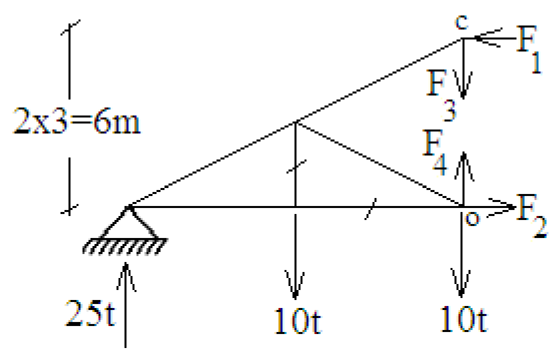


$\sum M_c = 25x6 - F_1x3 = 0$

$F_3 = 50t \rightarrow (tension)$

- <u>For Sec. C-C</u>

$\sum M_c = 10x6 - 25x12 + F_2x6 = 0$



$F_2 = 40\ (tension)$

$\sum M_0 = 25x12 - 10x6 - 6F_2 = 0$

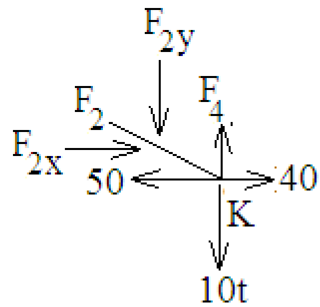
$F_2 \ = 40(comp)$

- At joint K

$\sum \mathrm{F_x} = 0 \ \ F_{2x} = 10\mathrm{t} \rightarrow$

$F_{2x} = 2F_{2y} \qquad F_{2y} = 5t \rightarrow$

$\sum \mathrm{F_y} = 0 \ \ F_4 = 15\mathrm{t} \uparrow$



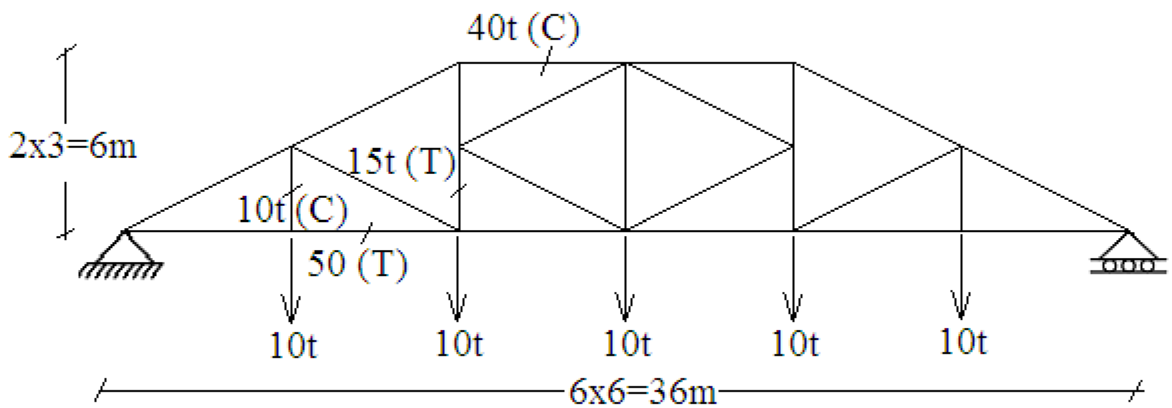


Fig. 5.21.b

Example (15)

Find analytically the forces in selected members of given truss in Fig. 5.23.a by the method of section

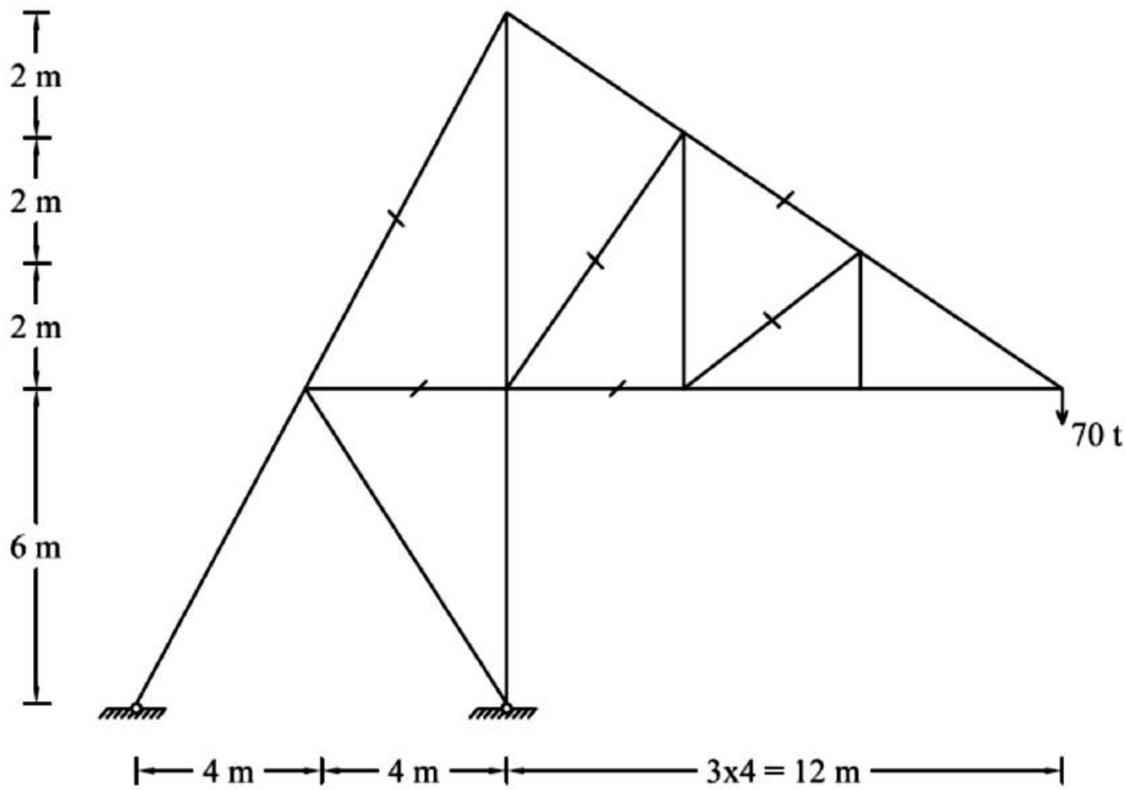
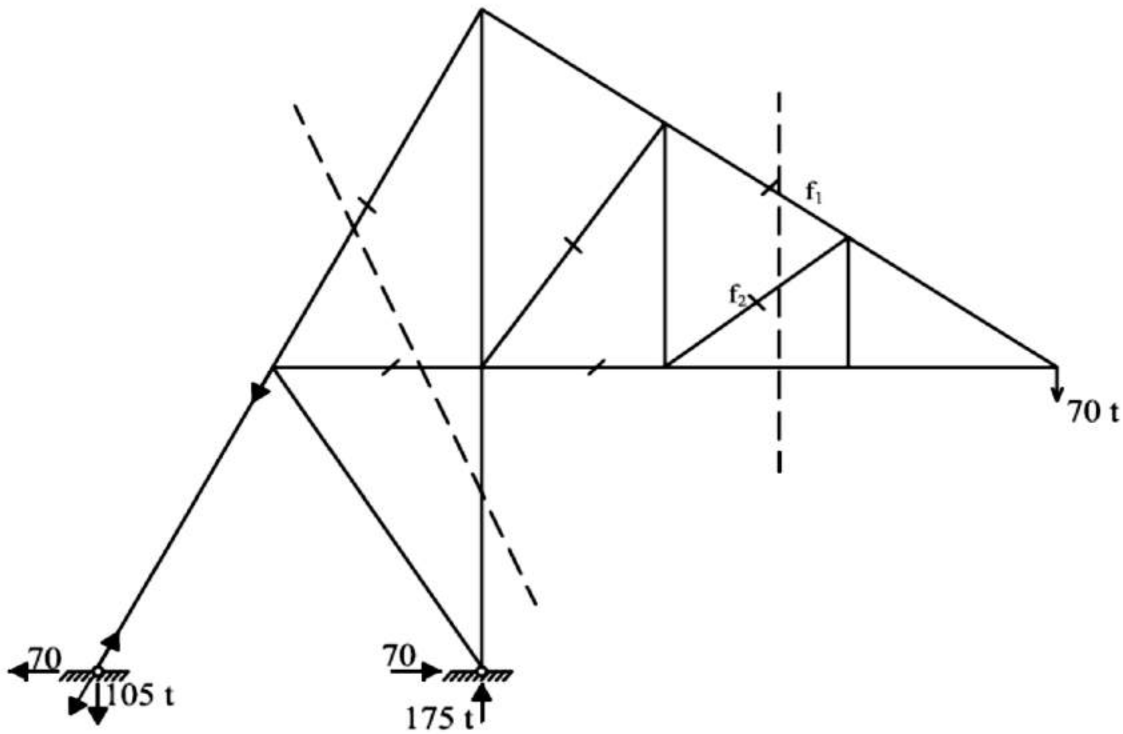
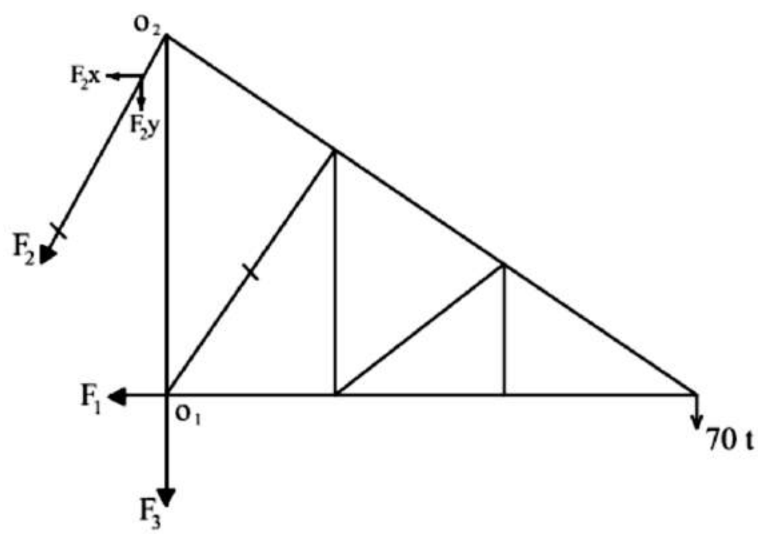


Fig. 5.23.a





$\sum M_{o1} = 0.0$

$70 \times 12 = F_2x = 6$

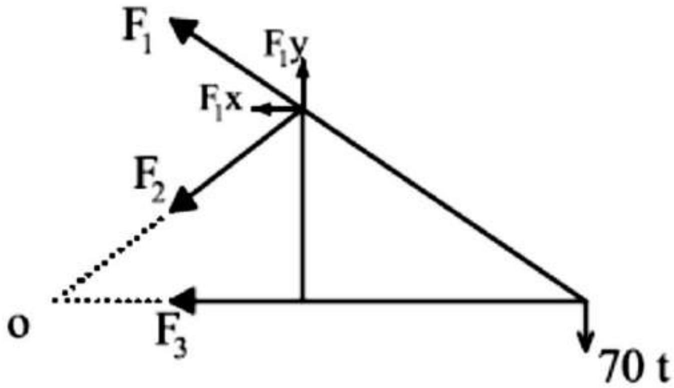
$F_2x = 140$ t (tension)

$\frac{F_{2y}}{F_{2x}} = \frac{4}{6} = \longrightarrow$ $F_2y =$ 93.33 t

$F_2 = 168.26$ t (tension)

$\sum M_{o2} = 0.0$

$F_1x = 140$ t (compression)



$\sum M_o = 0.0$

$F_1y \times 4 + F_1x \times 2 - 70 \times 8 = 0.0$

$4\,F_1y + 2\,F_1x - 70 \times 8 = 0.0$

$\frac{F_{1y}}{F_{1x}} = \frac{2}{4} = \longrightarrow \quad F_1y = 0.5$
F_1x

$2\,F_1x + 2\,F_1y - 70 \times 8 = 0.0$

$F_1x = 140$ t (tension)

$F_1y = 70$ t (tension)

$F_1 = 156.52$ t (tension)

Example (16)

Find graphically the forces in members of given truss in Fig.5.24

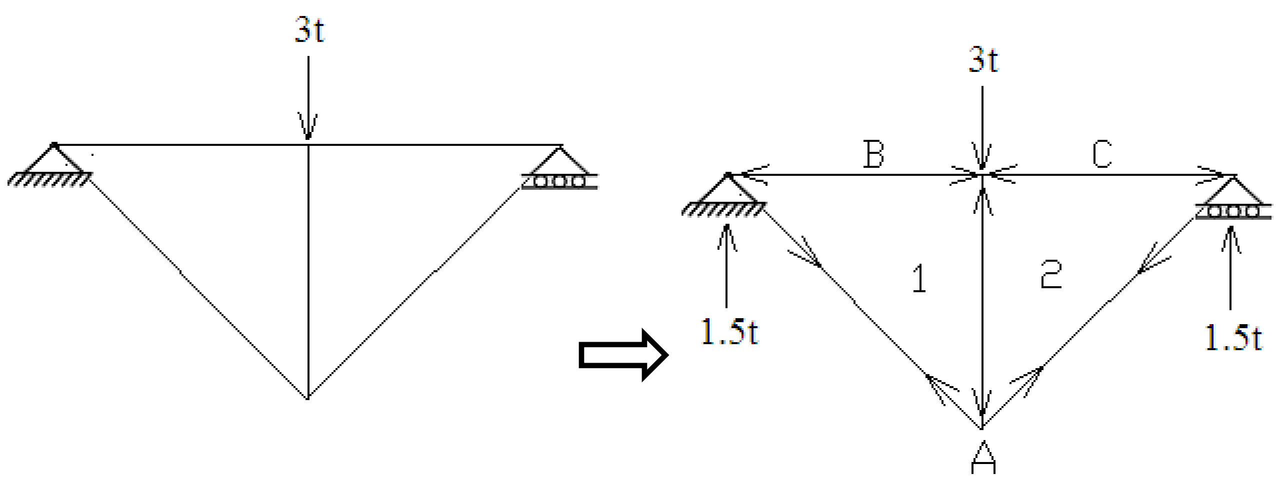
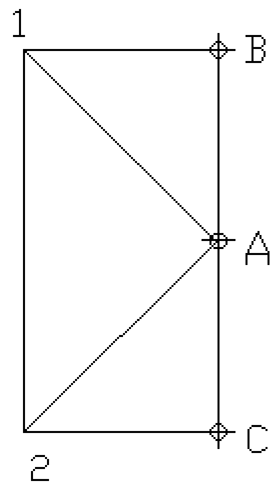


Fig.5.24

Member		C2	KL
Force	1		$1.5\sqrt{2}$
Type			T



Example (16)

Find graphically the forces in members of given truss in Fig.5. 24

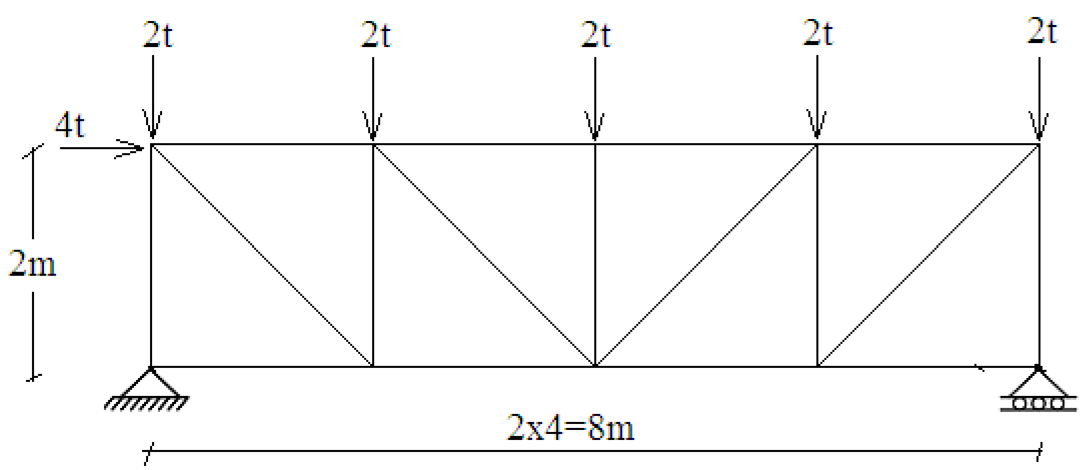


Fig. 5. 24.a

Solution

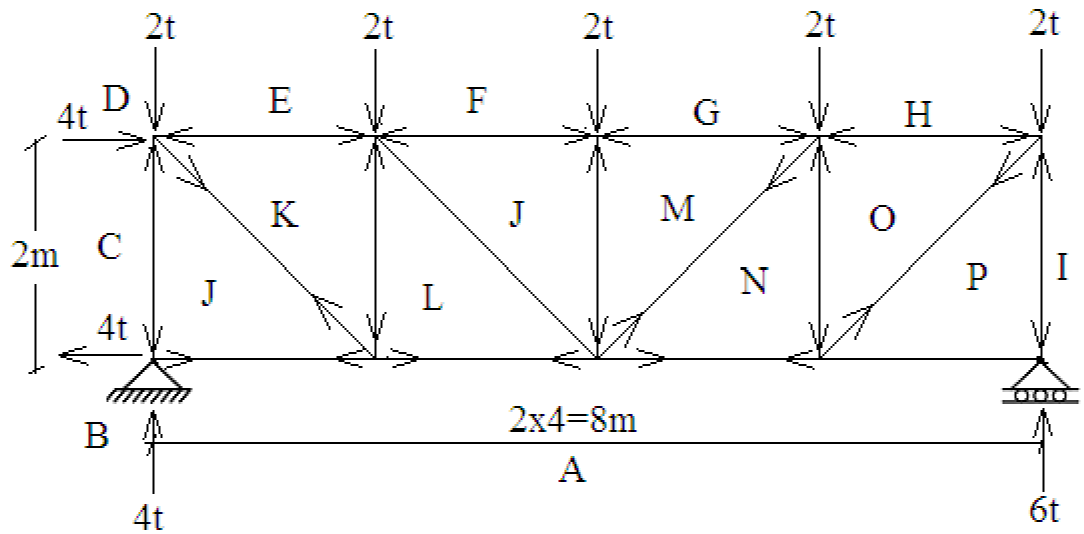
Finding the reactions

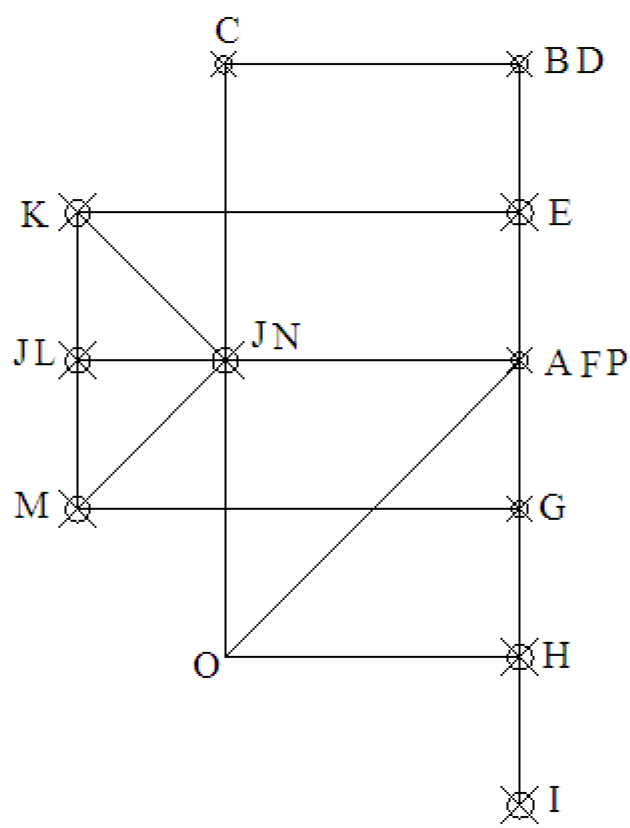
$\sum F_x = 0 \; X_a = 4 \leftarrow$

$\sum M_a = 2x2 + 2x4 + 2x6 + 2x8 + 4x2 - 8Y_b = 0 \qquad \therefore Y_b = 6t \uparrow$

$\sum M_b = 2x2 + 2x4 + 2x6 + 2x8 - 4x2 - 8Y_a = 0 \qquad \therefore Y_a = 4t \uparrow$

<u>*Check*</u> $\sum F_y = 0$ ok





Member	**CJ**	**JA**	**EK**	**KJ**	**KL**	**LA**	**LJ**	**JF**	**JM**	**GM**
Force	4	4	8	2√2	2	8	0	8	2	8
Type	C	T	C	T	C	T	…..	C	C	C
Member	MN	NA	NO	OH	OP	PA	PI			
Force	2√2	4	4	4	4√2	0	6			
Type	T	T	C	C	T	…	C			

Example (17)

Find graphically the forces in members of given truss in Fig. 5.25.

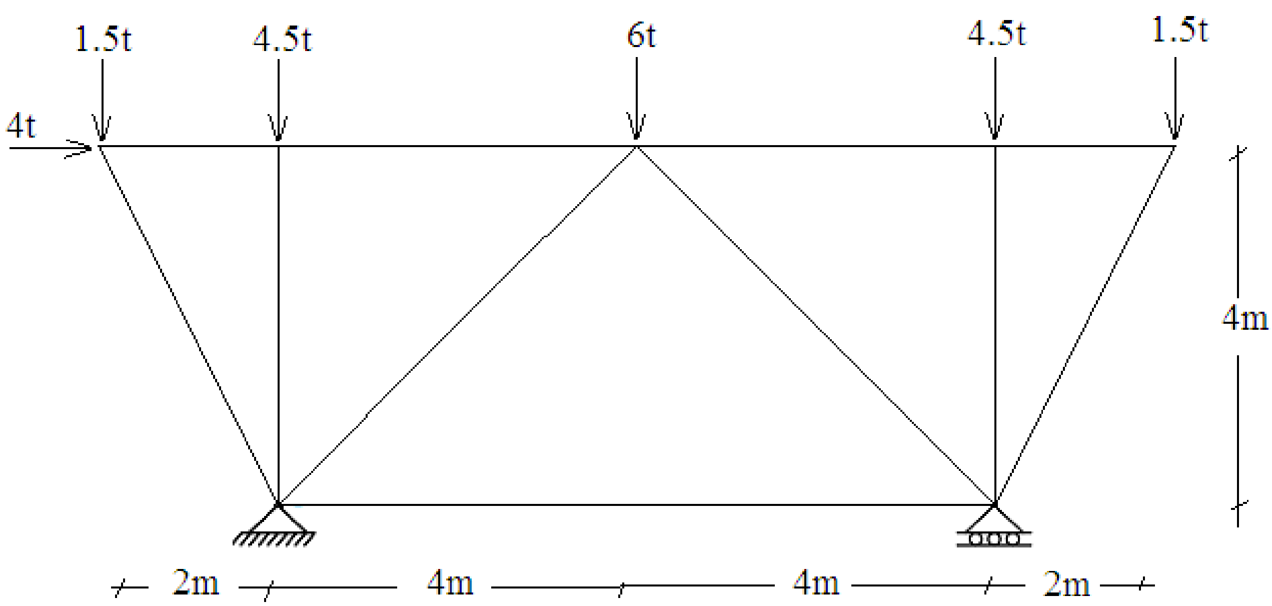


Fig. 5.25

Solution

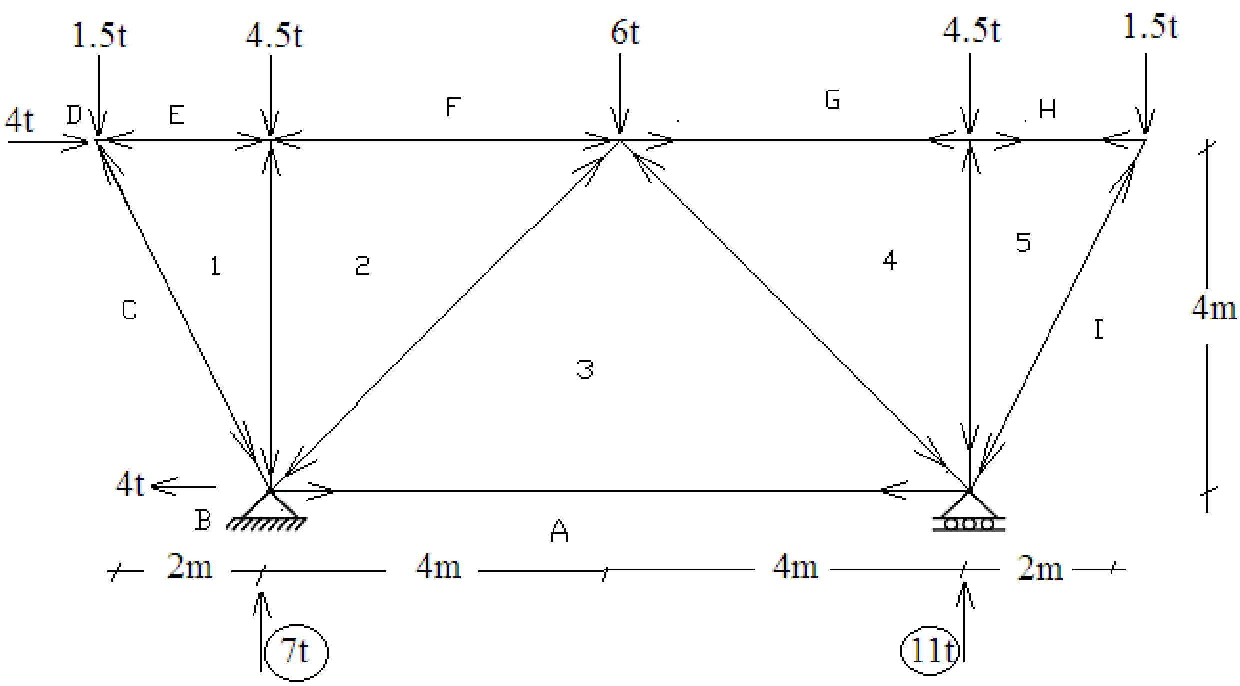
Finding the reactions

$$\sum F_x = 0 \quad X_a = 4 \leftarrow$$

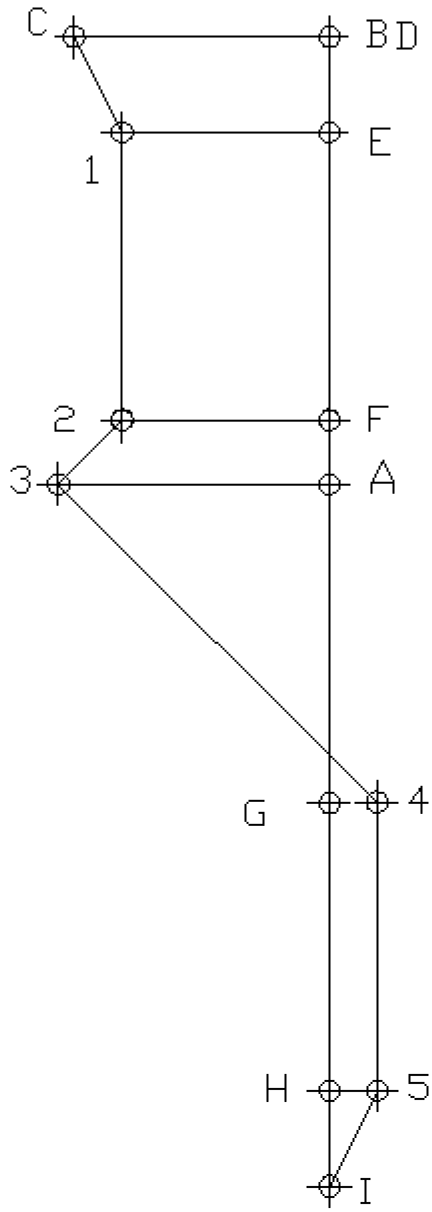
$$\sum M_a = 6x4 + 4.5x8 + 1.5x10 - 1.5x2 + 4x4 - 8Y_a = 0 \quad \therefore Y_b = 11t \uparrow$$

$$\sum M_b = 6x4 + 4.5x8 + 1.5x10 - 1.5x2 - 4x4 - 8Y_a = 0 \quad \therefore Y_a = 7t \uparrow$$

Check $\sum F_y = 0$ ok



Member	C1	E1	12	F2	23	34	G4	45	H5	5I	3A
Force	$0.75\sqrt{5}$	3.25	4.5	3.25	$\sqrt{2}$	$5\sqrt{2}$	0.75	4.5	0.75	$0.75\sqrt{5}$	4.25
Type	C	C	C	C	C	C	T	C	T	C	


C
BD
E
1
2
F
3
A
G
4
H
5
I

Example (18)

Find analytically the forces in members of given truss in Fig.5. 26

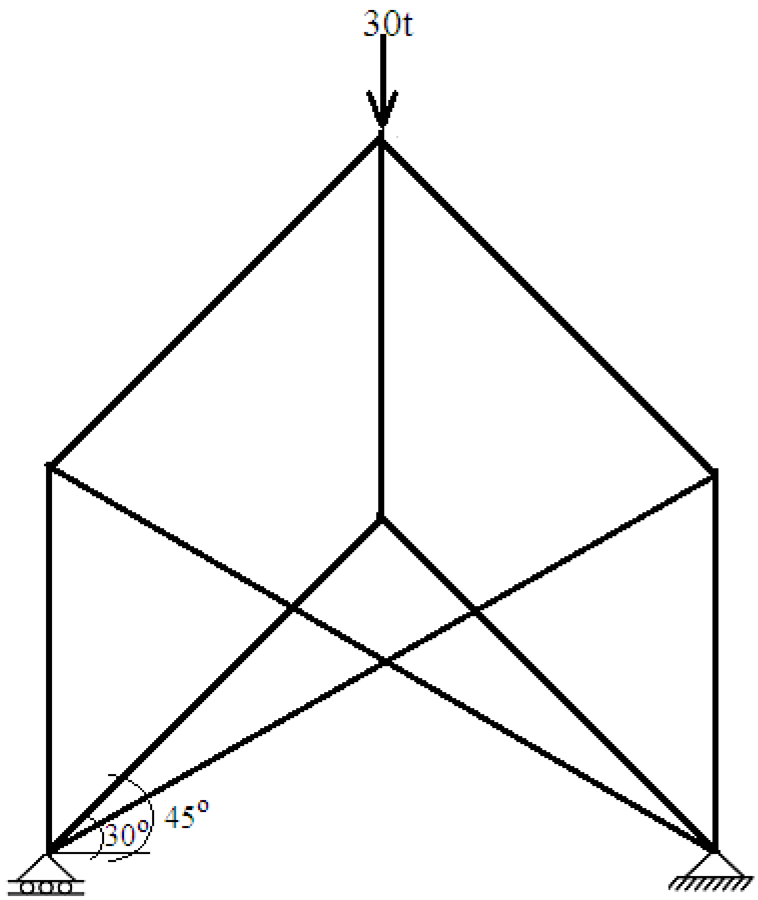
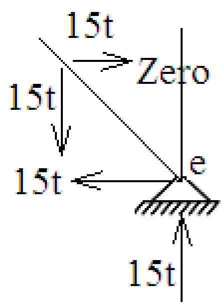
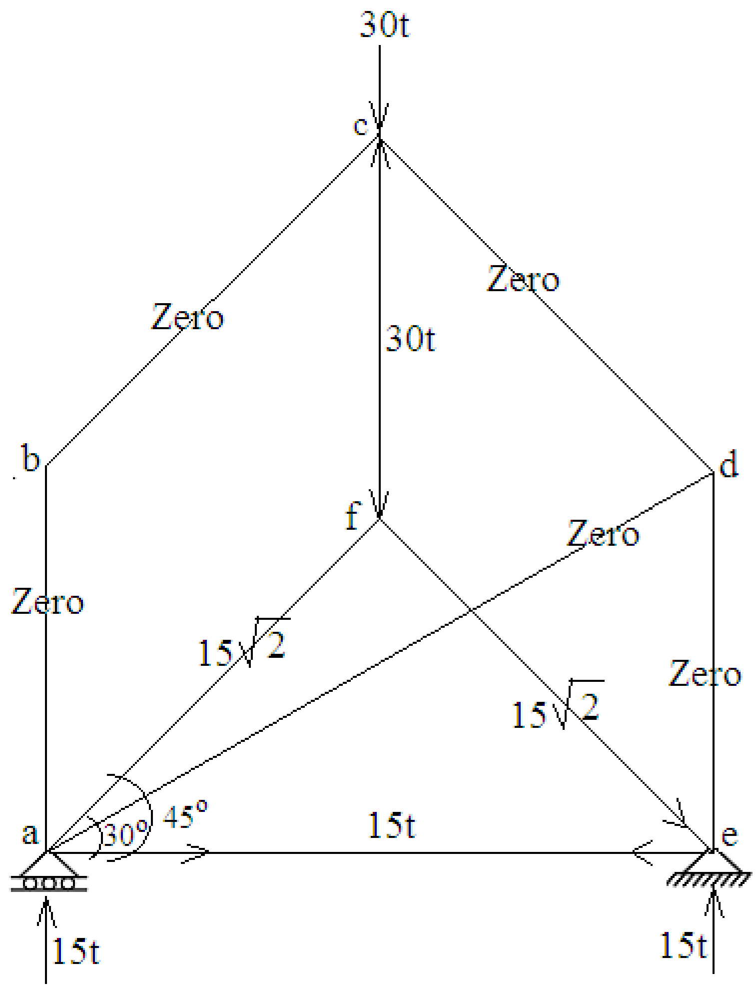


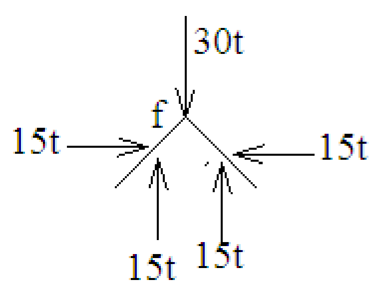
Fig. 5. 26

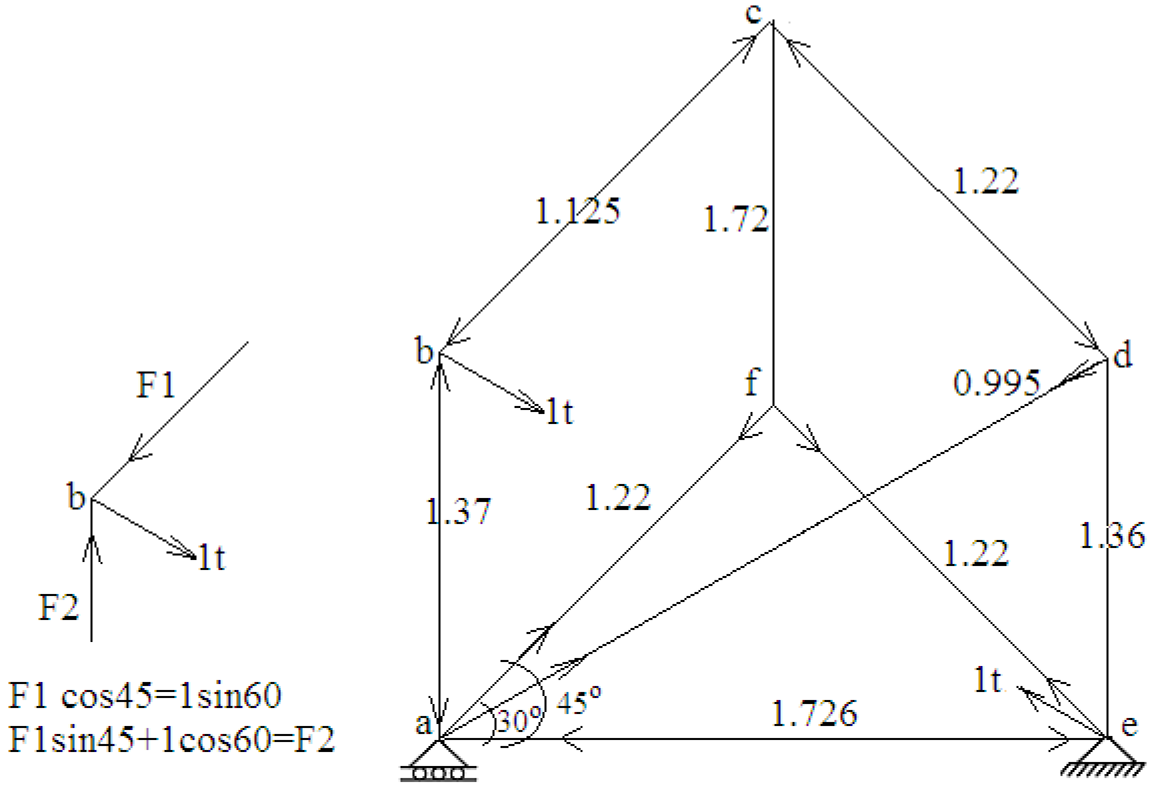
Solution

- For the main system









$$x = -\frac{15}{-1.726} = 8.69t$$

Total forces

Member	F_o	F_1	$F=F_o+xF_1$
AB	0	-1.37	-11.9
BC	0	-1.22	-10.614
CD	0	-1.22	-10.614
DE	0	-1.36	-11.83
EF	-15√2	1.22	-10.6
EB	0	1	8.69
EA	15	-1.726	0

AF	$-15\sqrt{2}$	1.22	-10.6
AD	0	0.99	8.7
CF	-30	1.72	-15.06

Example (18)

Find analytically the forces in members of given truss in Fig.5.27

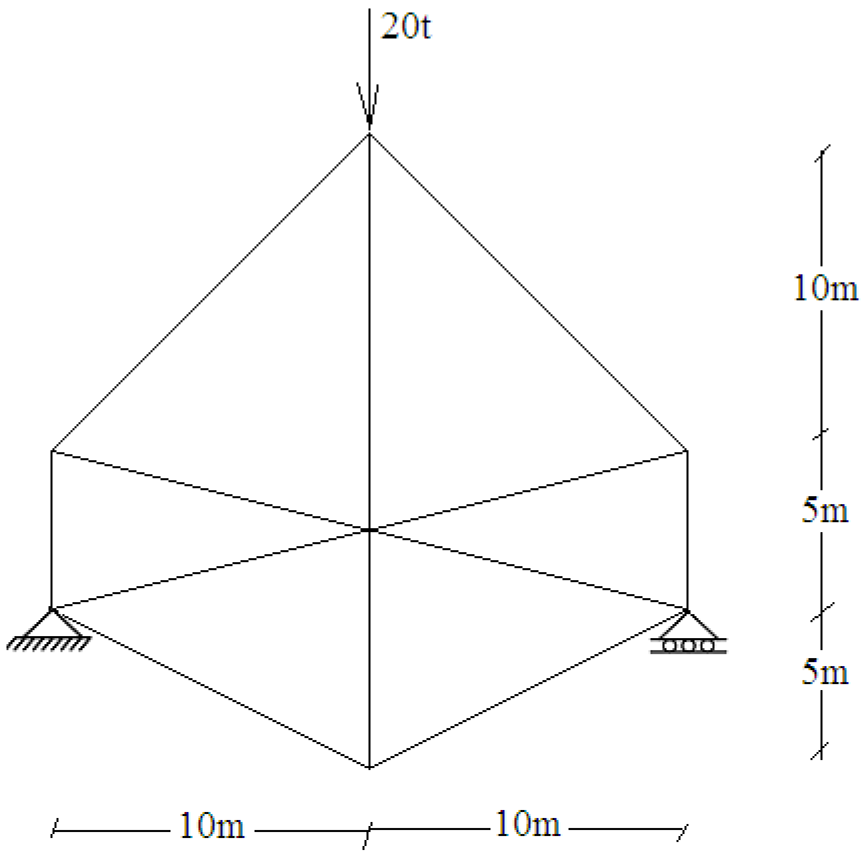


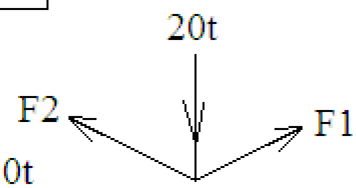
Fig.5.27

Solution

- For the main system

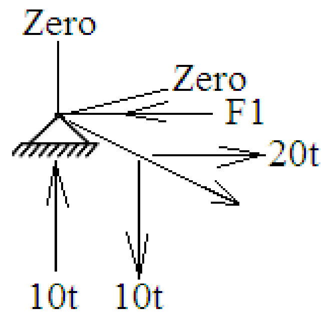
Joint f

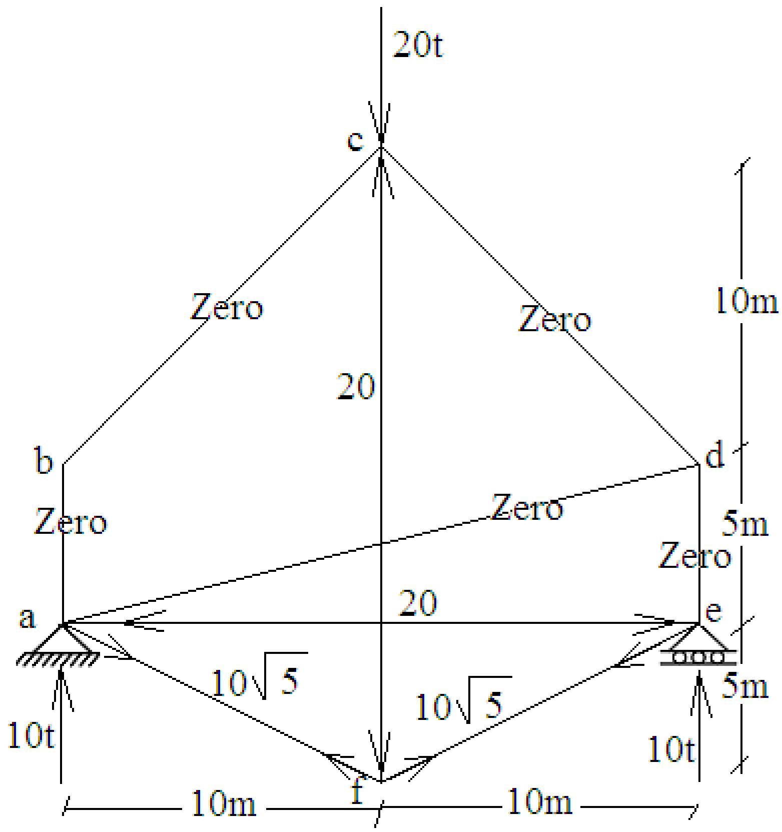
y1+y2=20
x1=x2
F1=F2
y1=10t and y2=10t
x1=2y1
$F1=F2=10\sqrt{5}$



F1=20t

Jointa



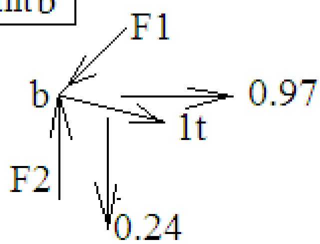


Main system

To find X1

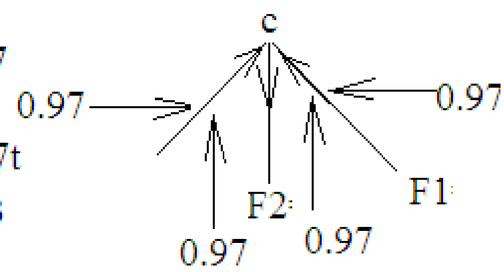
Joint b

x1=0.97
y1=0.97
F1=1.37t
F2=1.21t



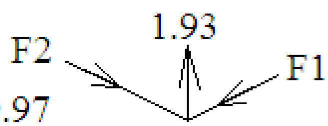
Joint c

x1=0.97
y1=0.97
F1=1.37t
F2=1.93



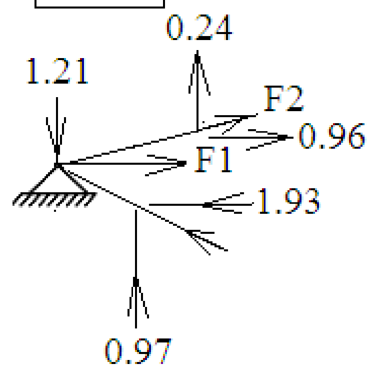
Joint f

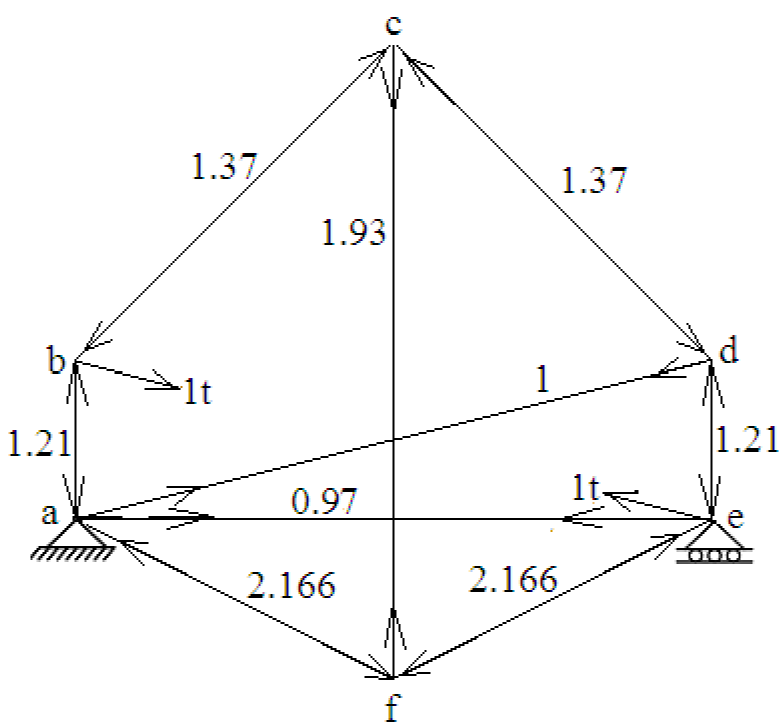
y1+y2=1.93
x1=x2
F1=F2
y1= 0.97 and y2= 0.97
x1=2y1
F1=F2=2.166



Jointa

y2=0.24
x2=0.96
F1=0.97





$$x = -\frac{-20}{0.97} = 20.62t$$

Total forces

Member	F_o	F_1	$F=F_o+xF_1$
AB	0	-1.21	-24.95
BC	0	-1.37	-28.25
CD	0	-1.37	-28.25
DE	0	-1.22	-25.16
EF	$10\sqrt{5}$	-2.166	-22.3
FC	-20	1.93	19.87
EB	0	1	20.62
EA	-20	0.97	0
AF	$10\sqrt{5}$	-2.166	-22.30
AD	0	1	20.62

Chapter (6)

INFLUENCE LINE

6.1. Introduction

Structures may be subjected to live loads whose position may vary on the structure as bridge structures. As the car moves across the bridge, the forces in the bridge members change with the position of the car. Also the maximum force in each member will varied at a different car location and the design of each member must be based on the maximum force in its.

Influence Line is defined as the behavior of a structure as a function of the position of a downward unit load moving across the structure. The behavior of the structure includes the reactions, shear force, normal force, bending moment and the torsion. To calculate the influence line follow the below equations:-

a. Consider a unit load to move over the structure from left to right
b. Calculate the values of reactions, shear force or bending moment, at the point under consideration, as the unit load moves over the structure from left to right
c. Plot the values of the reactions, shear force or bending moment, over the length of the beam, computed for the point under consideration

6. Solved examples

Example#1

Find the influence line for Y_a, Y_b, Q_{m-m}, Q_{s-s}, M_{s-s} and M_{m-m} as a result of moving unite loads from left to right on the shown beam in Fig. 6.1.

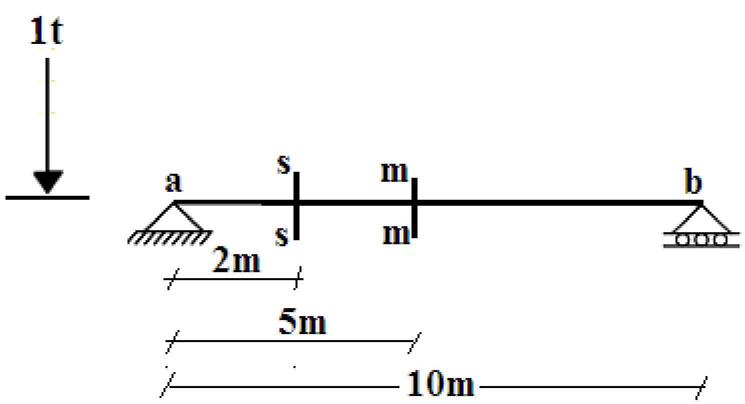


Fig. 6.1.a

Solution

- For I.L (Ya)
  When the unite load moves from a to b
  - The load at a $\rightarrow$ the reaction Y_a will be 1t upwards
  - The load at a distance 1mfrom a $\rightarrow$ the reaction Y_a will be 0.9t upwards
  - The load at a distance 2mfrom a $\rightarrow$ the reaction Y_a will be 0.8t upwards
  - The load at a distance 5m from a $\rightarrow$ the reaction Y_a will be 0.5t upwards
  - The load at a distance 10m from a (at b) $\rightarrow$ the reaction Y_a will be zero

- For I.L (Y_b)

  When the unite load moves from a to b

  - The load at a $\rightarrow$the reaction Y_b will be zero
  - The load at a distance 1m from a$\rightarrow$ the reaction Y_b will be 0.1t upwards
  - The load at a distance 2mfrom a $\rightarrow$ the reaction Y_b will be 0. 2t upwards
  - The load at a distance 5m from a $\rightarrow$ the reaction Y_b will be 0.5t upwards
  - The load at a distance 10m from a (at b)$\rightarrow$the reaction Y_b will be 1t upwards

- For I.L ($Q_{s\text{-}s}$)

  When the unite load moves from a to the locations of the sections $Q_{s\text{-}s}$ = Y_a-1

  - The load at a $\rightarrow$the shear $Q_{s\text{-}s}$ will be zero
  - The load at a distance 1m from a$\rightarrow$ the shear $Q_{s\text{-}s}$ =0.9-1=0.1t downwards
  - The load at a distance 2m from a $\rightarrow$ the shear $Q_{s\text{-}s}$ =0.8-1=0.2t downwards

  When the unite load moves from the locations of the sections to b $Q_{s\text{-}s}$ = Y_b-1a to

  - The load at b $\rightarrow$the shear $Q_{s\text{-}s}$will be zero
  - The load at a distance 1m from b $\rightarrow$the shear $Q_{s\text{-}s}$ =0.9-1=0.1t upwards

- The load at a distance 8m from b $\rightarrow$ the shear Q_{s-s} =0.2-1=0.8t upwards

- For I.L (Q_{m-m})
  When the unite load moves from a to the locations of the sections Q_{m-m} = Y_a-1

  - The load at a $\rightarrow$the shear Q_{m-m} will be zero
  - The load at a distance 1m from a$\rightarrow$ the shear Q_{m-m} =0.9-1=0.1t downwards
  - The load at a distance 5m from a $\rightarrow$ the shear Q_{m-m} =0.5-1=0.5t downwards

  When the unite load moves from the locations of the sections to b Q_{m-m} = Y_b-1a to

  - The load at b $\rightarrow$the shear Q_{m-m}will be zero
  - The load at a distance 1m from b $\rightarrow$the shear Q_{m-m} =0.9-1=0.1t upwards
  - The load at a distance 8m from b $\rightarrow$ the shear Q_{m-m} =0.2-1=0.8t upwards
  - The load at a distance 5m from a $\rightarrow$ the shear Q_{m-m} =0.5-1=0.5t upwards

- For I.L (M_{s-s})
  When the unite load moves from a to the locations of the section M_{s-s} = Y_a* the distance between the section and the support a-1*the distance between the location of the moving load and the section

  - The load at a $\rightarrow$the moment M_{s-s} will be zero

- The load at a distance 1m from a $\rightarrow$ the moment M_{s-s}

  =0.9x1=0.9mt

- The load at a distance 2m from a $\rightarrow$ the moment M_{s-s}

  =0.2x2=1.6mt

➢ For I.L (M_{m-m})

When the unite load moves from a to the locations of the sections M_{s-s} = Y_a* the distance between the section and the support a-1*the distance between the location of the moving load and the section

- The load at a $\rightarrow$ the shear M_{m-m} will be zero
- The load at a distance 1m from a $\rightarrow$ the moment M_{m-m}

  =0.9x1=0.9mt

- The load at a distance 5m from a $\rightarrow$ the moment M_{m-m}

  =0.5x5=2.5mt

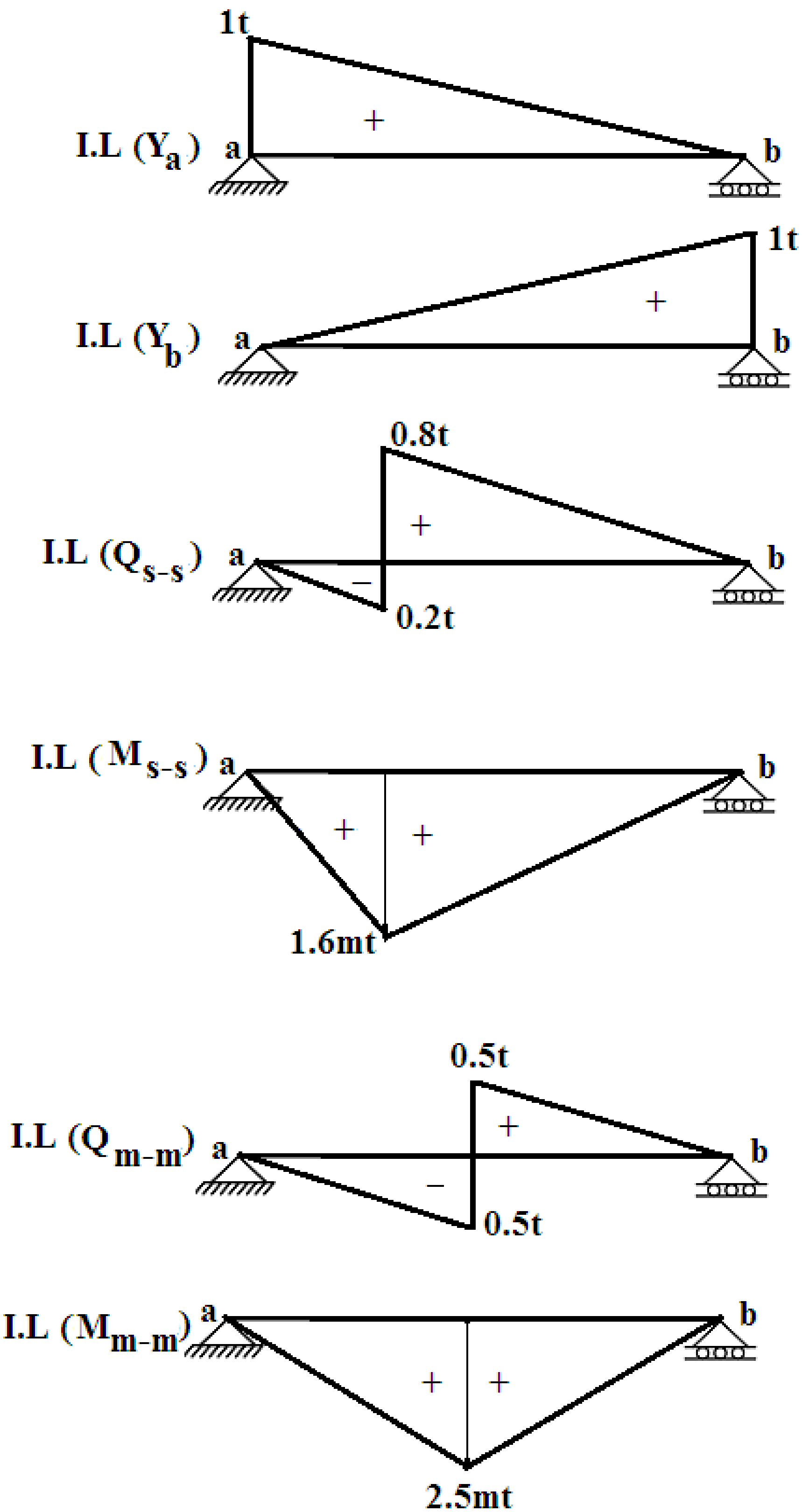


Fig. 6.1.b

Example#2

Find the influence line for Y_a, Y_b, Q_{m-m}, Q_{s-s}, M_{s-s}, M_{m-m}, Q_{n-n}, M_{n-n}, Q_{t-t} and M_{t-t} as a result of moving unite loads from left to right on the shown beam in Fig. 6.2.

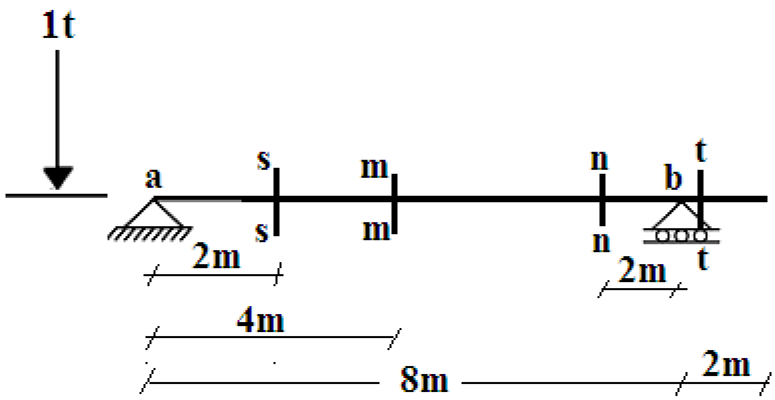


Fig. 6.2.a

Solution

- For I.L (Y_a)

When the unite load moves from a to b

- The load at a → the reaction Y_a will be 1t upwards
- The load at a distance 8m from a (at b)→the reaction Y_a will be zero
- The load at a distance 10m from a (at the free point) the reaction Y_a will be 2/8t=0.25t

- For I.L (Y_b)

When the unite load moves from a to b

- The load at a →the reaction Y_b will be zero
- The load at a distance 8m from a (at b) →the reaction Y_b will be 1t upwards

- The load at a distance 10m from a (at the free point) the reaction Y_b will be 10/8t=1.25t

➢ For I.L (Q_{s-s})

When the unite load moves from a to the locations of the sections $Q_{s-s} = Y_a - 1$

- The load at a $\rightarrow$the shear Q_{s-s} will be zero
- The load at a distance 2m from a (the location of the selected section)$\rightarrow$ the shear Q_{s-s} =0.75-1=0.25t downwards

When the unite load moves from the locations of the sections to b $Q_{s-s} = Y_b$-1a to

- The load at a distance 6m from b(the location of the selected section) $\rightarrow$ the shear Q_{s-s} =0.25-1=0.75t upwards
- The load at b $\rightarrow$the shear Q_{s-s}will be zero
- The load at a distance 10m from a (the location of the free point) $\rightarrow$the shear Q_{s-s} =10/8-1=0.25t downwards

➢ For I.L (Q_{m-m})

When the unite load moves from a to the locations of the sections $Q_{m-m} = Y_a - 1$

- The load at a $\rightarrow$the shear Q_{m-m} will be zero
- The load at a distance 4m from a $\rightarrow$ the shear Q_{m-m} =0.5-1=0.5t downwards

When the unite load moves from the locations of the sections to b $Q_{m-m} = Y_b$-1a to

- The load at a distance 4m from a $\rightarrow$ the shear Q_{m-m} =0.5-1=0.5t upwards
- The load at b $\rightarrow$the shear Q_{m-m} will be zero
- The load at a distance 10m from a (the location of the free point)

$\rightarrow$the shear Qs-s =10/8-1=0.25t downwards

➢ For I.L (M_{s-s})

When the unite load moves from a to the locations of the section M_{s-s} = Y_a* the distance between the section and the support a-1*the distance between the location of the moving load and the section

- The load at a $\rightarrow$the moment M_{s-s} will be zero
- The load at a distance 2m from a (the distance of the section)$\rightarrow$ the moment M_{s-s} =0.75x2=1.5mt
- The load at b $\rightarrow$the moment M_{s-s} will be zero
- The load at a distance 10m from a$\rightarrow$the moment M_{s-s} = 1x8-Y_bx6= -1x8+10/8x6=-0. 5mt

➢ For I.L (M_{m-m})

When the unite load moves from a to the locations of the sections M_{s-s} = Y_a* the distance between the section and the support a-1*the distance between the location of the moving load and the section

- The load at a$\rightarrow$ the shear M_{m-m} will be zero
- The load at a distance 4m from a$\rightarrow$the moment M_{m-m} =0.5x4=2mt
- The load at b $\rightarrow$the moment M_{m-m} will be zero
- The load at a distance 610m from a$\rightarrow$the moment M_{m-m} = -1x6+Y_bx4= -1x6+10/8x4=-1mt

- For I.L (Q_{n-n})

  When the unite load moves from a to the locations of the sections Q_{n-n} = Y_a-1

  - The load at a $\rightarrow$the shear Q_{n-n} will be zero
  - The load at a distance 6m from a $\rightarrow$ the shear Q_{m-m} =0.25-1=0.75t downwards

  When the unite load moves from the locations of the sections to b Q_{n-n} = Y_b-1a to

  - The load at a distance 6m from a $\rightarrow$ the shear Q_{n-n} =0.75-1=0.25t upwards
  - The load at b $\rightarrow$the shear Q_{n-n} will be zero
  - The load at a distance 10m from a (the location of the free point) $\rightarrow$the shear Qn-n =10/8-1=0.25t downwards

- For I.L (M_{n-n})

  When the unite load moves from a to the locations of the section M_{n-n} = Y_a* the distance between the section and the support a-1*the distance between the location of the moving load and the section

  - The load at a $\rightarrow$the moment M_{n-n} will be zero
  - The load at a distance 6m from a (the distance of the section)$\rightarrow$ the moment M_{s-s} =0.25x6=1.5mt
  - The load at b $\rightarrow$the moment M_{s-s} will be zero
  - The load at a distance 10m from a$\rightarrow$the moment M_{n-n} = 1x4-Y_bx2= -1x4+10/8x2=-1. 5mt

- For I.L (Q_{t-t})

- When the unite load moves from a to the locations of the section

  $Q_{t-t} = 0$

When the unite load moves from the locations of the sections to the free point Q_{t-t} = 1t upwards

➢ For I.L (M_{t-t})
- When the unite load moves from a to the locations of the section

  $M_{t-t} = 0$

  - The load at a distance 10m from a → the moment M_{t-t} = -2mt

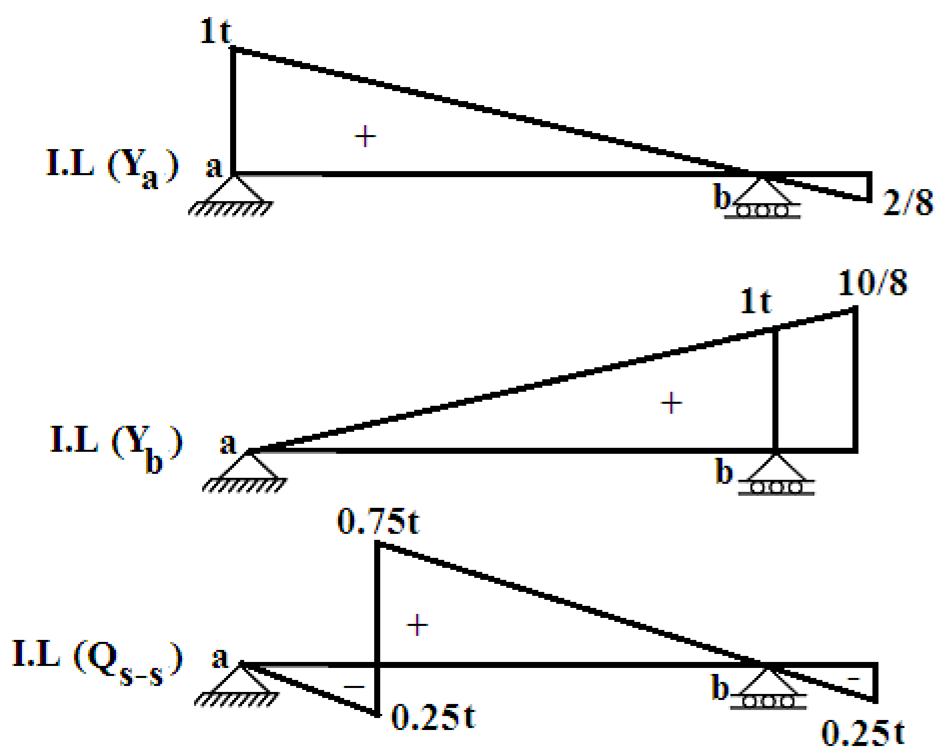


Fig. 6.2.b

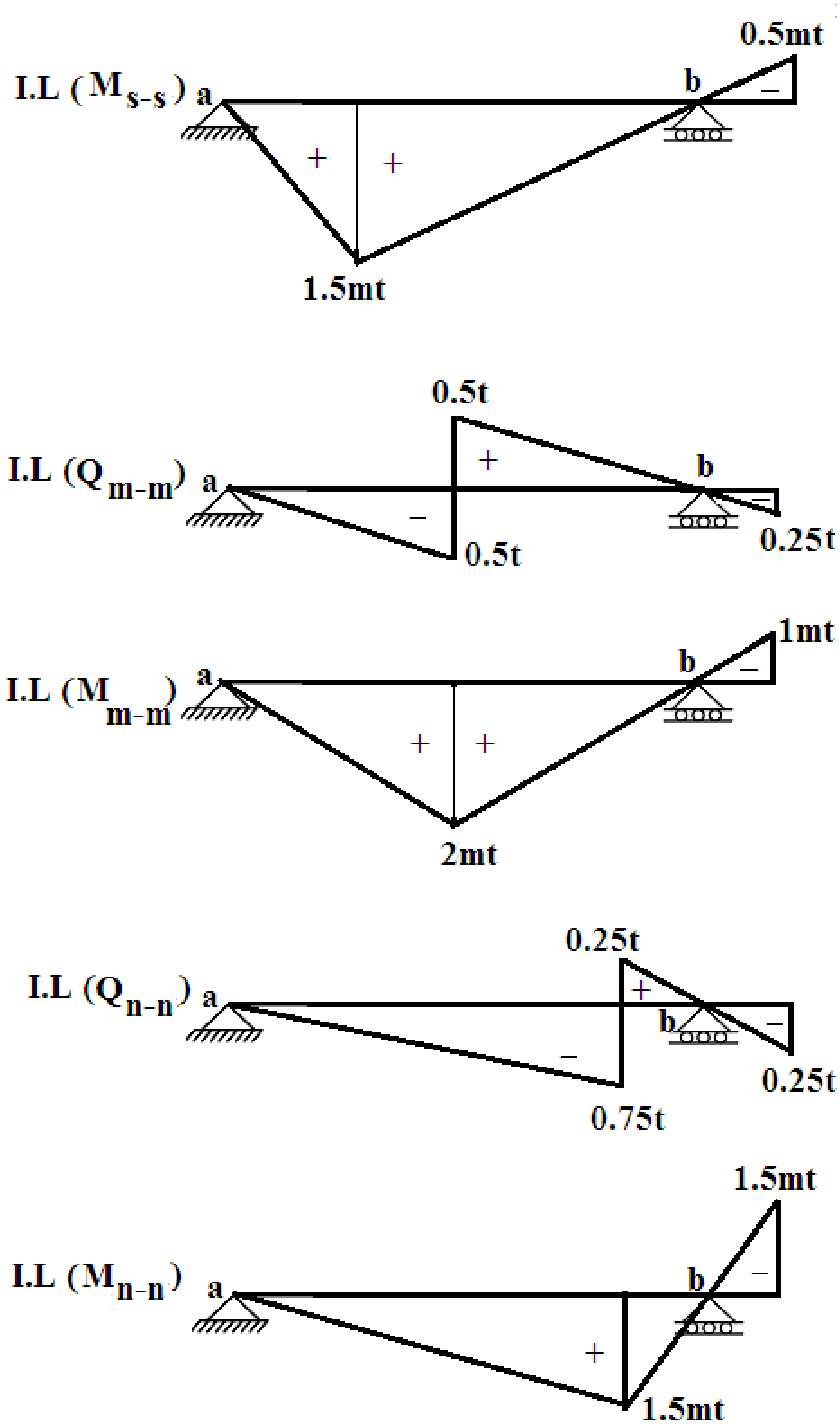

I.L (M s-s)
a
b
0.5mt
1.5mt
I.L (Q m-m)
a
b
0.5t
0.5t
0.25t
I.L (M m-m)
a
b
1mt
2mt
I.L (Q n-n)
a
b
0.25t
0.75t
0.25t
I.L (M n-n)
a
b
1.5mt
1.5mt

Fig. 6.2.c

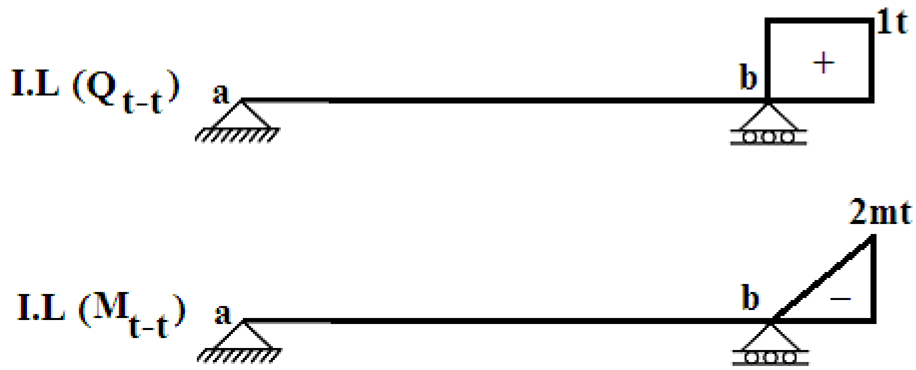


Fig. 6.2.d

Example#3

Find the influence line for Y_a, Y_b, $Q_{m\text{-}m}$, $Q_{s\text{-}s}$, $M_{s\text{-}s}$, $M_{m\text{-}m}$, $Q_{n\text{-}n}$ and $M_{n\text{-}n}$ as a result of moving unite loads from left to right on the shown beam in Fig. 6.3.

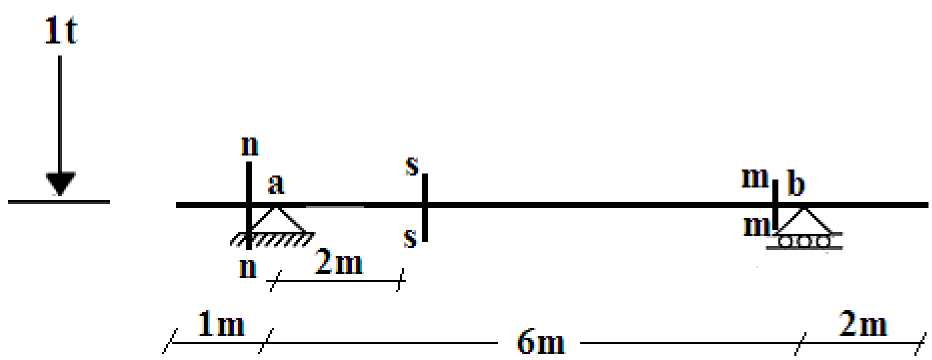


Fig. 6. 3.a

Solution

- For I.L (Y_a)

When the unite load moves from a to b

- The load at a $\rightarrow$ the reaction Y_a will be 1t upwards
- The load at a distance 6m from a (at b)$\rightarrow$the reaction Y_a will be zero

- The load at a distance 8m from a (at the right free point) the reaction Y_a will be 2/6t downward
- The load at a distance 1m from a (at the left free point) the reaction Y_a will be 7/6t upward

➢ For I.L (Y_b)
When the unite load moves from a to b

- The load at a $\rightarrow$the reaction Y_b will be zero
- The load at a distance 6m from a (at b) $\rightarrow$the reaction Y_b will be 1t upwards
- The load at a distance 8m from a (at the right free point) the reaction Y_b will be 8/6t=4/3t upwards

➢ For I.L (Q_{s-s})
When the unite load moves from a to the locations of the sections $Q_{s-s} = Y_a - 1$

- The load at a $\rightarrow$the shear Q_{s-s} will be zero
- The load at a distance 2m from a (the location of the selected section)$\rightarrow$ the shear Q_{s-s} =2/3-1=1/3t downwards
- The load at the location of the left free point the shear $\rightarrow Q_{s-s}$ =7/6-1=1/6t downwards

When the unite load moves from the locations of the sections to b $Q_{s-s} = Y_b$-1a to

- The load at a distance 4m from b (the location of the selected section) $\rightarrow$ the shear Q_{s-s} =1/3-1=2/3t upwards
- The load at b $\rightarrow$the shear Q_{s-s}will be zero

- The load at a distance 8m from a (the location of the right free $\rightarrow$point) the shear $Q_{s\text{-}s}$ =8/6-1=1/3t downwards

➢ For I.L ($Q_{m\text{-}m}$)

When the unite load moves from a to the locations of the sections $Q_{m\text{-}m}$ = Y_a-1

- The load at a $\rightarrow$the shear $Q_{m\text{-}m}$ will be zero
- The load at a distance 4m from a $\rightarrow$ the shear $Q_{m\text{-}m}$ =0-1=1t downwards

When the unite load moves from the locations of the sections to the free point $Q_{m\text{-}m}$ = Y_b-1a to

- The load at a distance 6m from a $\rightarrow$ the shear $Q_{m\text{-}m}$ =1-1=0 upwards
- The load at a distance 8m from a (the location of the right free point) $\rightarrow$ the shear Qs-s =8/6-1=1/3t downwards

➢ For I.L ($M_{s\text{-}s}$)

When the unite load moves from the left free point to the locations of the section $M_{s\text{-}s}$ = Y_a* the distance between the section and the support a-1*the distance between the location of the moving load and the section

- The load at a $\rightarrow$the moment $M_{s\text{-}s}$ will be zero
- The load at a distance 2m from a (the distance of the section) $\rightarrow$ the moment $M_{s\text{-}s}$ =2/3x2=4/3mt
- The load at the free left point $\rightarrow$ the moment $M_{s\text{-}s}$ =7/6x2-1x3=2/3mt

When the unite load moves from the left free point to the locations of the section $M_{s\text{-}s}$ = Y_a* the distance between the section and the support a-1*the distance between the location of the moving load and the section

- The load at b →the moment $M_{s\text{-}s}$ will be zero
- The load at a distance 6m from a (at the right free point)→ the moment $M_{s\text{-}s}$ = -1x6+Y_bx4= -1x6+8/6x4=-2/3mt

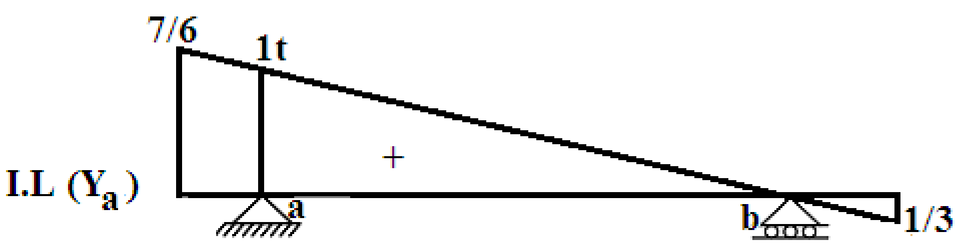
➢ For I.L ($Q_{n\text{-}n}$)

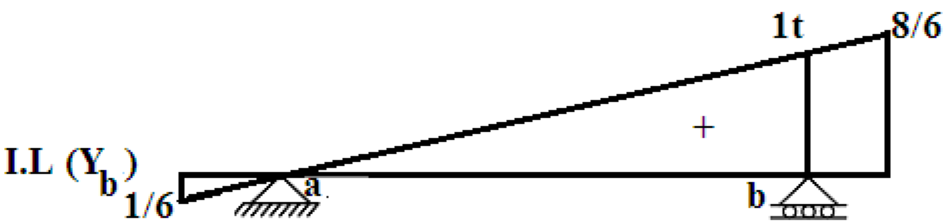
When the unite load moves from the locations of the sections to the left free point $Q_{t\text{-}t}$ = 1t downwards

➢ For I.L ($M_{t\text{-}t}$)

When the unite load moves from the left free point to the locations of the section $M_{n\text{-}n}$ = 1xthe distance

- The load at the free point →the moment $M_{n\text{-}n}$ = -1x1=-1mt
- The load at the location of the section $M_{n\text{-}n}$ = 0





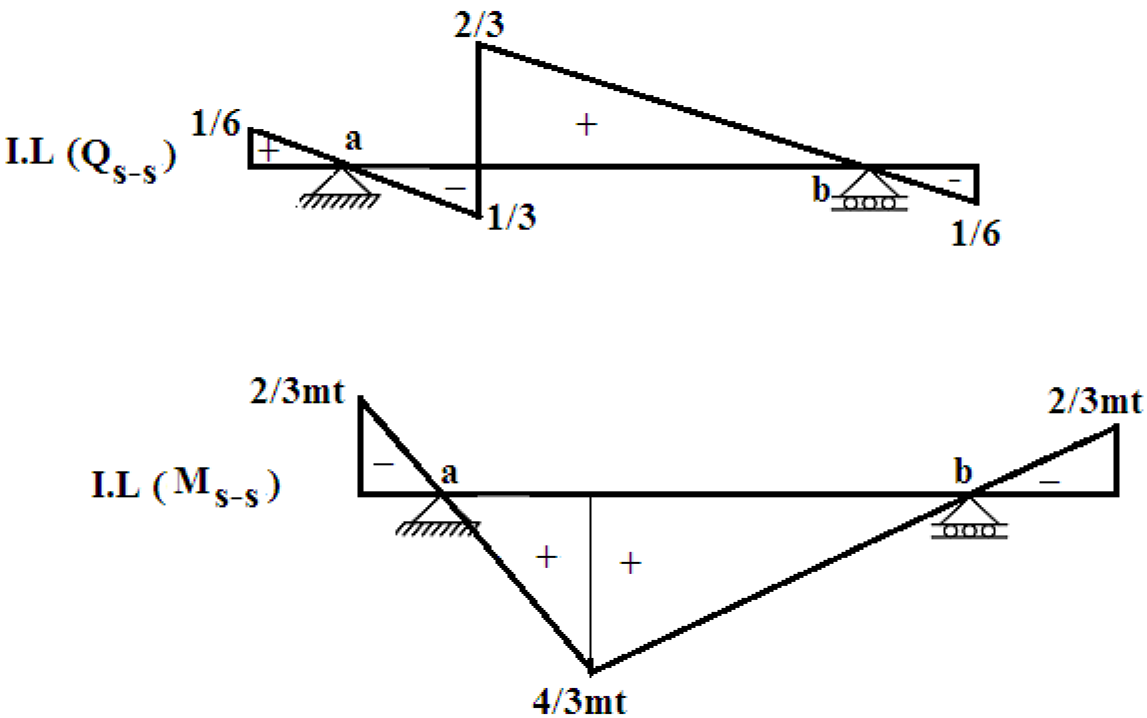


Fig. 6. 3.b

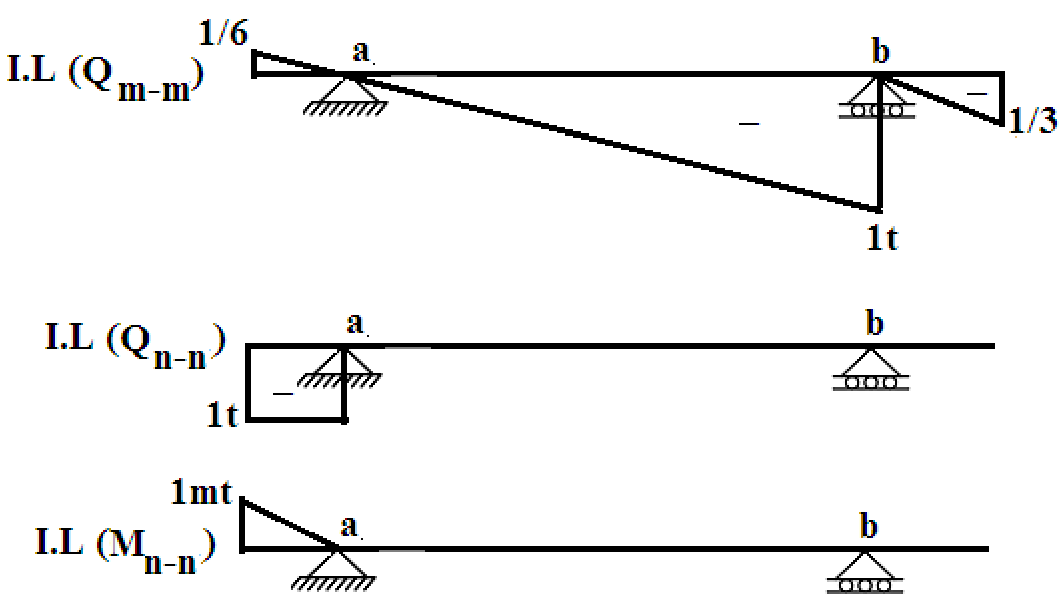


Fig.6. 3.c

Example#4

Find the influence line for Y_a, Y_b, $Q_{m\text{-}m}$, $Q_{s\text{-}s}$, $M_{s\text{-}s}$, $M_{m\text{-}m}$, $Q_{n\text{-}n}$ and $M_{n\text{-}n}$ as a result of moving unite loads from left to right on the shown beam in Fig. 6.4.

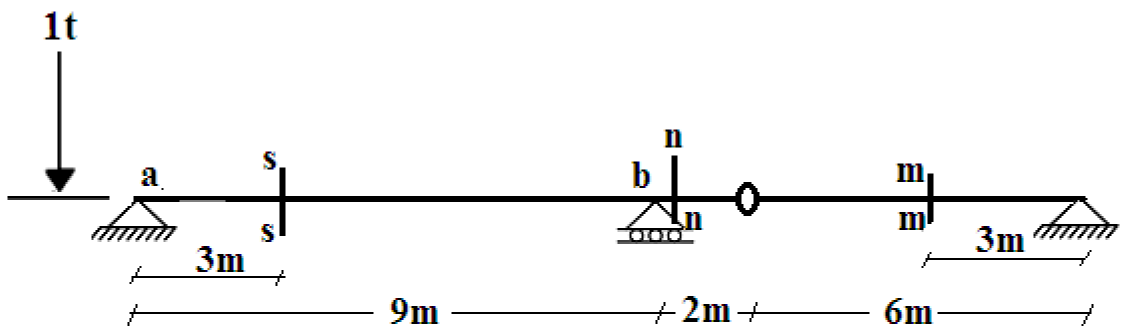


Fig. 6.4.a

Solution

- *For I.L Y_a*

Unite load at a Y_a=1t Unite load at b Y_a=0

Unite load at the intermediate hinge Y_a=-2/9t Unite load at c Y_a=0

- *For I.L Y_b*

Unite load at a Y_b=0 Unite load at b Y_b=1

Unite load at the intermediate hinge Y_b=11/9t Unite load at c Y_a=0

- *For I.L Y_c*

Unite load at a Y_c=0 Unite load at b Y_c=0

Unite load at the intermediate hinge Y_c=0 Unite load at c Y_c=1t

- *For I.L $Q_{s\text{-}s}$*

Unite load at a $Q_{s\text{-}s}$=0 Unite load at s-s left $Q_{s\text{-}s}$=2/3-1=-1/3t

Unite load at s-s right $Q_{s\text{-}s}$=1-1/3=2/3t

Unite load at the intermediate hinge Q_{s-s}=1-11/9=-2/3t

Unite load at c Q_{s-s} =0

- *For I.L M_{s-s}*

Unite load at a M_{s-s}=0 Unite load at s-s M_{s-s}=2/3x3=2mt

Unite load at the intermediate hinge M_{s-s}=-1x8+11/9x6=-2/3t

Unite load at c M_{s-s} =0

- *For I.L Q_{m-m}*

Unite load at a Q_{m-m}=0 Unite load at the intermediate hinge Q_{m-m}=0

Unite load at m-m left Q_{m-m}=0.5-1=-0.5t

Unite load at m-m right Q_{m-m}=1-0.5=0.5t Unite load at c Q_{m-m} =0

- *For I.L M_{m-m}*

When the unite load moves from a to the intermediate hinge the moment at m-m equal zero (the section in secondary element)

Unite load at m-m M_{m-m}=-0.5x3=1.5mt Unite load at c M_{m-m} =0

- *For I.L Q_{n-n}*

When the unite load moves from a to the location of the section, the shear at n-n equal zero (the section in secondary element)

Unite load at n-n left Q_{n-n}=0 Unite load at n-n right Q_{n-n}=1t

Unite load at the intermediate hinge Q_{n-n}=1t Unite load at c Q_{n-n} =0

- *For I.L M_{n-n}*

When the unite load moves from a to the location of the section, the moment at n-n equal zero (the section in secondary element)

Unite load at n-n $M_{m-m}=0$ Unite load at the intermediate hinge $Q_{n-n}=-1x2=-2mt$ Unite load at c $M_{n-n}=0$

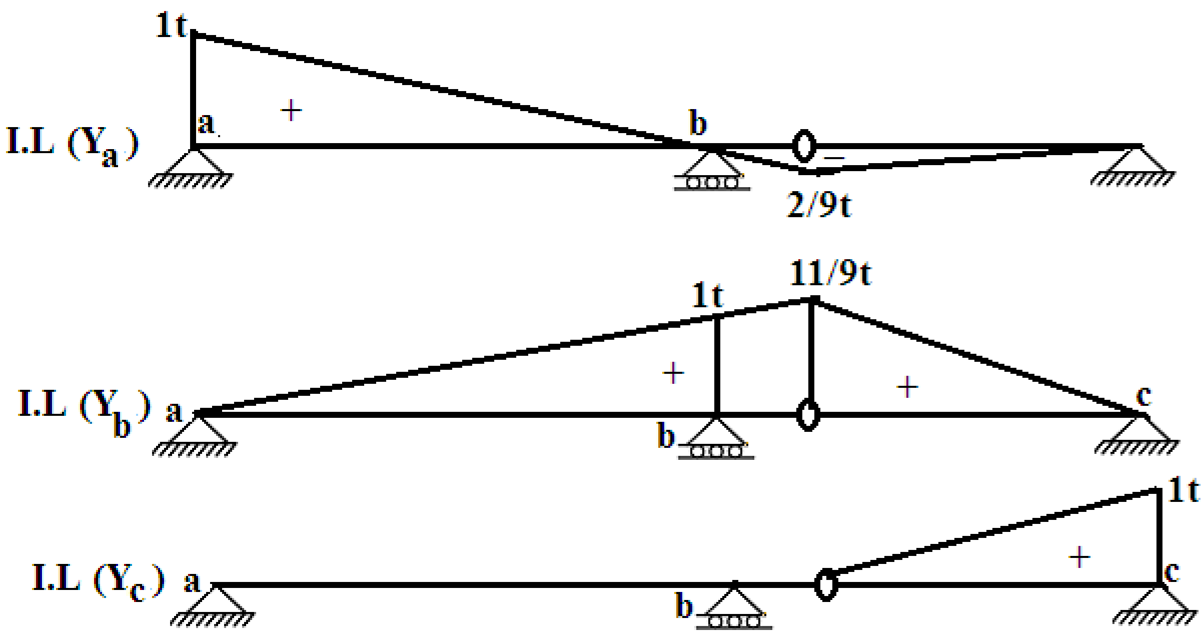


Fig. 6.4.b

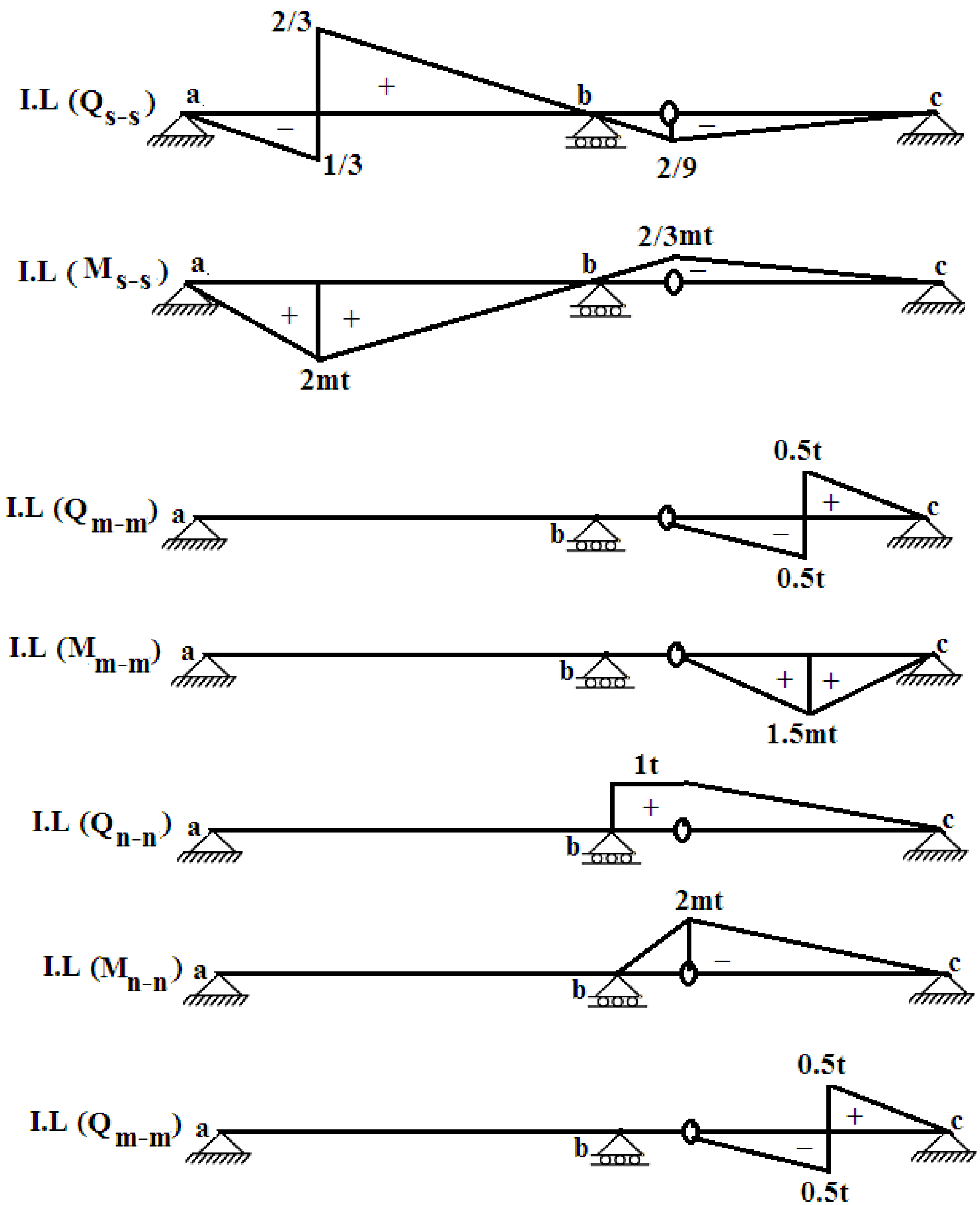

I.L (Q s-s)
a
b
c
2/3
1/3
2/9
+
−
−
I.L (M s-s)
a
b
c
2/3mt
2mt
+
+
−
I.L (Q m-m)
a
b
c
0.5t
0.5t
+
−
I.L (M m-m)
a
b
c
1.5mt
+
+
I.L (Q n-n)
a
b
c
1t
+
I.L (M n-n)
a
b
c
2mt
−
I.L (Q m-m)
a
b
c
0.5t
0.5t
+
−

Fig. 6.4.c

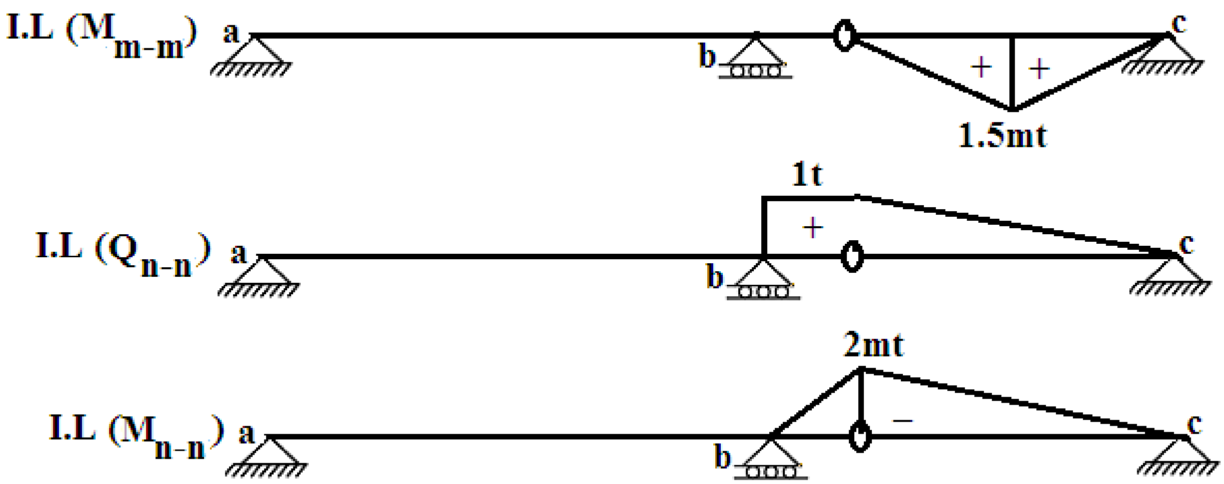


Fig. 6.4.d

Example#5

Find the influence line for Y_a, Y_b, $Q_{m\text{-}m}$, $Q_{s\text{-}s}$, $M_{s\text{-}s}$, $M_{m\text{-}m}$, and $Q_{m\text{-}m}$ as a result of moving unite loads from left to right on the shown beam in Fig. 6.5.

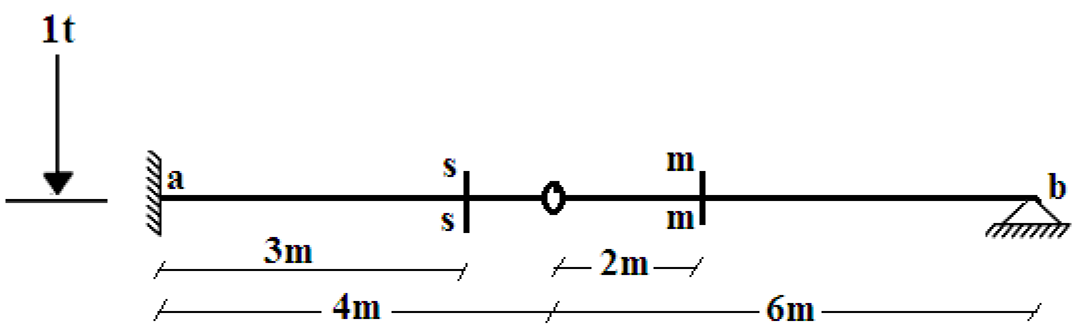


Fig. 6.5.a

Solution

- *For I.L Y_a*

Unite load at a Y_a=1t　　　　Unite load at b Y_a=0

Unite load at the intermediate hinge $Y_a=1$

- *For I.L M_a*

Unite load at a $M_a=1t$ Unite load at the intermediate hinge $M_a=-4mt$

Unite load at b $M_a=0$

- *For I.L Y_b*

When the unite load moves from a to the intermediate hinge the response at part c-b equal zero (the secondary element) Unite load at c $Y_a=0$

- *For I.L Q_{s-s}*

Unite load at a $Q_{s-s}=0$ Unite load at s-s left $Q_{s-s}=0$

Unite load at s-s right $Q_{s-s}=1t$

Unite load at the intermediate hinge $Q_{s-s}=1t$ Unite load at c $Q_{s-s}=0$

- *For I.L M_{s-s}*

Unite load at a $M_{s-s}=0$ Unite load at s-s $M_{s-s}=0$

Unite load at the intermediate hinge $M_{s-s}=-1x1=-1mt$

Unite load at c $M_{s-s}=0$

- *For I.L Q_{m-m}*

Unite load at the intermediate hinge $Q_{m-m}=0$

Unite load at m-m left $Q_{m-m}=2/3-1=-1/3t$

Unite load at m-m right $Q_{m-m}=1-1/3=2/3t$ Unite load at c $Q_{m-m}=0$

- *For I.L M_{m-m}*

Unite load at m-m $M_{m-m}=2/3x2=4/3mt$ Unite load at c $M_{m-m}=0$

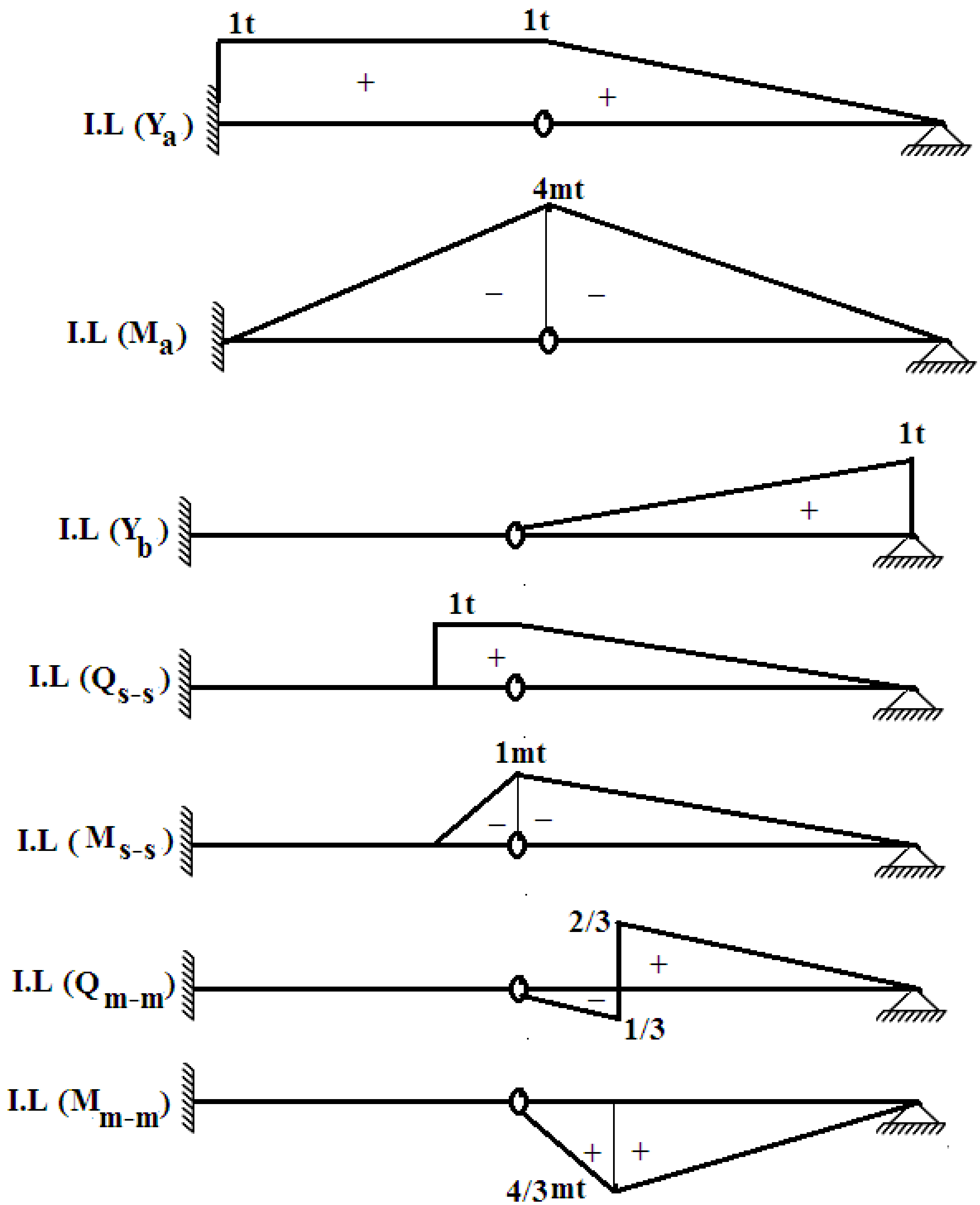

1t
1t
+
+
I.L (Y_a)
4mt
−
−
I.L (M_a)
1t
+
I.L (Y_b)
1t
+
I.L (Q_{s-s})
1mt
−
−
I.L (M_{s-s})
2/3
+
−
1/3
I.L (Q_{m-m})
I.L (M_{m-m})
+
+
4/3mt

Fig. 6.5.b

Example#6

Find the influence line for Y_a, Y_b, Q_{m-m}, Q_{s-s}, M_{s-s}, M_{m-m}, Q_{n-n} and M_{n-n} as a result of moving unite loads from left to right on the shown beam in Fig. 6.6.

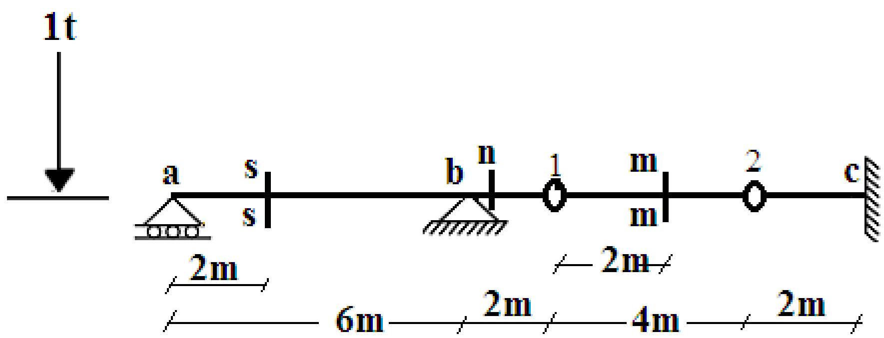


Fig. 6.6.a

Solution

- *For I.L Y_a*

  Unite load at a Y_a=1t Unite load at b Y_a=0

  Unite load at the intermediate hinge #1 Y_a=-2/6t

  Unite load at the intermediate hinge #2 Y_a=0 Unite load at c Y_a=0

- *For I.L Y_b*

  Unite load at a Y_b=0 Unite load at b Y_b=1

  Unite load at the intermediate hinge#1 Y_b=8/6t

  Unite load at the intermediate hinge #2 Y_b=0 Unite load at c Y_b=0

- *For I.L Y_c*

  When the unite load moves from a to the intermediate hinge#1, the responses in part 2-c are zero

  Unite load at the intermediate hinge#1 Y_c=0

  Unite load at the intermediate hinge#2 Y_c=1

Unite load at c Y_c=1

- *For I.L M_c*

Unite load at the intermediate hinge#1 M_c=0

Unite load at the intermediate hinge#2 M_c=-1x2=-2mt

Unite load at c M_c=0

- *For I.L Q_{s-s}*

Unite load at a Q_{s-s}=0 Unite load at s-s left Q_{s-s}=2/3-1=-1/3t

Unite load at s-s right Q_{s-s}=1-1/3=2/3t

Unite load at the intermediate hinge#1 Q_{s-s}=1-8/6=-2/6t

Unite load at the intermediate hinge#2 Q_{s-s} =0

- *For I.L M_{s-s}*

Unite load at a M_{s-s}=0 Unite load at s-s M_{s-s}=2/3x2=4/3mt

Unite load at the intermediate hinge#1 M_{s-s}=-1x6+8/6x4=-2/3t

Unite load at c M_{s-s} =0

- *For I.L Q_{m-m}*

When the unite load moves from a to the location of sec. m-m the second intermediate hinge, the responses at the section are zero

Unite load at m-m left Q_{m-m}=0 Unite load at m-m right Q_{m-m}=1t

Unite load at the intermediate hinge#1 Q_{m-m}=1

Unite load at the intermediate hinge#2 Q_{m-m} =0

- *For I.L M_{m-m}*

Unite load at m-m M_{m-m}=0

Unite load at the intermediate hinge#1 M_{m-m} =-1x2=-2mt

Unite load at the intermediate hinge#2 M_{m-m} =0

➢ *For I.L Q_{n-n}*

When the unite load moves from a to the first intermediate hinge and the second intermediate hinge, the response at n-n equal zero (the section in secondary element)

Unite load at n-n left Q_{n-n}=-0.5t   Unite load at n-n right Q_{n-n}=0.5t

➢ *For I.L M_{n-n}*

Unite load at n-n M_{m-m}=0.5x2=1mt

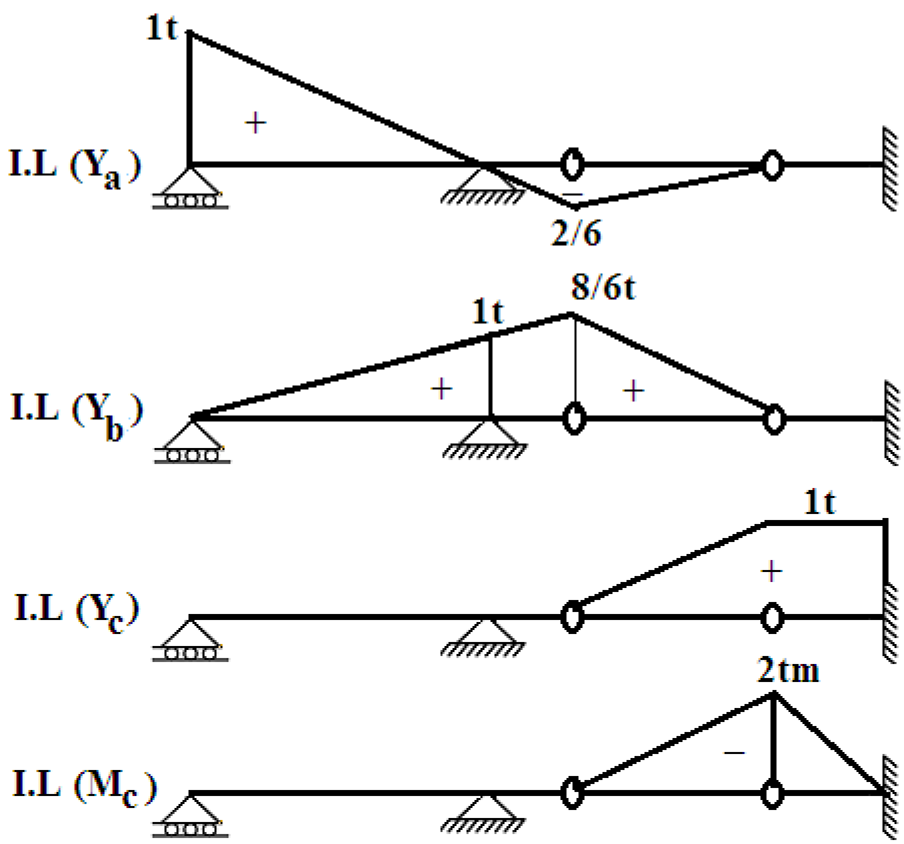


Fig. 6.6.b

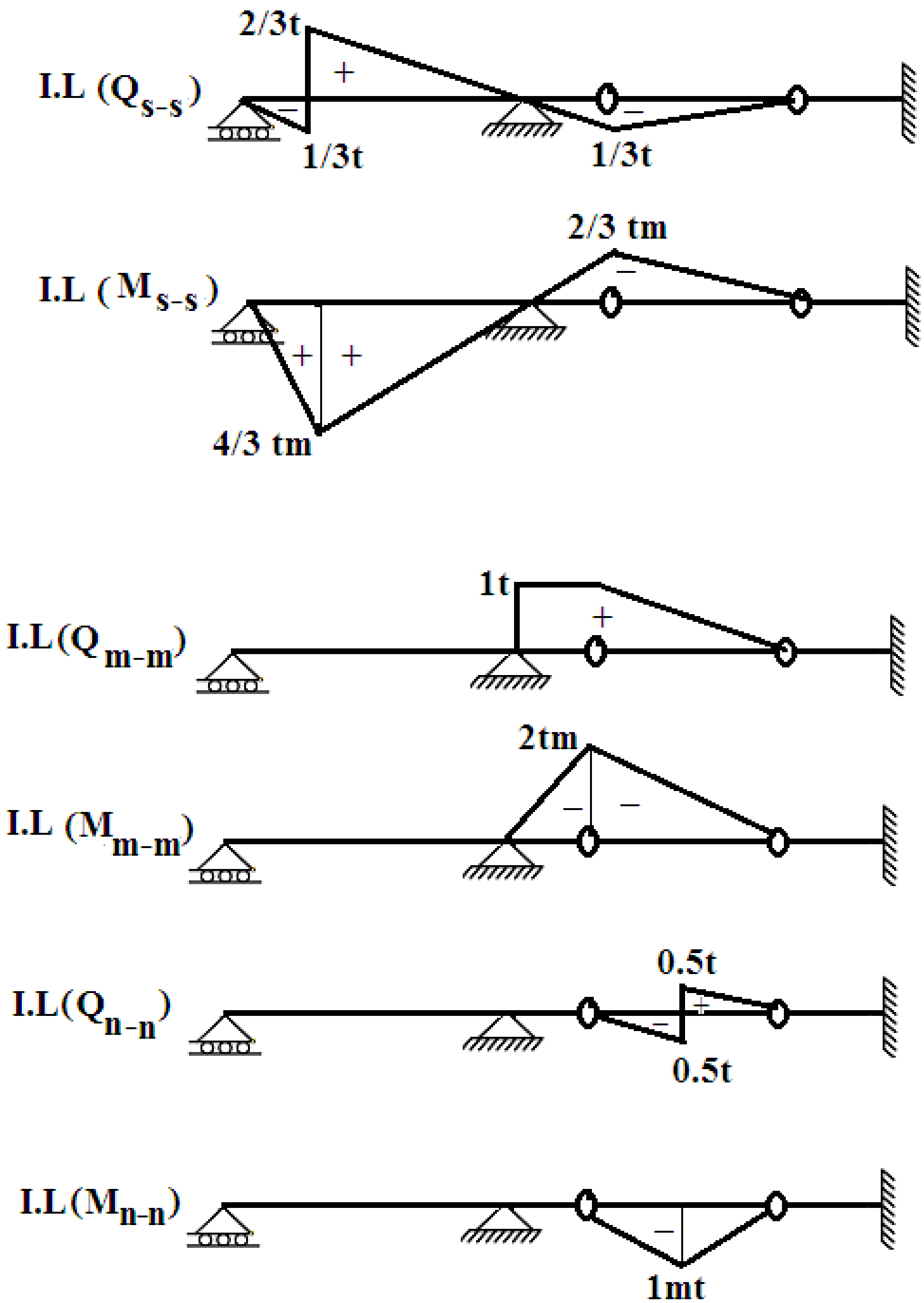


Fig. 6.6.c

Example#7

Find the influence line for Y_a, Y_b, $Q_{s\text{-}s}$ and $M_{s\text{-}s}$ as a result of moving unite loads from left to right on the shown beam in Fig. 6.7.

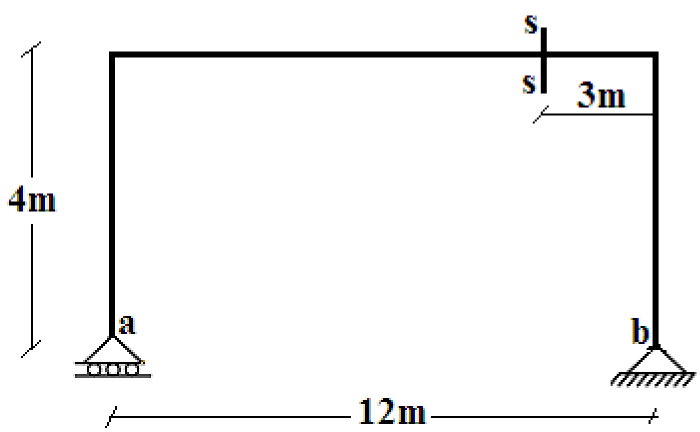


Fig. 6.7.a

Solution

- *For I.L Y_a*

  Unite load at a Y_a=1t Unite load at b Y_a=0

- *For I.L Y_b*

  Unite load at a Y_b=0 Unite load at b Y_b=1

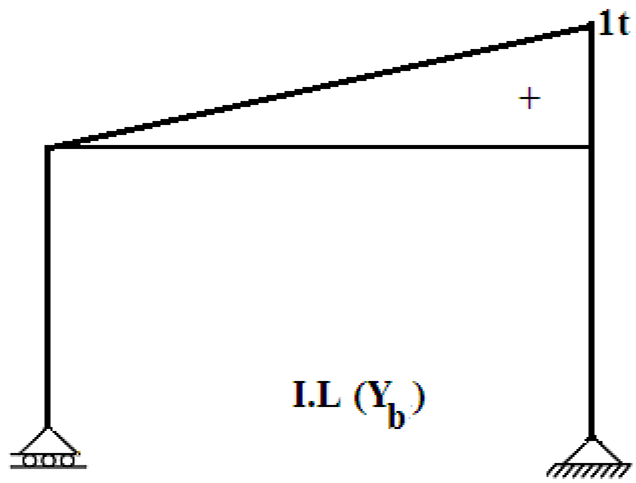
- *For I.L $Q_{s\text{-}s}$*

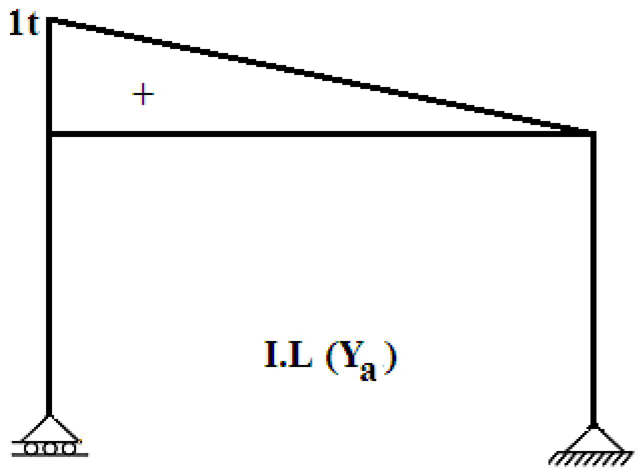
  Unite load at a $Q_{s\text{-}s}$=0 Unite load at s-s left $Q_{s\text{-}s}$=3/12-1=-3/4t (upwards)

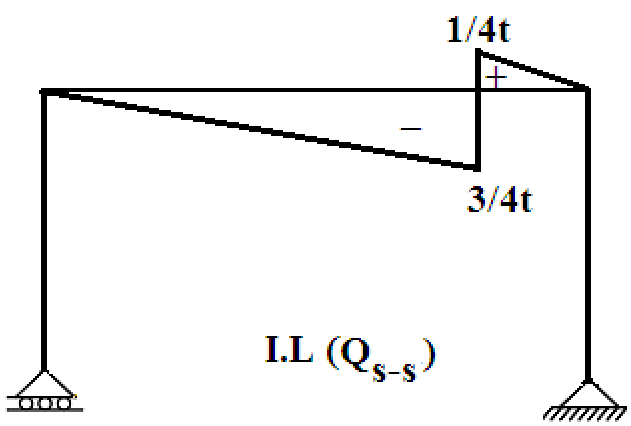
  Unite load at s-s right $Q_{s\text{-}s}$=9/12-1=1/4t (downward)

- *For I.L $M_{s\text{-}s}$*

  Unite load at a $M_{s\text{-}s}$=0 Unite load at s-s $M_{s\text{-}s}$=3/12x9=2.25mt


1t
+
I.L (Y_b)


1t
+
I.L (Y_a)


1/4t
+
−
3/4t
I.L (Q_{s-s})

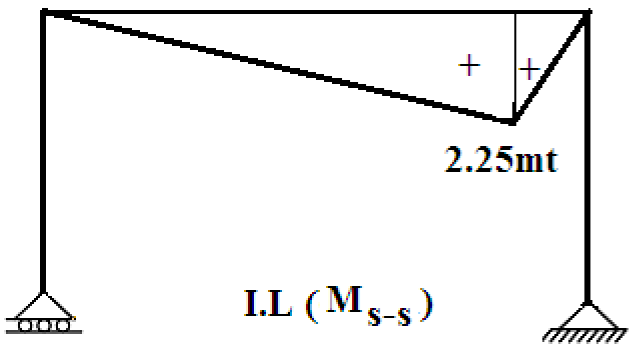


Fig. 6.7.b

Example#8

Find the influence line for Y_a, Y_b, Q_{m-m}, Q_{s-s}, M_{s-s}, M_{m-m}, Q_{n-n} and M_{n-n} as a result of moving unite loads from left to right on the shown beam in Fig. 6.8.

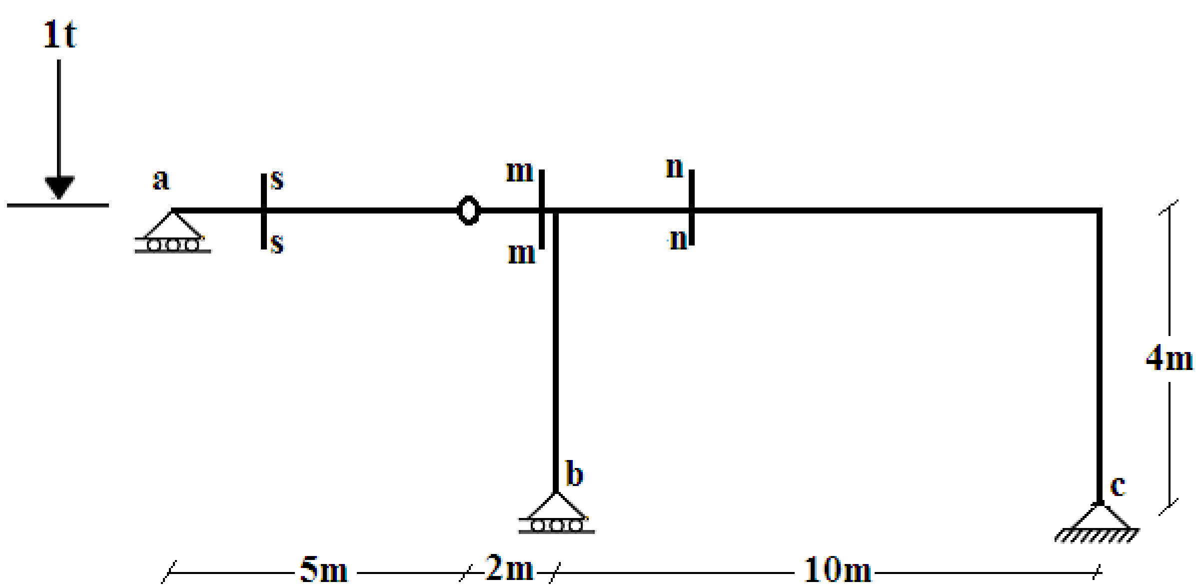


Fig. 6.8.a

- *For I.L Y_a*

When the unite load moves from the intermediat hinge to the end of the frame, the responses in part a-1 (secondary) are zero

Unite load at a Y_a=1t

- *For I.L Y_b*

Unite load at a Y_b=0 Unite load at column c Y_b=0

Unite load at column b Y_b=1t

Unite load at the intermediate hinge Y_b=1.2t

- *For I.L Y_c*

Unite load at a Y_c=0 Unite load at column c Y_c=1t

Unite load at column b Y_c=0 Unite load at the intermediate hinge Y_c=-0.2t

- *For I.L Q_{s-s}*

Unite load at a Q_{s-s}=0 Unite load at s-s left Q_{s-s}=3/5-1=-2/5t

Unite load at s-s right Q_{s-s}=1-2/5=3/5t

Unite load at the intermediate hinge Q_{s-s}=0

- *For I.L M_{s-s}*

Unite load at a M_{s-s}=0 Unite load at s-s M_{s-s}=3/5x3=6/5mt

Unite load at the intermediate hinge#1 M_{s-s}=0

- *For I.L Q_{m-m}*

When the unite load moves from the location of sec. m-m to column *c*, the responses at the section are zero

Unite load at m-m right $Q_{m-m}=0$ Unite load at m-m left $Q_{m-m}=-1t$

Unite load at a $M_{m-m}=0$

- *For I.L M_{m-m}*

Unite load at m-m $M_{m-m}=0$

Unite load at the intermediate hinge1 $M_{m-m}=-1x2=-2mt$

Unite load at a $M_{m-m}=0$

- *For I.L Q_{n-n}*

Unite load at a $Q_{n-n}=0$

Unite load at the intermediate hinge $M_{n-n}=0.2$

Unite load at column b $Q_{n-n}=0$

Unite load at n-n left $Q_{n-n}=0.6-1=-0.4t$

Unite load at n-n right $Q_{n-n}=1-0.4=0.6t$ Unite load at column c $Q_{n-n}=0$

- *For I.L M_{n-n}*

Unite load at a $M_{n-n}=0$

Unite load at the intermediate hinge $M_{n-n}=-1x6+1.2x4=-1.2mt$

Unite load at column b $M_{n-n}=0$

Unite load at n-n $M_{n-n}=0.6x4=2.4mt$ Unite load at column c $Q_{n-n}=0$

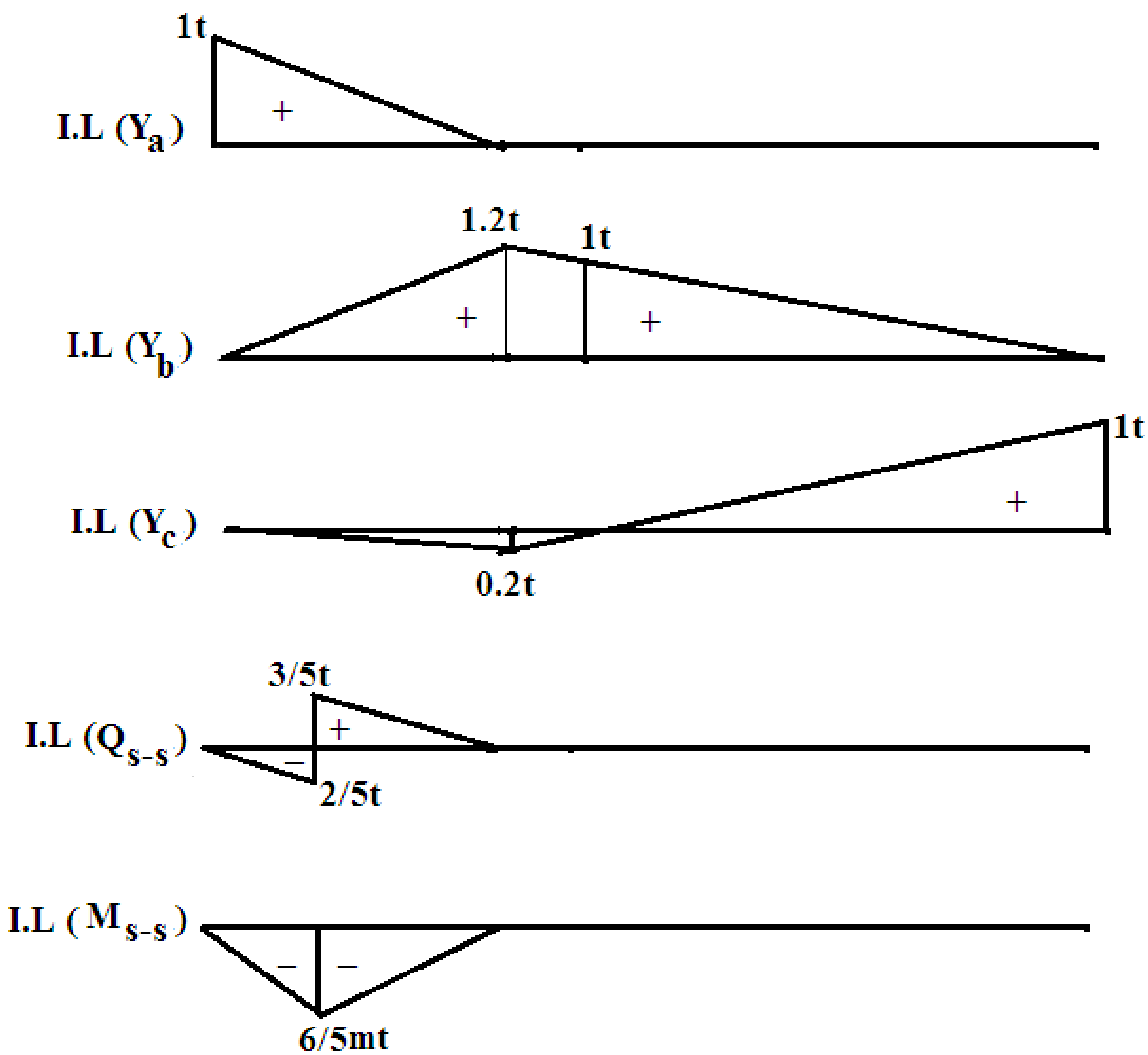
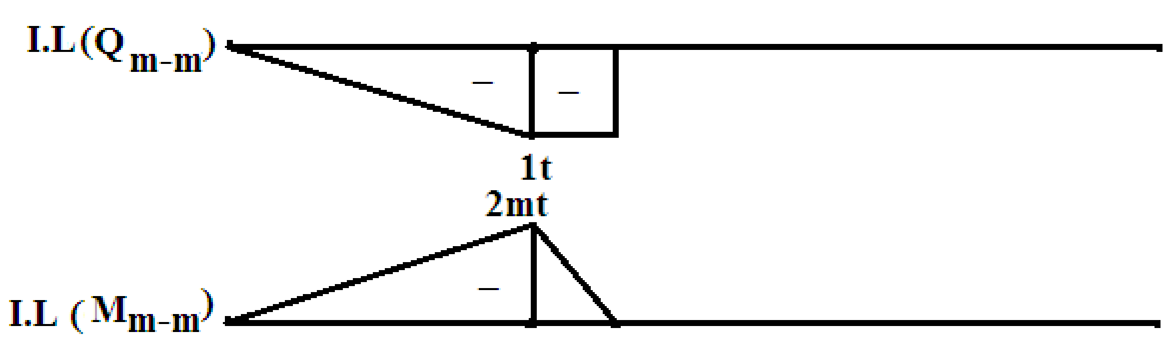

1t
I.L (Y_a)
+
1.2t
1t
I.L (Y_b)
+
+
1t
I.L (Y_c)
+
0.2t
3/5t
I.L (Q_{s-s})
+
–
2/5t
I.L (M_{s-s})
–
–
6/5mt

Fig. 6.8.b


I.L(Q_{m-m})
–
–
1t
2mt
I.L (M_{m-m})
–

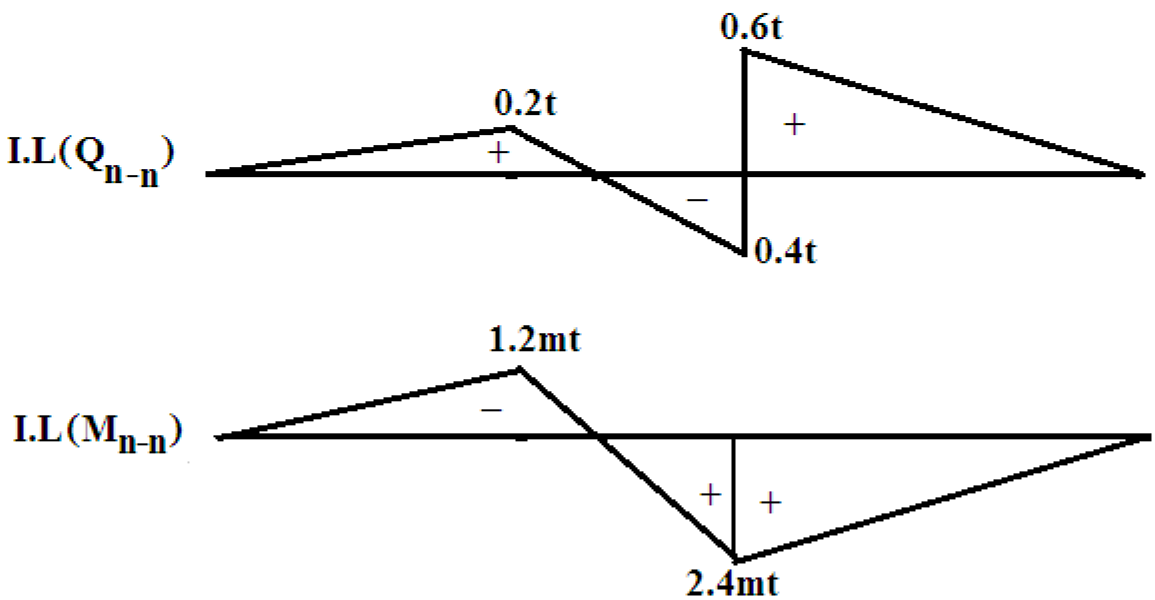
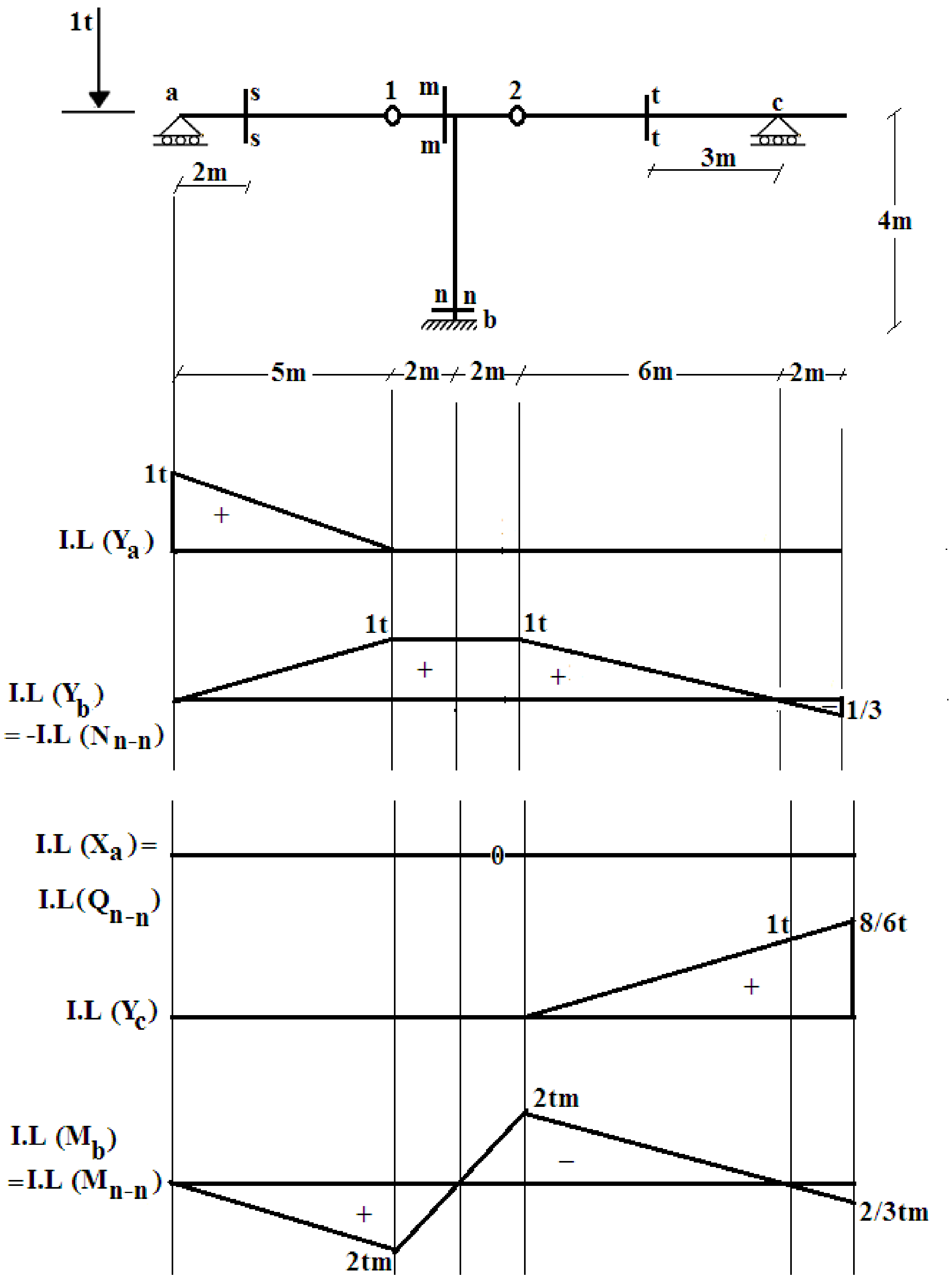


Fig. 6.8.c

Example#9

Find the influence line for Y_a, Y_b, Q_{m-m}, Q_{s-s}, M_{s-s}, M_{m-m}, Q_{n-n} and M_{n-n} as a result of moving unite loads from left to right on the shown beam in Fig. 6.9.



1t
a
s
s
1
m
m
2
t
t
c
3m
2m
4m
n
n
b
5m
2m
2m
6m
2m
1t
+
I.L (Y_a)
1t
1t
+
+
I.L (Y_b)
= -I.L (N_{n-n})
1/3
I.L (X_a)=
0
I.L(Q_{n-n})
1t
8/6t
+
I.L (Y_c)
2tm
I.L (M_b)
=I.L (M_{n-n})
–
+
2/3tm
2tm

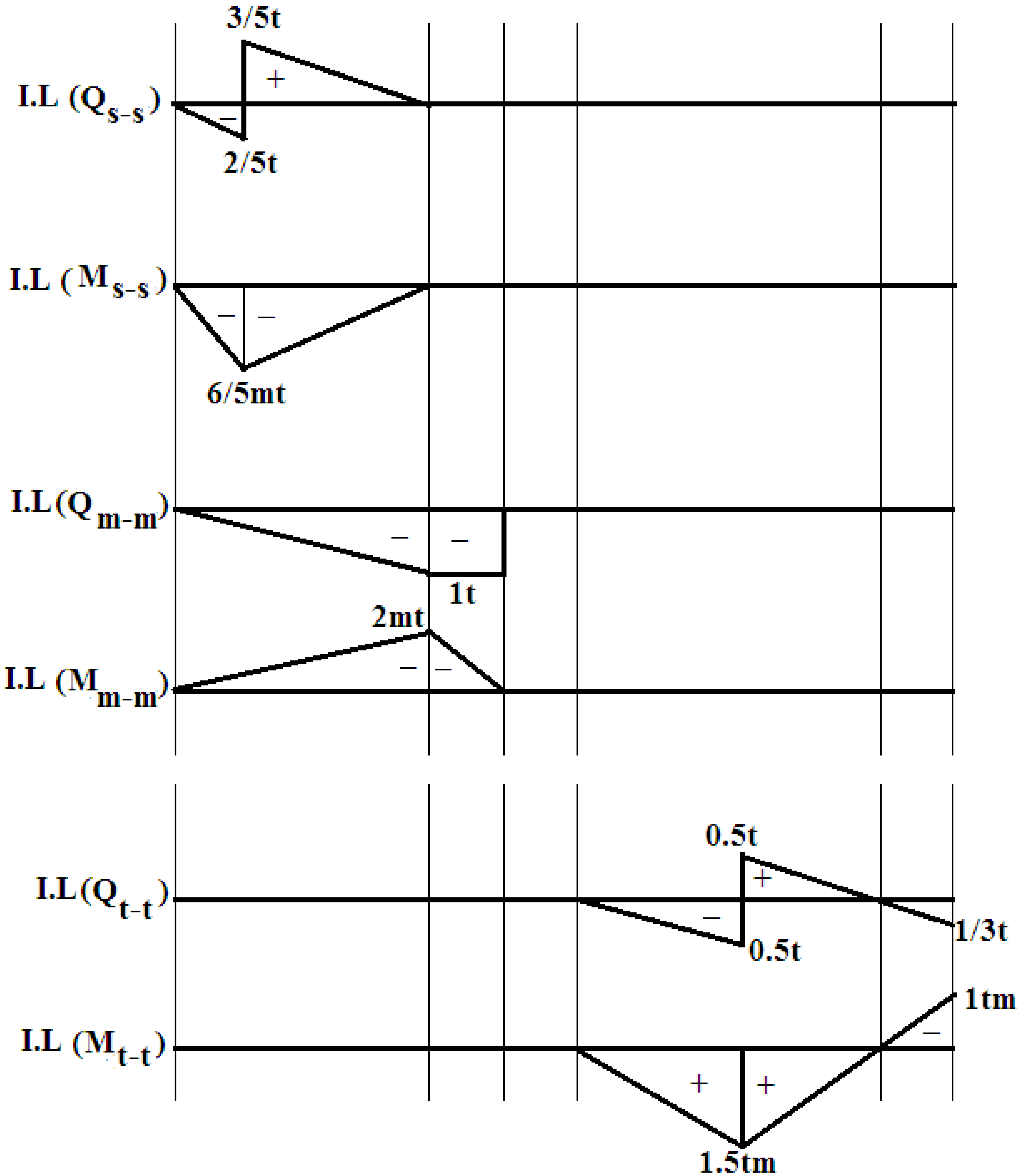

I.L (Q_{s-s})
3/5t
+
−
2/5t
I.L (M_{s-s})
−
−
6/5mt
I.L(Q_{m-m})
−
−
1t
2mt
−
−
I.L (M_{m-m})
I.L(Q_{t-t})
0.5t
+
−
0.5t
1/3t
1tm
I.L (M_{t-t})
−
+
+
1.5tm

Fig. 6.9

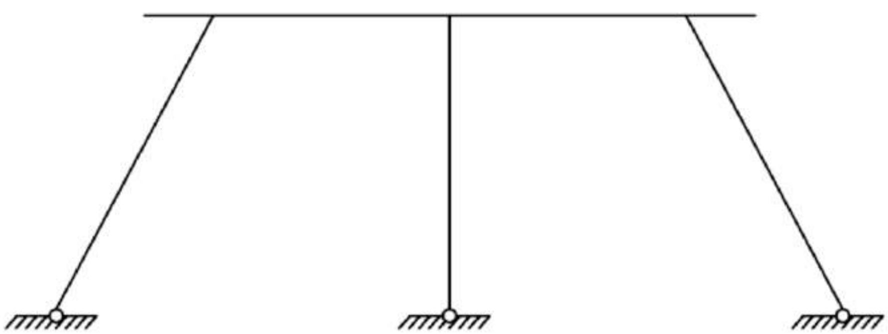
Sheet 1

State whether the structures shown in the following figures are **stable** or **unstable** and **statically determinate** or **statically indeterminate.**

1

2

3



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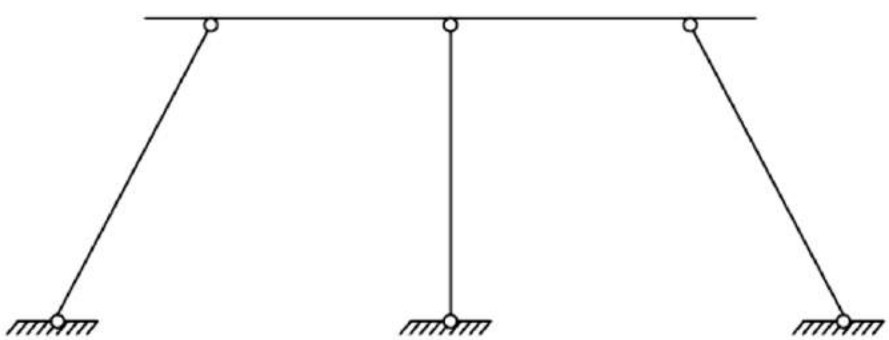

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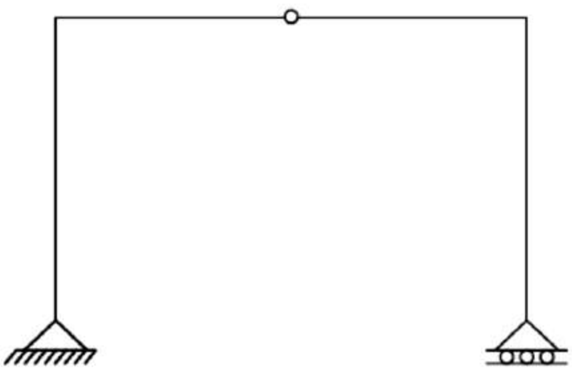
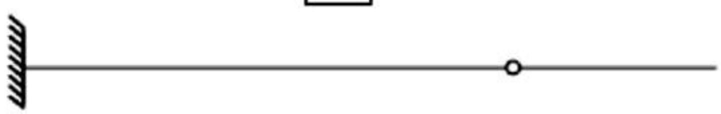


6

7



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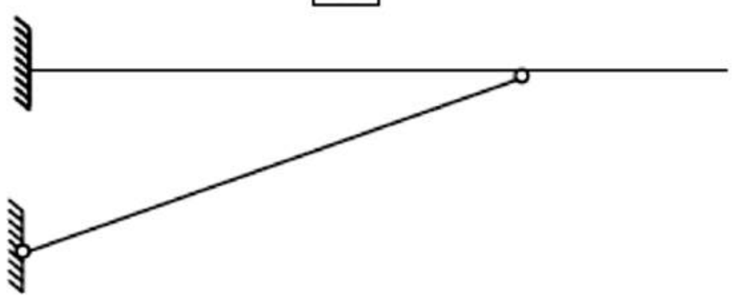

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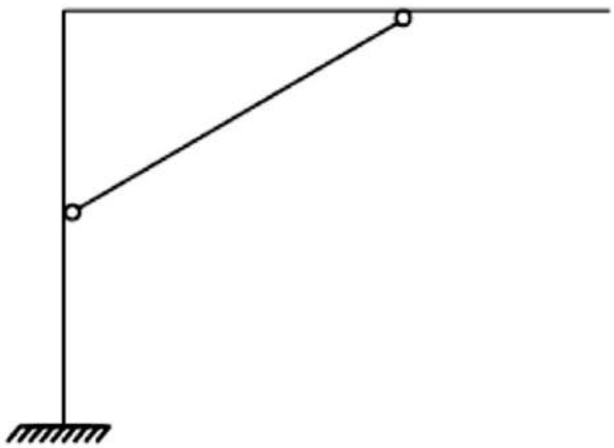

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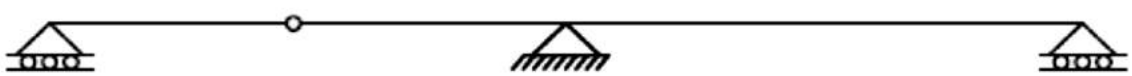

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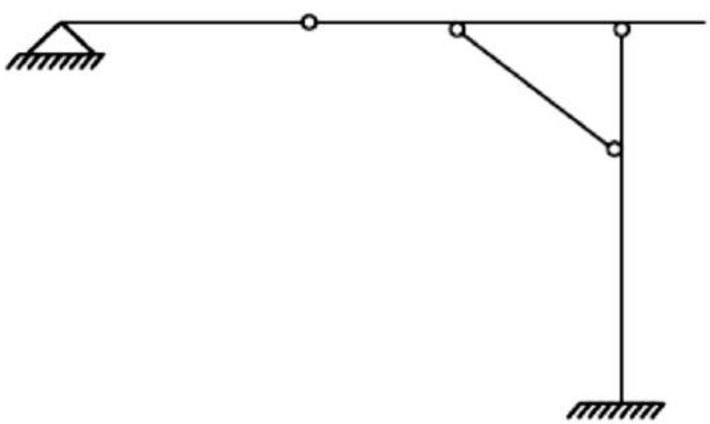
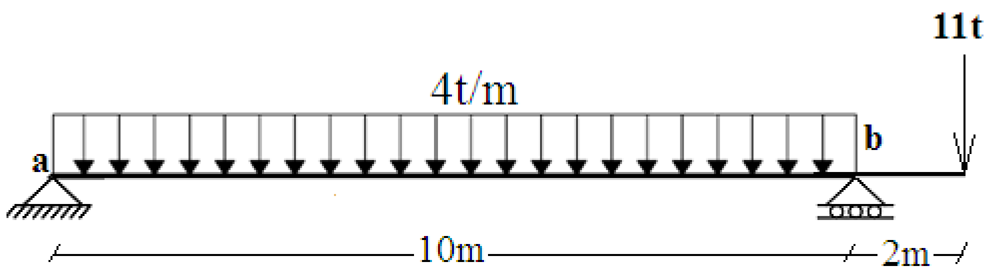
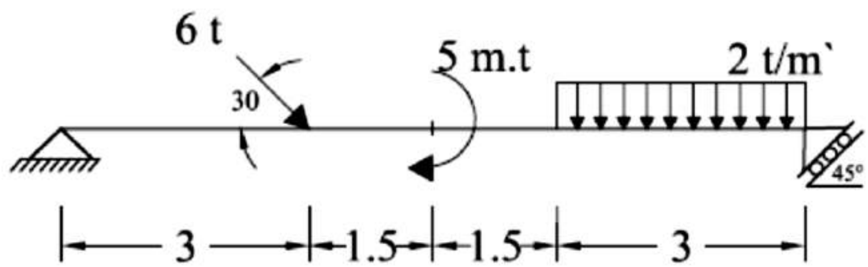


Sheet (2)

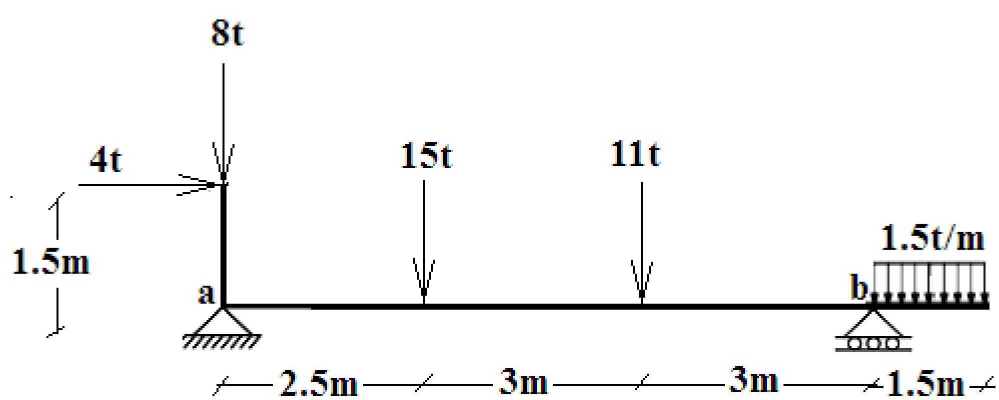
For the given beams and frames, find the reactions at supports.



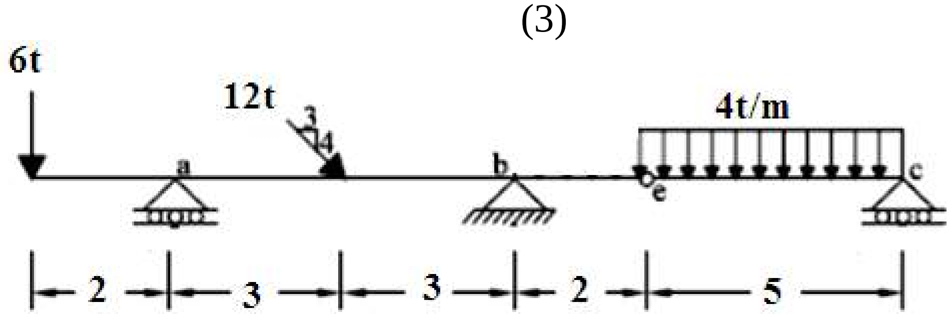
(1)



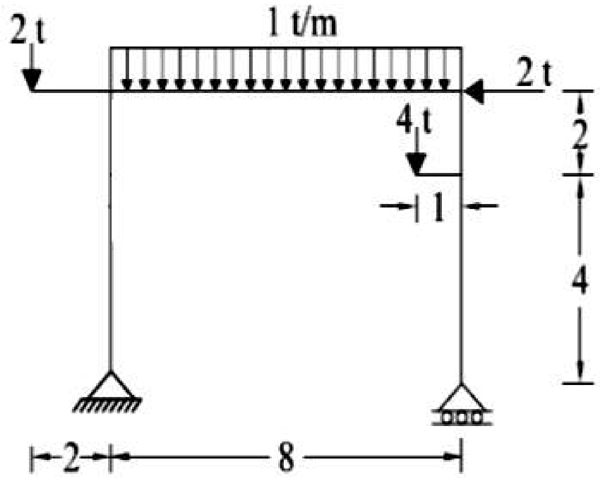
(2)



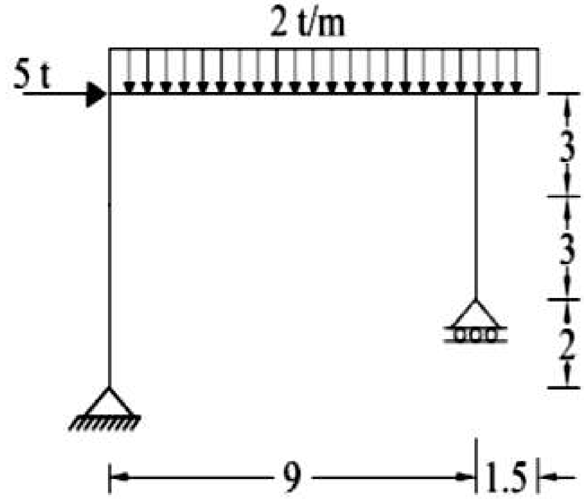
(3)



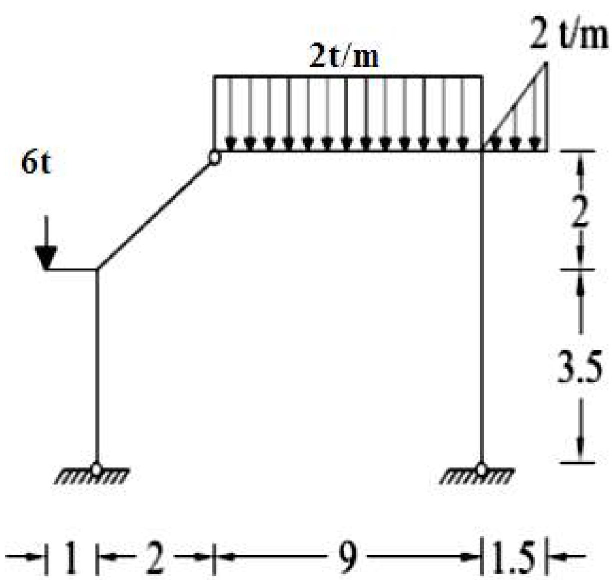
(4)


2 t
1 t/m
2 t
4 t
2
1
4
2
8

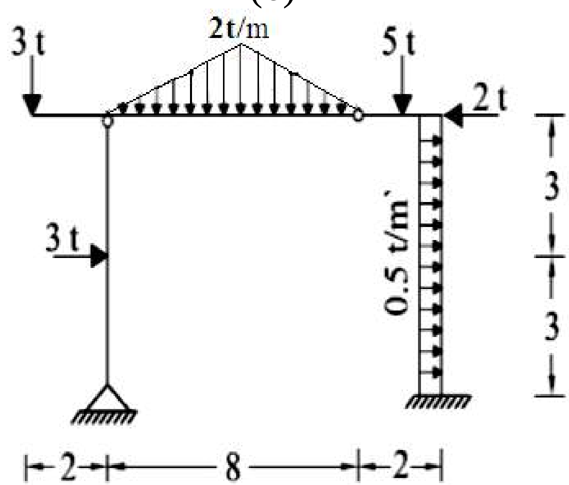
(5)


2 t/m
5 t
3
3
2
9
1.5

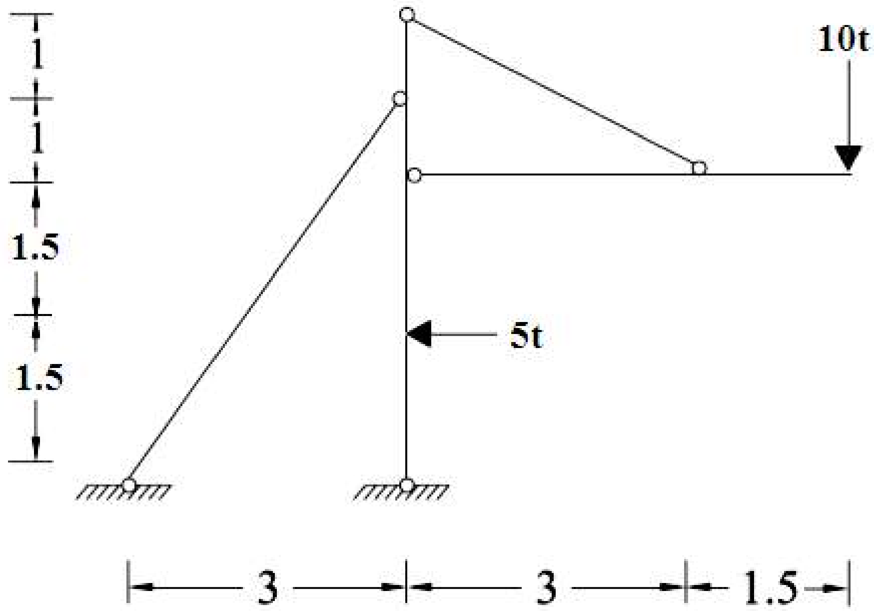
(6)


2 t/m
2t/m
6t
2
3.5
1
2
9
1.5

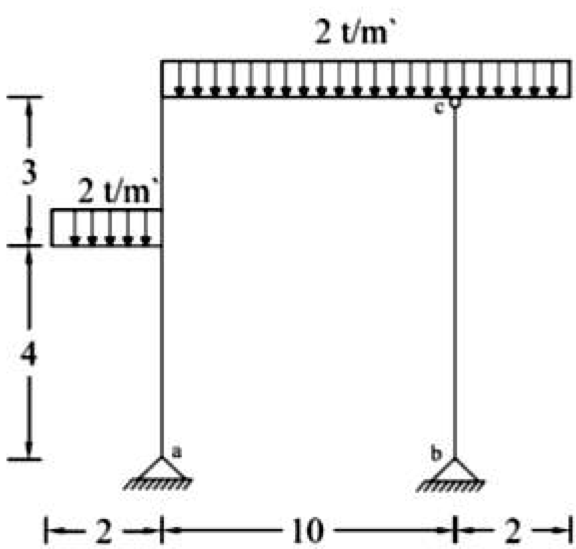
(7)


3 t
2t/m
5 t
2 t
3 t
0.5 t/m`
3
3
2
8
2

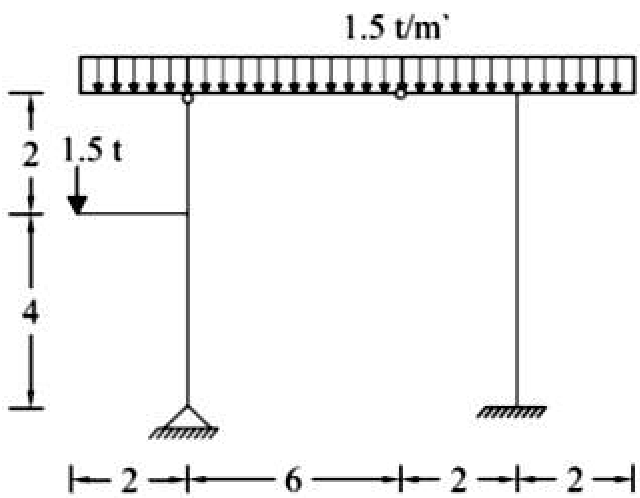
(8)



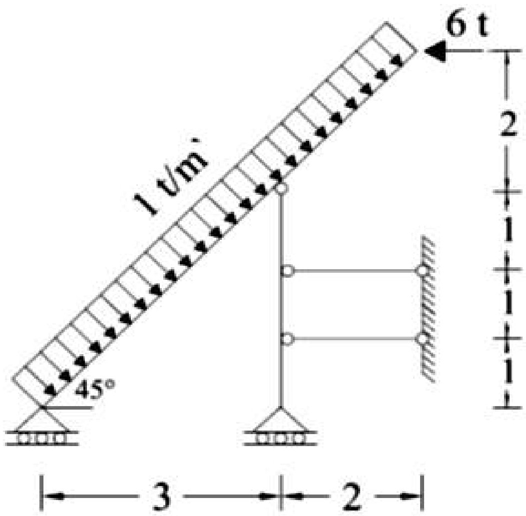
(9)



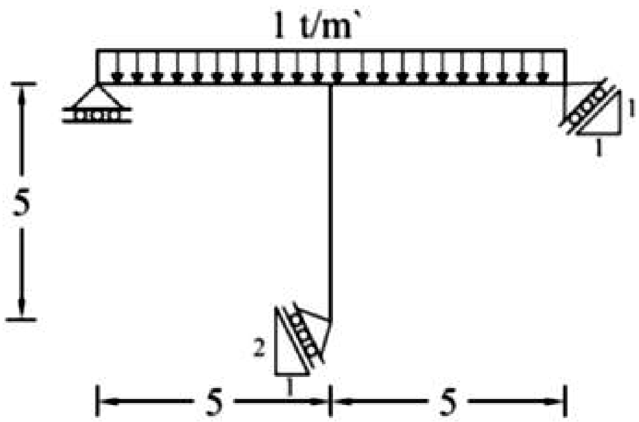
(10)



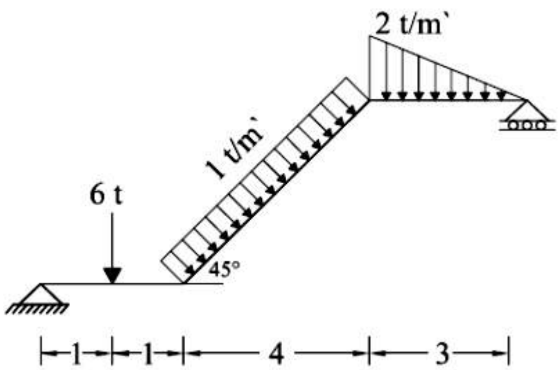
(11)



(12)



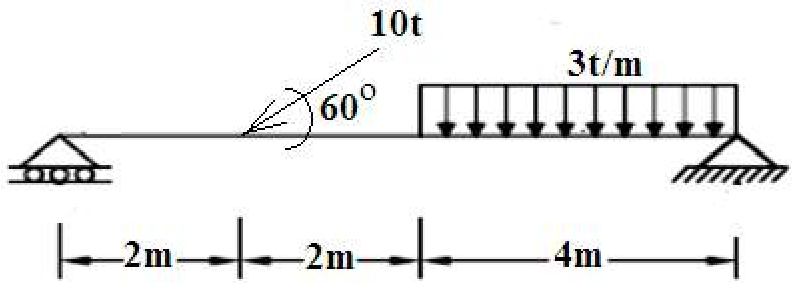
(13)



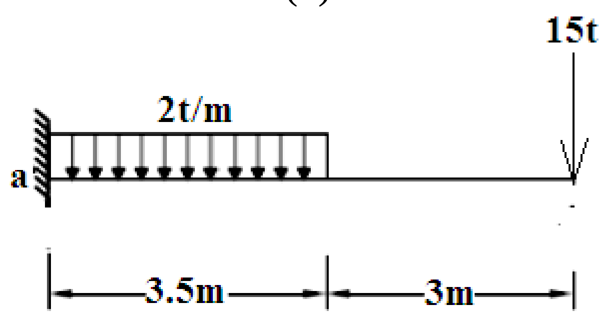
(14)

Sheet#3

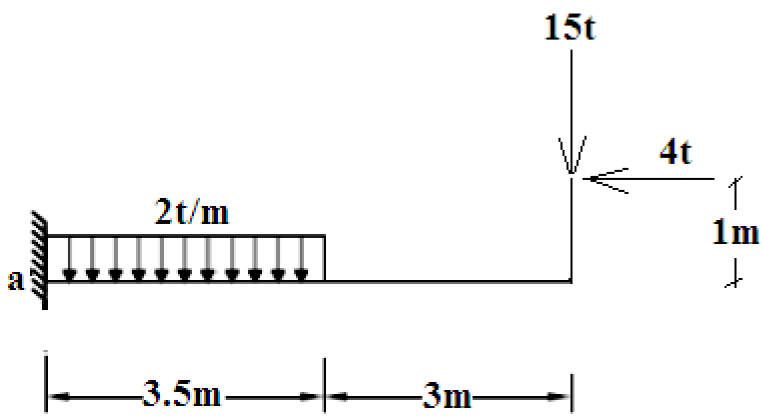
[1]For the below beams, draw normal, shear forces and bending moment diagrams



(1)



(2)



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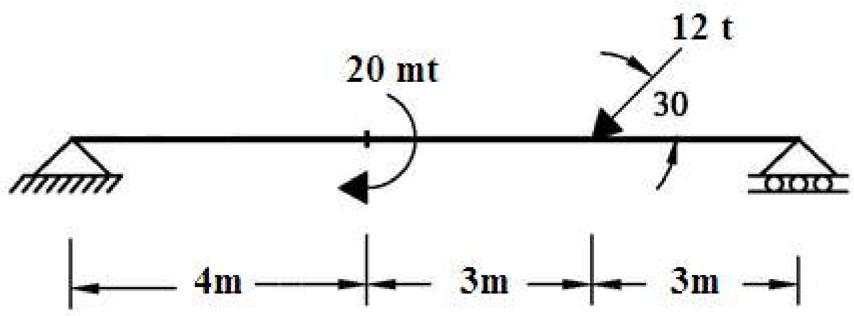
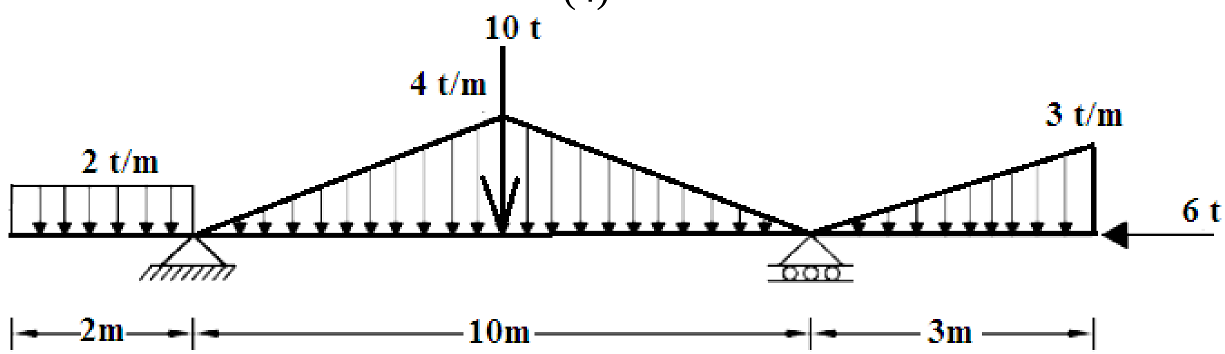
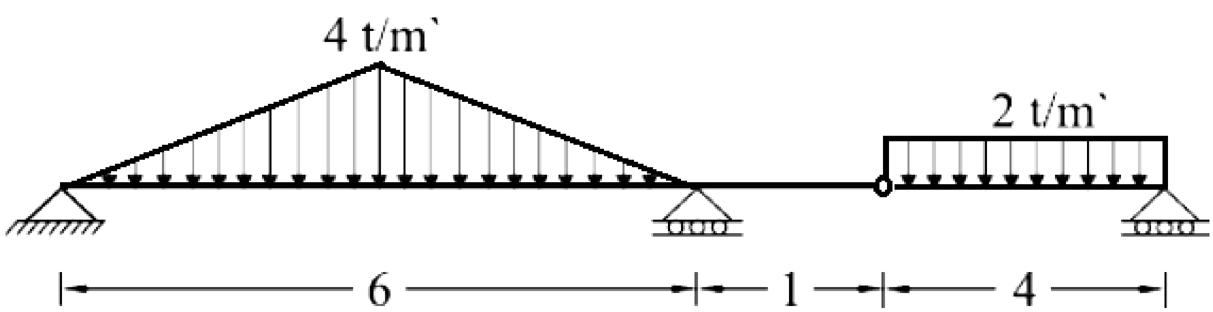


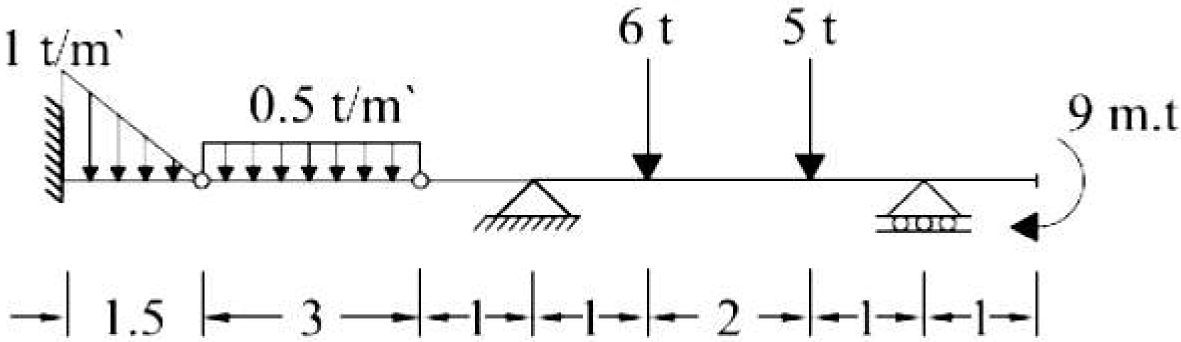
(4)



(5)

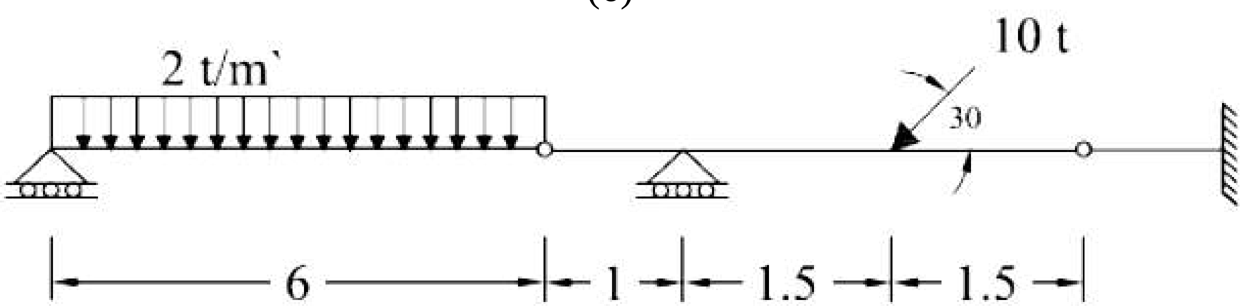


(6)



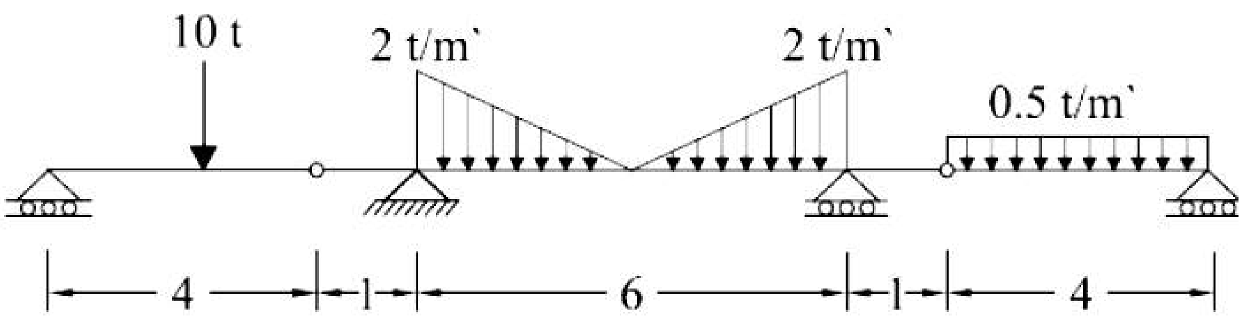
1 t/m`
0.5 t/m`
6 t
5 t
9 m.t
1.5
3
1
1
2
1
1

(6)



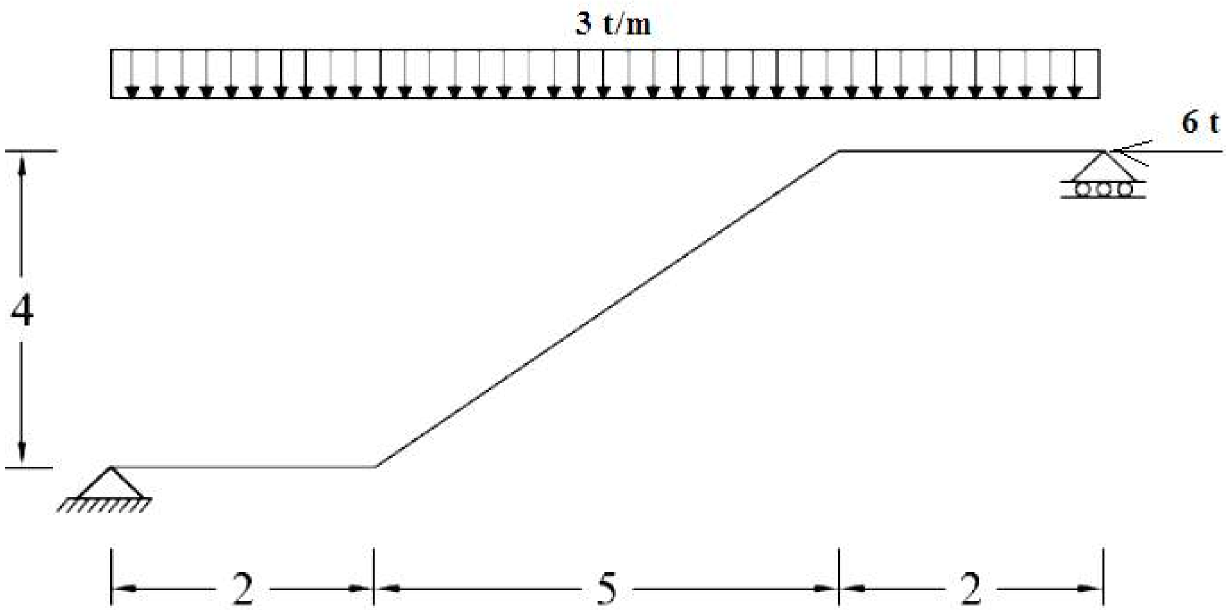
2 t/m`
10 t
30
6
1
1.5
1.5

(7)

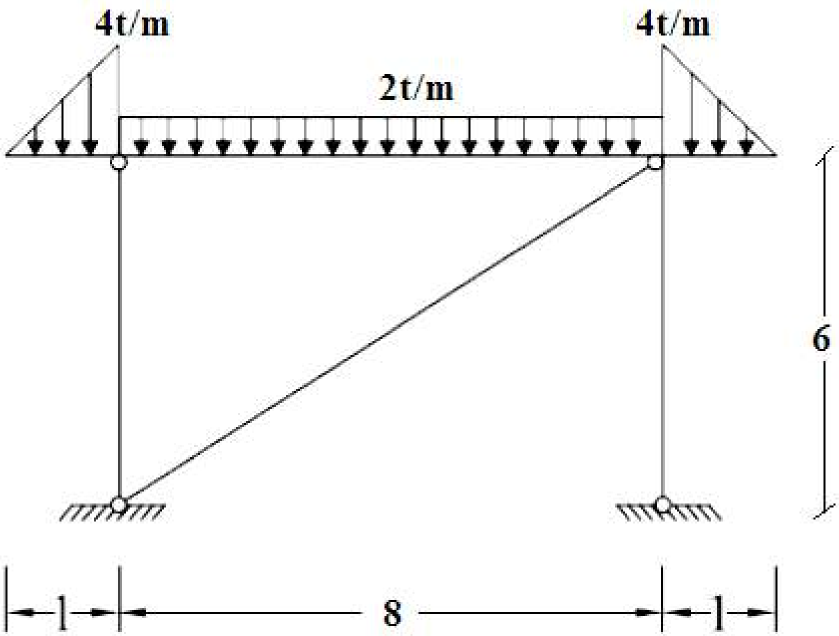


10 t
2 t/m`
2 t/m`
0.5 t/m`
4
1
6
1
4

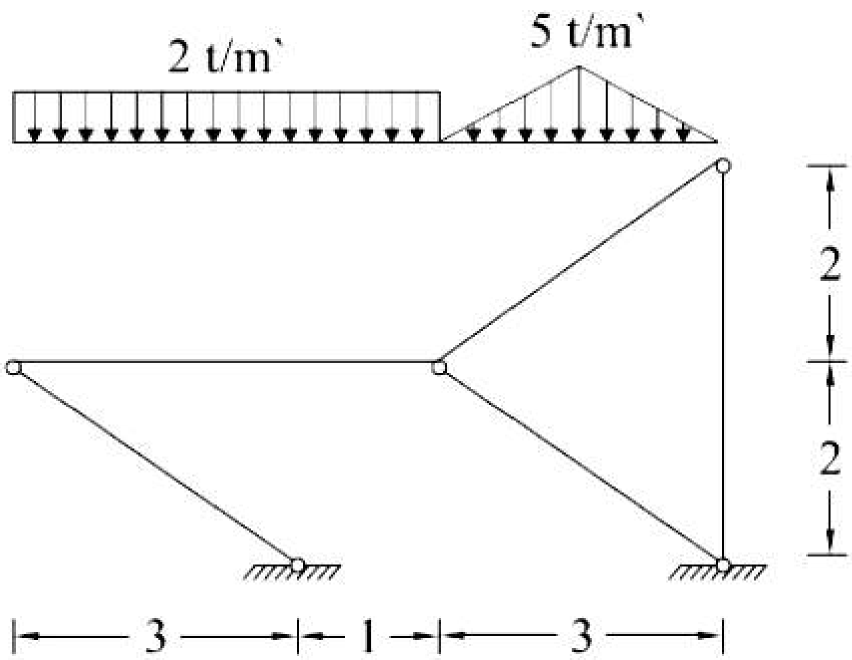
(8)



(9)

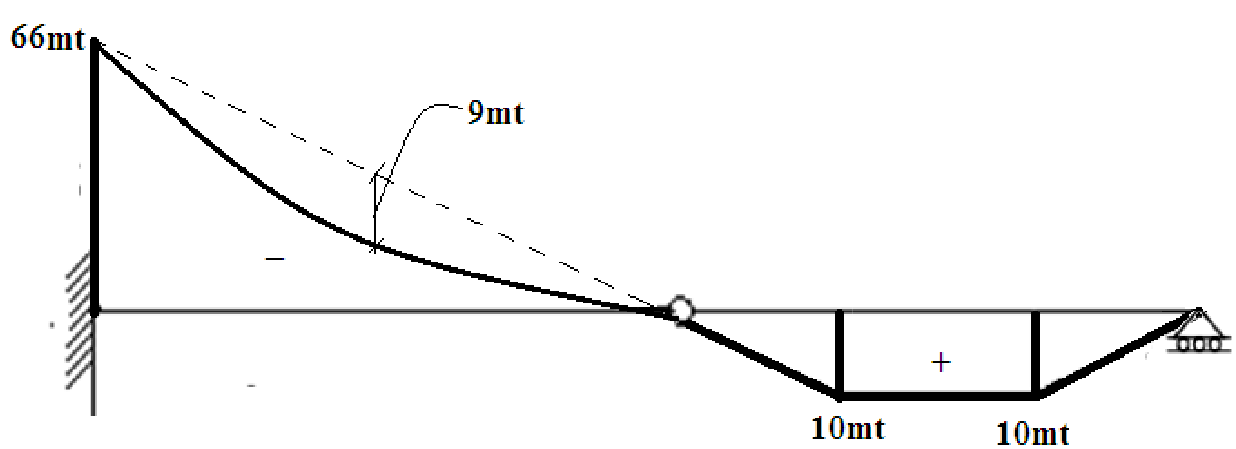


(10)



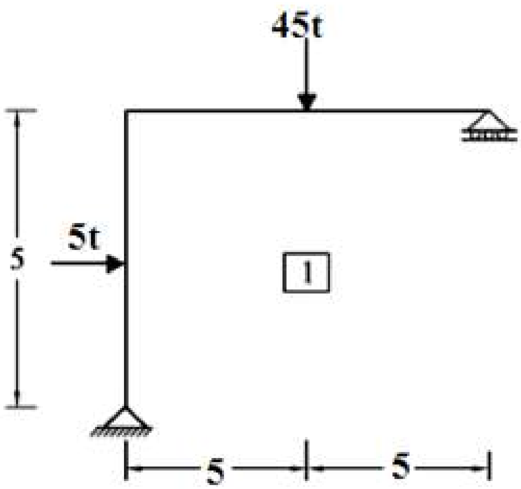
(11)

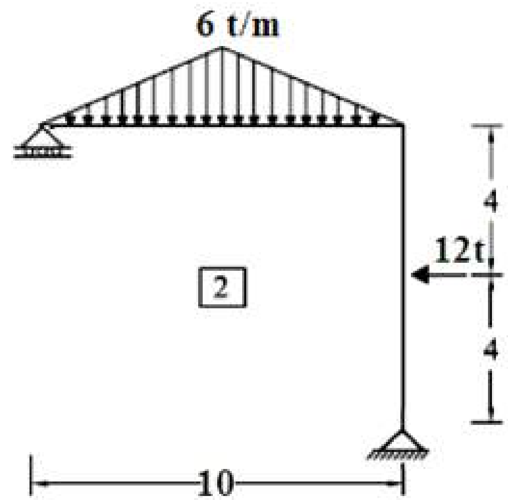
2] For the given bending moment diagram, find the applied load then draw the shear force diagram

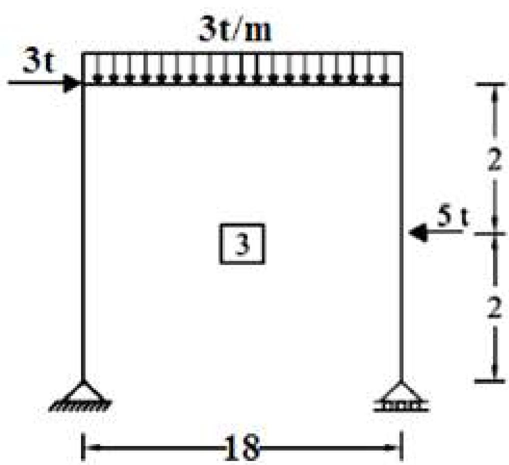


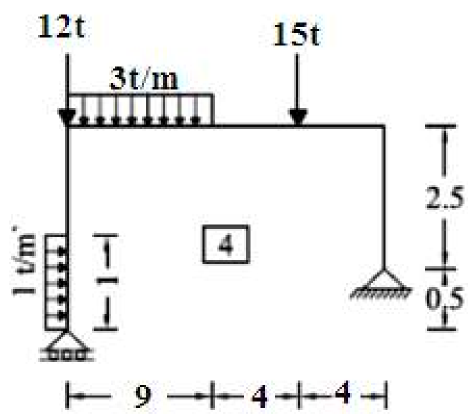
Sheet#4

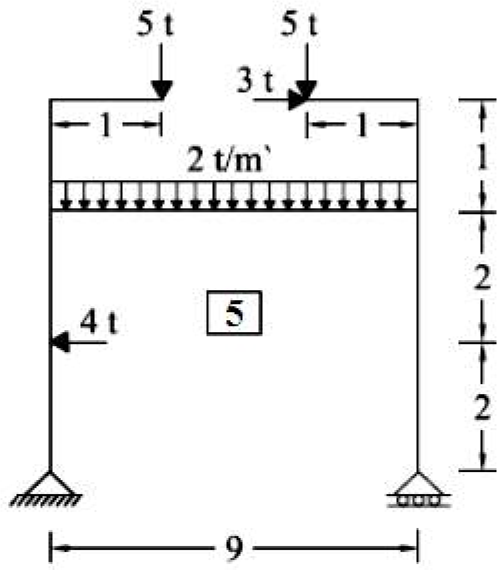
For the below beams, draw normal, shear forces and bending moment diagrams

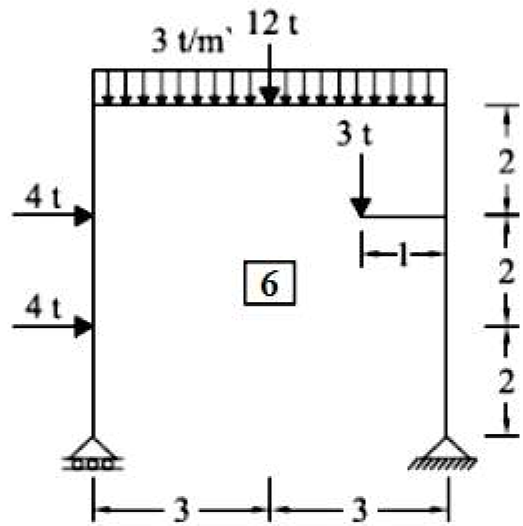


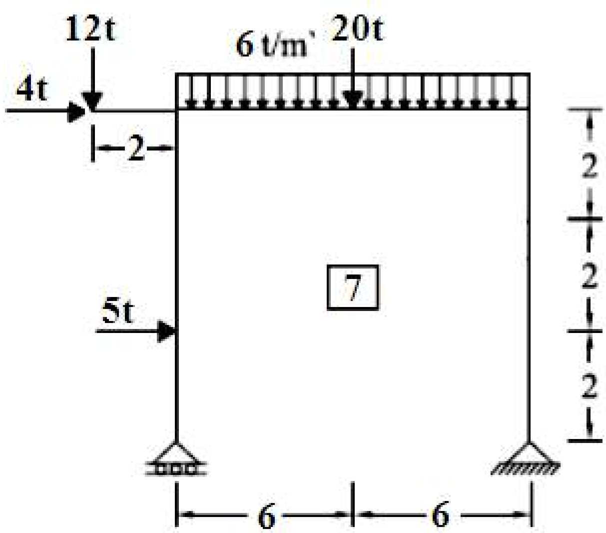


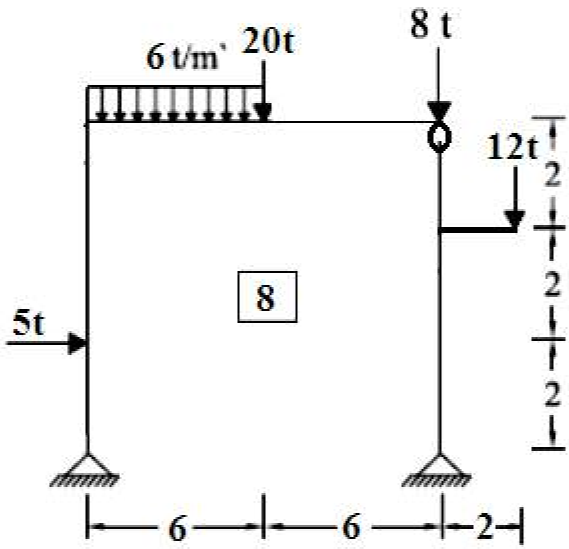




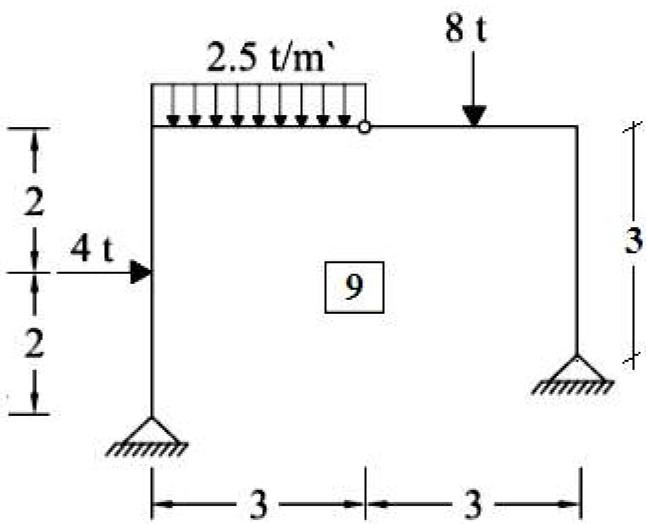

5 t
5 t
3 t
1
1
2 t/m`
4 t
5
1
2
2
9


3 t/m`
12 t
3 t
4 t
4 t
6
1
2
2
2
3
3


12t
4t
6 t/m`
20t
2
5t
7
2
2
2
6
6



6 t/m`
20t
8 t
12t
5t
8
2
2
2
6
6
2



2.5 t/m`
8 t
4 t
9
2
2
3
3
3